Analogue VLSI Implementations of Two Dimensional, Nonlinear, Active Cochlea Models

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Abstract

This thesis presents my work from the last 4 years on active, nonlinear two dimensional (2D) silicon cochlea modelling. It begins with an introduction on what is currently understood about the active and nonlinear characteristics of the mammalian cochlea and proceeds to develop an active, 2D cochlea model which incorporates these characteristics. Of particular importance in the model is the idea that the cochlear amplifier (CA) has dynamics governed by the Hopf equation. The realisation of the active 2D model leads to several hardware implementations that are based on two slightly different but equivalent approaches.

In the first approach I propose an implementation of the Hopf equation based on its equivalence with changing the quality factor (Q) in a second-order band-pass filter. This implementation is called Automatic Quality Factor Control (AQC) and we show that it not only has the dynamics of a system that is governed by the Hopf equation but also that it represents a type of parametric amplification. Parametric amplification is characterised by its robustness to both noise and a wide range of phase shifts. AQC is an intuitive approach where the feedback loop explicitly adds or subtracts energy from the system based on the intensity of the input signal. Two silicon cochleae were made based on this approach. Both of these silicon cochleae were the first silicon cochleae to exhibit large-signal compression, two-tone suppression and the creation of combinational, odd-ordered distortion products. The second silicon cochlea with AQC also facilitated a study into the effects of coupling between basilar membrane (BM) resonator sections.

In the second approach I propose an implementation based on implicitly modelling the Hopf equation as a Hopf Oscillator. In this approach there is no visible control-loop, however, the Hopf oscillator possesses the same dynamics as the AQC implementation. A silicon cochlea based on this approach was built and tested and, as expected, it exhibits the same nonlinear, active characteristics as the silicon cochlea with AQC. With this implementation I was able to further explore coupling in the cochlea. I was also able demonstrate masking as well as high quality factors comparable with biology.
Together this body of work provides the foundations for a silicon cochlea that can be used to better understand the biological cochlea as well as explore higher auditory centres.
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Chapter 1. Introduction

This thesis explores two dimensional (2D) models of the nonlinear, active cochlea by constructing silicon cochleae and comparing their performance with biological results. Silicon cochleae have been constructed for over 20 years while scientists have been creating models of the cochlea for almost a century. The cochlea is the organ where transduction from acoustic vibrations to neural (electrical) activity takes place. The human cochlea has a frequency range of approximately ten octaves (20 Hz – 20 kHz) and an input dynamic range of 120 dB sound pressure level (SPL). This is very impressive given the cochlea’s size, power consumption and the apparent fragility of the main sensory cells known as the hair cells.

The silicon cochlea models that precede this work have been successful in modelling a number of features of the biological cochlea; however, none have demonstrated all of its active and nonlinear characteristics. In this thesis three silicon cochleae are developed that demonstrate the majority of nonlinear and active characteristics of the mammalian cochlea. They can be used to further understand biology as well as facilitate research into higher auditory centres.

As with the majority of silicon cochleae that have come before, the silicon cochleae in this work were constructed in the analogue circuit domain. The main reason for using analogue circuits was to allow a continuation of the work on 2D silicon cochleae that had already been undertaken in this laboratory. The more philosophical reason is that analogue circuits are inherently noisy and poorly matched and as a result might be considered better analogues for biological building blocks such as neurons and cells which suffer from some of the same drawbacks. As such, these drawbacks constitute a reality test for the models – a realistic model of the cochlea must not depend on perfect matching and infinite precision.

The content of this thesis is organised as follows: in Chapter 2 the physiology of the mammalian cochlea is discussed with particular emphasis on its nonlinear and active properties. Chapter 2 will also briefly discuss several computational models of the cochlea that provided inspiration for the work presented here. A comprehensive review of previous silicon cochleae is given in Chapter 3. Here silicon cochleae are categorised
based on their dimensionality and whether they have included circuitry to model the nonlinear, active cochlea. In Chapter 4 a log-domain circuit known as the Tau Cell (which is the main building block used to create filters in the silicon cochleae) is discussed with an emphasis on techniques to improve its matching, dynamic range and distortion performance. The 2D cochlea model used as the basis for the silicon cochleae in this work is discussed in Chapter 5. Here we begin with a discussion of the passive 2D cochlea model followed by the additions made to this model which incorporate the cochlea’s active behaviour.

The first of the active 2D silicon cochleae is presented in Chapter 6 along with a comprehensive analysis of its performance. Chapter 7 presents an improved silicon cochlea based on what we learned from the one presented in Chapter 6. Using improved circuit design, the silicon cochlea in Chapter 7 facilitates an exploration into coupling between sections of the cochlea which results in predictions about the operation of the biological cochlea. A third silicon cochlea, based on a slightly different model to that used in Chapters 6 and 7 is presented in Chapter 8. Here we further explore coupling in the cochlea and demonstrate high quality factors.

In Chapter 9 we discuss some of the results and predictions from Chapter 6, 7 and 8 along with recommendations for future work and concluding remarks. Enjoy!
Note: the use of “we”. Throughout this thesis I have used the personal pronoun “we”. This has been done for stylistic reasons (rather than a manifestation of multiple personality disorder). All the work presented here has been performed by me unless otherwise acknowledged.

Chapter 2. The Active Cochlea

2.1 Introduction

In this chapter the biological cochlea and its active characteristics are outlined and discussed. We begin with a brief summary of the anatomy and physiology of the human cochlea followed by a discussion of its nonlinear properties. We then look at computational models that include the cochlea’s nonlinear behaviour.

We use the terminology *active* to emphasise the fact that the cochlea is a dynamic organ that actively changes its properties to maximise its signal to noise ratio (SNR). For a long time the cochlea was thought to be a passive organ that simply divides sound into frequency components. In (Gold 1948) it was theorised that this was not the full story, however, it was only with improvements in measurement equipment and techniques that the active behaviour of the cochlea was observed and recorded. The first major evidence of nonlinear, active processes in the cochlea was found in 1971 when Rhode discovered compression in the response of the basilar membrane and the inner hair cells (Rhode 1971). Since this time a number of related discoveries have reinforced the notion that the cochlea is, in fact, a dynamic organ.

Despite a number of attempts, a mathematical model of the cochlea that comprehensively covers all of its nonlinear properties in a biologically plausible manner is yet to be proposed.

2.2 The Human Cochlea

The human cochlea is a fascinating transduction organ that illustrates the ingenious way in which engineering problems are solved in nature. It has a dynamic range of approximately 120 dB (Plack 2005) – allowing us to hear from the slightest whisper to the roar of a Boeing 747 flying overhead. The cochlea is a bony, fluid-filled, spiral structure that forms the majority of the inner ear. It performs the transduction between the pressure signal representing the acoustic input and the neural signals that carry information to brain. Figure 2.1 shows the location of the cochlea relative to other key features in the human ear.
The cochlea is spiralled from the base (lowest turn) to the apex (highest turn) containing approximately 2.5 turns. The main features of the cochlea are shown in Figure 2.2. Here we see two cross-sectional views of the cochlea. In Figure 2.2 (a) we see the cross-section of a single duct showing three separate chambers: the scala vestibuli, scala media and scala tympani. The scala media is isolated from both the scala vestibuli and the scala tympani via Reissner’s membrane and the basilar membrane, respectively. As well as being physically separated from the other two chambers, the fluid in the scala media is also different in composition to that in the scala vestibuli and scala tympani. Specifically, the fluid in the scala vestibuli and the scala tympani is called perilymph while the scala media is filled with a fluid known as endolymph. Perilymph and endolymph are very similar except that they have different ionic compositions. The scala vestibuli and the scala tympani are connected together via a small hole in the cochlea wall at the apex of the cochlea called the helicotrema (see Figure 2.3). Thus the endolymph-filled scala media is completely contained within the perilymph-filled scala vestibuli and scala tympani. The different ionic composition of the endolymph and perilymph result in battery-like chemical reactions and is further evidence of the active processes within the cochlea.
Figure 2.2 (a) A cross-sectional view of a single duct in the cochlea (b) A cut-away view of the organ of Corti showing both hair cells and supporting cells. The numbered features are: 1 basilar membrane, 2 Hensen’s cells, 3 Deiters’ cells, 4 endings of spiral afferent fibres on the outer hair cells, 5 outer hair cells, 6 outer spiral fibres, 7 outer pillar cells, 8 tunnel of Corti, 9 inner pillar cells, 10 inner phalangeal cells, 11 border cell, 12 tectorial membrane, 13 type I spiral ganglion cell, 14 type II spiral ganglion cell, 15 bony spiral lamina, 16 spiral blood vessel, 17 cells of the tympanic lamina, 18 auditory nerve fibres, 19 radial fibre, 20 inner hair cell, 21 reticular lamina. Adapted from (Kelly 1991) and (van Schaik 1998).
The organ of Corti (Figure 2.2 (b)) sits atop the basilar membrane. It contains both the inner and outer hair cells (IHCs and OHCs, respectively). The tips of these cells are hair-like, stereocilia. Deflections in the stereocilia of the IHCs generate neural signals (afferents) that travel to the brain. Neural signals from the brain (efferents) can alter the length and width of the OHC stereocilia.

Figure 2.3 shows the simplified, uncoiled cochlea with only its key features. Here we see both the oval window, which is connected to the stapes (see Figure ), and the round window, which is a membrane that allows the pressure within the cochlea duct to be equalised. The fluid within the cochlea is assumed to be incompressible. When the oval window moves both Reissner’s membrane (not shown along with the scala media in Figure 2.3) and the basilar membrane (BM) are deflected as a result and the round window moves in the opposite direction to the initial movement in the oval window.

The BM changes in width (illustrated in Figure 2.3) and elasticity from narrow and stiff at the base to wide and flexible at the apex. These changes in the physical characteristics of the BM assist in the way in which the cochlea divides an input signal into its frequency components. At the base of the cochlea the physical characteristics of the BM are such that it responds better (i.e. greater movement is produced) to high frequency stimuli whereas the apex responds better to low frequency stimuli. The characteristic frequency at a particular place along the BM is said to be the frequency that produces the greatest deflection at that place. In 1960 Georg von Békésy observed the
movement of the BM in response to a pure tone (von Bekesy 1960). He noted that the displacement of the BM had a characteristic pattern when pure tones with different frequencies were presented to the ear. The displacement pattern builds up at the base of the cochlea until it reaches a maximum amplitude at the place along the BM where the pure tone frequency is characteristic. It then falls off quickly as it travels towards the apex of the cochlea. If one were to observe this phenomenon in an uncoil cochlea (as in Figure 2.3) the displacement pattern would look much like an ocean wave. Hence, the theory that sound propagates in the cochlea as a *travelling wave* was established. The travelling wave arises from the physical characteristics of the BM that result in increased phase delay as a signal travels from the base to the apex.

When the basilar membrane moves it produces a shearing force between the reticular lamina (see 21 in Figure 2.2 (b)) and the tectorial membrane. The stereocilia of the hair cells (shown in Figure 2.2 (b)) are deflected as a result of this shearing force. The reticular lamina is the delicate membrane from which the stereocilia of the hair cells protrude. As it is part of the organ of Corti it moves with the motion of the BM. The greater the shearing force, the greater the deflection of the hair cells and the greater the neural response.

![Diagram of the cochlea with labels](image)

*Figure 2.4 Innervation of the hair cells afferent and efferent fibres (van Schaik 1998) adapted from (Watts 1992).*
There are approximately 3500 inner hair cells and 12000 outer hair cells in the human cochlea. The inner hair cells (shown as a single row in Figure 2.2 (b)) are responsible for transducing BM vibrations into electrical signals. Thus, if an input signal is at a particular frequency the place where this frequency is the characteristic frequency will produce the greatest movement of the BM, shearing force and subsequent deflection of the inner hair cells. The stereocilia of the outer hair cells (shown as three rows in Figure 2.2 (b)) are embedded in the tectorial membrane and are believed to be involved in the active operation of the cochlea. The outer hair cells will be discussed in more detail in section 2.3.

The inner hair cells and outer hair cells are connected to the auditory nerve via afferent (towards the brain) and efferent (from the brain) fibres. These fibres travel in bundles and enter the organ of Corti through perforations, known as habenula perforata, in the spiral lamina (see 15 in Figure 2.2 (b)). Connections between the two hair cell types and the fibres are very different. Afferent fibres connect to a single inner hair cell (about 10 fibres per hair cell constituting about 95% of all afferent fibres in the cochlea) whereas a single afferent fibre connects to approximately ten outer hair cells connecting basally from the habenula perforata. The majority of efferent fibres are connected to the outer hair cells (about 80%). They connect radially outwards from the habenula perforata. The remaining 20% of efferent fibres constitute the inner spiral fibres. These fibres synapse below the inner hair cells rather than with them directly (Zemlin 1998). The density of efferent connections to the OHCs is greatest at the base of the cochlea while the density of inner spiral fibres is greatest towards the apex. The innervation of the hair cells by afferent and efferent fibres is shown in Figure 2.4.

### 2.3 Nonlinear Properties of the Active Cochlea

Active amplification in the cochlea was proposed in (Gold 1948) but was not proven until thirty years later when Kemp discovered otoacoustic emissions (Kemp 1978). This discovery by Kemp followed shortly after the discovery of the nonlinear behaviour of the basilar membrane in (Rhode 1971). Both of these discoveries are related to the action of the outer hair cells. The discovery of the motile properties of the outer hair cells (Brownell, Bader et al. 1985) further reinforced the key role that the outer
hair cells play in the active cochlea. We will briefly discuss active amplification, nonlinear amplification and the mechanical properties of the outer hair cells followed by a summary of our current understanding of the nonlinear, active cochlea.

2.3.1 Active Amplification

In his paper on “The physical basis of the action of the cochlea”, Gold hypothesised that there must exist some form of active amplification given the fact that the viscosity associated with the cochlea fluid would not allow for the quality factor (Q) values which seemed to be present in the biological cochlea (Gold 1948). This hypothesis was not confirmed until Kemp discovered that when an acoustic impulse was presented to the ear a response was detected over 5 ms post-stimulation (Kemp 1978). After ruling out the presence of a noise artefact, Kemp was able to attribute these post-stimulation responses to the cochlea. This hypothesis was reinforced further by the absence of similar post-stimulation responses from tests performed on cochlea deaf subjects (i.e. subjects whose deafness was as a result of defects in their cochlea).

It was later (Kemp 1981) found that the emitted response can have greater energy than the original stimulus and sometimes a tone is emitted without the presence of a stimulus at all (called spontaneous otoacoustic emissions). The discovery that the ear can emit sounds is significant since it suggests that there is active amplification taking place within the cochlea.

2.3.2 Nonlinear Amplification

In (Rhode 1971) it was found that transduction in the cochlea is nonlinear. Specifically it was observed that the gain at the characteristic frequency increases as the input decreases and vice versa. While at frequencies lower than the characteristic frequency the relationship between input and output was observed to be linear. These observations show that the response of the cochlea, at a particular place along the BM, is saturated at the characteristic frequency for high input levels.

Nonlinear amplification is controlled or regulated by the efferent fibres from the brain (Mountain 1980), (Siegel and Kim 1982). More recent findings (Maison, Micheyl et al. 2000) suggest that the bandwidth and level of a the contralateral stimulus can influence the efferent controlled suppression of amplification in the cochlea. The level of
the otoacoustic emissions decreases somewhat when noise is applied to the contralateral ear (Collet, Kemp et al. 1990). Hence we can see a connection between nonlinear amplification, efferent control and otoacoustic emissions.

### 2.3.3 Mechanical Properties of the Outer Hair Cells

For a long time it was believed that the source of active, nonlinear amplification in the cochlea was the outer hair cells (OHCs). It was not until 1985, however, when Brownell and colleagues discovered certain unusual mechanical properties of the OHCs, that a direct link was made between the OHCs and amplification within the cochlea (Brownell, Bader et al. 1985). Brownell found that depolarising currents applied to the synaptic end of the OHCs reduced their length and increased their width whereas a hyperpolarising current applied to the synapses of the OHCs resulted in an increase in length and decrease in width. The OHCs’ mechanical response is frequency selective with some cells becoming shorter in response to one frequency and longer in response to another.

The dominant theory linking the motile behaviour of the OHCs with active, nonlinear amplification is that OHCs add in-phase energy to low-level input signals and increase damping to high-level input signals. The importance of the OHCs is clear given their physical characteristics and the fact that they are predominately connected to the efferent fibres, which have been shown to influence the active behaviour of the cochlear. It appears that the OHCs provide the positive feedback required to overcome the influence of the damping within the cochlear partition and are thus responsible for active amplification. Active amplification within the cochlear and its supporting mechanisms is sometimes referred to as the cochlear amplifier.

In 2000 Eguíluz and colleagues proposed that the cochlear amplifier has the dynamical characteristics of a Hopf bifurcation although the source of this nonlinearity was not ascribed to any particular cochlear mechanism (Eguíluz, Ospeck et al. 2000). Camalet and colleagues made an almost simultaneous conjecture, specifying that the Hopf bifurcation was a result of self-tuned critical oscillations of the hair cells (Camalet, Duke et al. 2000). It seems reasonable that it is the hair cells that are the source of the dynamics of the cochlear amplifier, however, there is no real consensus on the operating
point of the cochlear amplifier in relation to the Hopf bifurcation nor whether the Hopf bifurcation correctly models the cochlear amplifier’s dynamics.

### 2.3.4 Summary

Combining the results of sections 2.3.1, 2.3.2, and 2.3.3 we can obtain a coherent picture of the active cochlea shown in Figure 2.5 (Plack 2005). Here we see that the selectivity of the response along with gain is increased at low input levels whereas high input levels show a compressed response.

The cochlea is an active organ which has a nonlinear response. While the exact mechanisms behind the cochlear amplifier have not been fully described we know that the OHCs are involved in providing mechanical changes to the cochlear partition. Recent work suggests that the OHCs poise themselves on or close to a dynamical instability known as a Hopf bifurcation, although the exact way in which this is achieved is not yet known.

![Figure 2.5](image.png)  
**Figure 2.5** The effects of the nonlinear, active behaviour of the cochlea on basilar membrane velocity.

### 2.4 Characteristics of the Nonlinear, Active Cochlea

In this section we review some of the main characteristics of the active cochlea that are used to evaluate the “goodness” of a cochlea model. Of course these are not the only characteristics of the nonlinear, active cochlea. They are, however, some of its more interesting features and those which are believed to have the greatest impact on the way we hear.
2.4.1 Nonlinear gain and large-signal compression

Figure 2.6 (a) shows the frequency response at a particular place along the BM of a chinchilla cochlea. Here the input to the cochlea (in dB SPL) is given next to each frequency response curve. For low inputs we see that the frequency response is very sharp. As the input is increased, however, the response becomes more damped and the best frequency shifts towards lower frequencies. Figure 2.6 (b) shows the same data except the frequency is now plotted against the gain at each input level. Here we can clearly see large-signal compression and nonlinear gain.

The phase response of the chinchilla BM is shown in Figure 2.7 for input intensities varying between 10 dB SPL and 100 dB SPL at a characteristic frequency of 10kHz (Ruggero, Rich et al. 1997). In this data higher intensity input signals have a greater delay (phase accumulation) than low intensity signals. Specifically the group delay at the centre frequency is 0.99 ms and 0.61 ms at 10 dB and 90 dB respectively. The slope of the phase curve increases after the centre frequency until it plateaus.
2.4.2 Two-tone Suppression

Physiological experiments with the live cochlea have shown that the magnitude of the output signal in response to a test tone is reduced in the presence of another tone. This phenomenon is called two-tone suppression. Figure 2.8 shows evidence of two-tone suppression in the cochlea of a chinchilla for both higher suppressor tones (a) and lower suppressor tones (b) (Ruggero, Robles et al. 1992).

The probe tone represents the centre frequency at the measurement place along the basilar membrane. In Figure 2.8 these frequencies are 8 kHz and 6.8 kHz for (a) and (b) respectively. Each curve represents the magnitude of the probe tone versus BM velocity as the intensity of the suppressor is varied. The data clearly shows that as the intensity of the suppressor tone is increased the intensity of the probe tone must also increase to maintain the same BM velocity. Two-tone suppression is one of the clearest indicators that interaction occurs between the active elements along the BM.
Figure 2.8 Two-tone suppression in the chinchilla cochlea for both higher suppressor tones (a) and lower suppressor tones (b) (Ruggero, Robles et al. 1992).

Figure 2.9 Frequency spectra at a place along the basilar membrane show odd-order distortion products from a chinchilla cochlea.

### 2.4.3 Combinational tones

In 1714 composer and violinist Giuseppe Tartini discovered that when listening to a pair of tones he could hear pitches which corresponded to frequencies that were not being played. These so-called combinational tones or distortion products are by-products of the nonlinear action of the cochlea. When the cochlea is exposed to two tones which are close to the centre frequency at a particular place along the BM the frequency spectra of the BM in response to these tones contains prominent, odd-order distortion products.
Odd-order distortion products include $2f_1 - f_2$, $2f_2 - f_1$, $3f_1 - 2f_2$, $3f_2 - 2f_1$, and so on, where $f_1$ and $f_2$ are the two input tones.

Figure 2.9 shows the frequency spectra at a place along the BM where the centre frequency was 8 kHz. Here $f_1$ and $f_2$ were chosen to be 7.6 kHz and 8.4 kHz so that they closely flanked the centre frequency. The odd-ordered distortion products can clearly be seen in Figure 2.9 and the distortion products for $2f_1 - f_2$ and $2f_2 - f_1$ have been indicated on the diagram. It is interesting to note that these distortion products are not present in an unhealthy cochlea and as such we can say that if a cochlea is healthy it produces distortion!

### 2.5 Computational Models of the Active Cochlea

Given the difficulty in making measurements on a small organ such as the cochlea, computational models can be very useful. The majority of cochlea models to date have dealt with the cochlea as a mechanical structure and made appropriate mathematical or electrical substitutions as required. Recent work on the Hopf bifurcation, however, has focused more on the underlying mathematics rather than the initiating mechanical processes.

Looking at Figure 2.5 we see that as well as an increase in gain at low input levels the tuning of the cochlea also sharpens. The difference between this and automatic gain control (AGC) is that AGC only increases the gain in the system having no impact on its tuning. The increase in the selectivity of the tuning of the cochlea is one way that it deals with noise. By simply increasing the gain the noise associated with the signal will also be amplified. Increasing the gain along with the selectivity of the response, however, filters out much unwanted noise, amplifying only in-band noise, and improves the signal to noise ratio (SNR). Most models of the active cochlea include some form of feedback that tracks changes in the input signal and adds or takes energy away appropriately. Here we look at the models of Lyon and Zweig and discuss recent Hopf bifurcation-related issues as they are particularly relevant to our 2-D active cochlea model (see Chapter 5).

In 1990 Richard Lyon discussed the use of AGC (in which he included changes in the sharpness of the tuning of the cochlea in his definition of AGC) in modelling the active cochlea (Lyon 1990). Lyon favoured the use of a dynamic gain-control loop in
In order to implement the AGC rather than a compressive nonlinearity as it opened up the possibility for the inclusion of efferent connections into the model. In these models the AGC is necessary to compress the large, real-world dynamic range into one which can be handled by the neural connections in the ear. One of the main emphases of Lyon’s cochlea models is the need for AGC to be coupled between resonators. Figure 2.10 shows the algorithmic block diagram from (Summerfield and Lyon 1992) where Lyon’s 1-D cochlea model was implemented on a digital ASIC. A similar coupled-AGC model was implemented in the earlier computational model of (Lyon 1982).

![Algorithmic block diagram of Lyon's cross-coupled AGC 1-D cochlea. Redrawn from (Summerfield and Lyon 1992).](image)

Lyon asserted that coupling between AGCs was necessary to enable reproduction of sharp tuning curves and nonlinearities such as two-tone suppression in computational models. The coupling between AGCs was weighted so that changes in gain of adjacent filters were passed-on by a reduced amount (Lyon 1991).
The Lyon cochlea model favoured the view that compression in the cochlea was as a result of both instantaneous nonlinearities and adaptive, efferent-mediated feedback; both ipsilateral and contralateral (i.e. from the same ear and the other ear). This is because sharp tuning curves and two-tone suppression are as a result of lateral attenuation. Hence, active gain is insufficient in reproducing realistic cochlea curves. In this model the notion of adaptive feedback is primarily from mechanical processes (such as deflections of the OHCs) although other biological features can affect the instantaneous nonlinearities.

Many models have incorporated the idea of negative damping in a feedback loop to obtain active gain. In 1991 George Zweig proposed a double-pole oscillator model of the cochlea that included both negative damping and time-delay feedback (Zweig 1991). The negative damping provided the system with active amplification while the time-delay feedback was necessary to stabilize the system. This model was derived from biological data and was one of the first models that poised individual oscillators close to instability in order to obtain biologically plausible results.

In this model the strength of both the negative damping and feedback is determined by the BM velocity, $V$, and its derivatives. A block diagram illustrating the feedback system is given in Figure 2.11. In this diagram,

$$\frac{1}{s^2 + \delta s + 1}$$

(2.1)
represents the double-pole transfer function, $\delta$ is the damping constant, $P$ is the differential pressure across the scala media, $\omega_{c0}$ is the characteristic angular frequency of the oscillator, and $M_0$ is the effective inertia between a section of the cochlea and transverse motion. The feedback is represented by:

$$\rho e^{\psi s}$$

where $\rho$ is the feedback strength, and $\psi$ is a constant determined by empirical data. The time-delay of the feedback is proportional to the oscillator’s time constant which varies along the BM. The individual sections of the cochlea are coupled together only through the cochlear fluid. Hence, the parameters $\psi$ and $\rho$ are independent for each section.

Following the assertion in 2000 that the hair cells are governed by the Hopf equation there have been a number of attempts to build this behaviour into an updated version of Zweig’s cochlear amplifier. Kern and Stoop (Kern and Stoop 2003; Stoop and Kern 2004) have developed a coupled structure that suggests that sections of the cochlea (which embody supercritical stability) cannot be considered in isolation, as their dynamics affect the propagation of sound through the cochlea as a whole. In their model considerable importance has been placed on the need for a biomorphic coupling between cochlear nonlinearities (Kern and Stoop 2003), (Martignoli, van der Vyver et al. 2007).

There is no real consensus on the operating point of the cochlear amplifier in relation to the Hopf bifurcation. Kern et. al. (Kern and Stoop 2003; Stoop and Kern 2004) suggested a static operating point, which would require subcritical operation for stability, while Camalet et. al. (Camalet, Duke et al. 2000) have suggested that the amplifier is self-tuned to supercritical stability. Supercritical stability suggests that the amplifier is on the cusp of self-oscillation (and therefore at the point of maximum gain) when the input signal is vanishingly small. Operation at this point would result in limit-cycling in the event of even very minor mis-tuning; therefore, a self-tuning loop is almost mandatory if the amplifier is to operate in supercritical stability. Zweig argues that it is not plausible to consider the action of the OHCs as governed by a general nonlinear equation, such as the Hopf equation, as it is based on asymptotic rather than transient behaviour (Zweig 2003). Zweig’s model in (Zweig 1991) is based on an oscillator that is close to instability which is a similar concept to Hopf oscillators. The difference is that Zweig’s model
utilises a “double pole” rather than a single pole as is the case with the Hopf equation. Whether or not this is a significant difference is still unknown.

2.6 Conclusions

In this chapter we have briefly outlined the anatomy and physiology of the cochlea. In particular we discussed the nonlinear properties of the active cochlea and its physiological characteristics. We followed this with a discussion of some of the computational models that have incorporated the active, nonlinear behaviour of the cochlea. This discussion is by no means an exhaustive literature review, but rather a look at several models similar to that which we have implemented in Chapter 5.

2.7 References


Chapter 3. Silicon Cochleae

3.1 Introduction

Researchers have been building, improving, and studying silicon cochleae for almost 20 years now and prior to integrated circuit models, electronic cochleae used discrete components (Allen 1985). The challenge and attraction of building electronic cochleae lies in the design and implementation of a complex signal processing system that follows basic principles of biological cochleae. With the introduction of low-cost analogue very large scale integration (VLSI) and the promise of being able to implement relatively complex signal processing systems with real-time operation, the first silicon cochleae were produced. Richard Lyon and Carver Mead (Lyon and Mead 1988) built the first silicon cochlea and there have been many variations and improvements since this initial design. Figure 3.1 shows a tree diagram illustrating the progression of silicon cochlea modelling since the days of Lyon and Mead’s first silicon cochlea.

All silicon cochleae discretise the BM using a number of filters or resonators with an exponentially decreasing characteristic frequency (from base to apex). In this chapter, we classify silicon cochleae based on the details of the coupling between the cochlear filter elements and whether or not the gain and frequency selectivity of the cochlear filters adapts dynamically with changes in input intensity. Silicon cochleae may be classified as either one-dimensional (1D) or two-dimensional (2D) based on the coupling between the cochlear elements. One-dimensional silicon cochleae model the longitudinal wave propagation of the BM from base to apex (the x-direction in Figure 3.2), while two-dimensional silicon cochleae model the fluid within the cochlea duct as well as the BM taking both the longitudinal and vertical wave propagation into account. The systematic changes in the properties of the BM with longitudinal position, such as stiffness and width, are generally modelled by systematic changes in the parameters of the cochlear filter elements. Silicon cochleae are classified as active when the gain and/or selectivity of the cochlear filter elements change dynamically based on the intensity of the input, essentially increasing the gain and frequency tuning at low intensities and reducing these at high intensities. Ideally, the more features that a silicon cochlea has, the closer the
results should match those from biology. However, as more features usually demands greater complexity and cost in the implementation, silicon cochleae are generally designed with only those features required for a specific application. Applications for silicon cochleae include speech processing, sound localisation, and sound scene analysis, to name but a few. Experimenting with silicon cochleae in real time allows researchers to isolate individual components of the model and gain better insight as to how they work. In this regards, it is important in the design of silicon cochleae to use realistic biological models. From a signal processing point of view, the biological cochlea is interesting to engineers because of its large dynamic range, stability and noise immunity.

Despite over 20 years of research there has yet to be a silicon cochlea with a dynamic range and noise immunity that comes close to matching that of the real biological cochlea. This needs to be put in perspective, however, considering that we are attempting to build and understand a complex system that has evolved over millions of years. In this chapter we introduce many of the interesting designs, developments, and
improvements in the silicon cochlea over the years and show that, while progress has been slow, models are becoming more complex, matching biology more closely as we attempt to include more and more of the functionality found in the biological cochlea. In 20 years we have gone from a simple passive model of the cochlea as a cascade of filters to silicon cochleae that model the coupling within the cochlea fluid and active cell structures such as outer hair cells (OHCs). Silicon cochlea design is a stimulating and fascinating area of research which has the potential to improve our understanding of the biological cochlea and harness its many advantages in modern signal processing applications.

![Figure 3.2 The uncoiled cochlea.](image)

### 3.2 1D Cochlea Models

#### 3.2.1 Lyon and Mead’s Silicon Cochlea

3.2.1.1 The 1D Cochlea Model (Lyon and Mead 1988)

A rigorous explanation of the 1D cochlea model is given in (Lyon and Mead 1988), however, a good understanding of Lyon and Mead’s silicon cochlea and subsequent 1D silicon cochleae depends mainly on understanding the reasoning behind the use of a filter cascade to model the BM.

As sound enters the cochlea it initiates a fluid displacement, velocity and membrane deflection wave which travels along the length of the BM from base to apex. The
physical properties of the BM change from base to apex in such a way that the various
time components of the pressure wave result in maximum displacement at varying
positions along the BM. The design of the 1D silicon cochlea is based on the
consideration of changes in the properties of the basilar membrane along its length, the x-
direction, and implicitly includes ideal 2D and 3D effects. Specifically, measurements of
the biological cochlea in a number of animals, including humans, have shown that the
characteristic frequency along the BM is exponentially decreasing, i.e., it is linear on a
logarithmic scale: high frequencies at the base to low frequencies at its apex. To create
the silicon cochlea, the BM is discretised into segments of equal length, \( \Delta x \), and the array
of segments is subsequently modelled by a cascade of filters with a characteristic
frequency, or rather its inverse, a time constant, \( \tau \), that scales according to the
characteristic frequency of each BM segment. Each filter in the cascade is identical,
except for its characteristic frequency, and has a low-pass or band-pass characteristic,
such that the higher frequency signal components are selectively filtered out as the signal
travels through the cascade. This removal of high-frequency components results in a
steep roll-off in the response curves at the output of each filter after the characteristic
frequency. This steep roll-off in the high frequency signal components is seen in the
biological cochlea as the pressure wave travels along the BM. Thus, despite its
simplifications, this model provides a good first-order approximation of the signal
processing within the biological cochlea.

Figure 3.3 shows the output of a software implementation (Slaney 1988) of an early
version of Richard Lyon’s cochlea model (Lyon 1982). This model included a number of
cascaded notch filters with resonators attached to the output taps of each stage and is
sometimes referred to as a 1D parallel-cascade model because the resonators are in
parallel with each other and the notch filters are in a cascade. The notch filter and
resonator are combined to form a single filter in the cascade-only model in (Lyon and
Mead 1988). These models are mathematically equivalent with the only difference being
a reduction in the computational effort required for the cascade-only model. The main
features shown in Figure 3.3 include the steep roll-off in gain after resonance, the
logarithmic spacing of the outputs, and the attenuation of the curves as the signal moves
from the basal or high-frequency end of the BM filter cascade toward the apical or low
frequency end. The attenuation of the signal at low frequencies is due to the inclusion of a preemphasis filter. The preemphasis filter is a high-pass filter which models the approximate frequency response of the outer and middle ear. In this example the corner frequency of the preemphasis filter is approximately 300 Hz. In this implementation, the input signal to the cochlea, which corresponds to a pressure wave, is differentiated to give a signal representation of BM velocity. It is more common to differentiate the output which gives the same result in a linear system except perhaps for a DC offset. The differentiation results in the sloped frequency response seen at frequencies below the resonant peak (+20 dB/dec). Without the differentiation, the frequency response below resonance is approximately flat which indicates that the BM displacement is constant at frequencies below the best frequency.

![Figure 3.3 Output response of a software version of Lyon’s cochlear model.](image)

### 3.2.1.2 The Silicon Cochlea

Richard Lyon and Carver Mead created the first silicon cochlea in 1988 based on the model described in Section 3.2.1.1 (Lyon and Mead 1988). In their design the BM is
modelled by a cascade of 480 second-order filter sections. The time constant, $\tau$, of each stage increases exponentially from the beginning of the cascade to the end. Output taps were taken, not from every second-order section, but at equal distances along the cascade due to limitations in the number of output pins available. Figure 3.4 shows a sketch of the cascade structure employed by Lyon and Mead and subsequently by many others in building 1D silicon cochleae.

![Figure 3.4 Floorplan of the Lyon and Mead Cochlea showing the cascade of second-order (from Lyon and Mead 1988).](image-url)
Each second-order section (Figure 3.5) is comprised of three wide-range transconductance amplifiers. Two amplifiers are connected together in a feed-forward, follower-integrator configuration and the third one is connected as a positive feedback amplifier. The positive feedback amplifier controls the Q-value of the second-order circuit. The small signal transfer function for the second-order section is given in the Laplace domain by:

$$\frac{V_{out}}{V_{in}} = \frac{1}{\tau_2 s^2 + \frac{\tau_1}{\tau} s + 1}$$

(3.1),

where, $G = G_1 = G_2$ is the transconductance of an amplifier and $G_1$, $G_2$, $G_3$ are the transconductances of wide-range amplifiers 1, 2, and 3 respectively, $\tau = C/G$ is the time constant, $s = j\omega$ is the Laplacian parameter, $C$ is the capacitance, $Q = \frac{1}{2(1-\alpha)}$ is the quality factor, and $\alpha = \frac{G_3}{G_1 + G_2}$. From (3.1) we can see that each second-order section has the frequency response of a low-pass filter. The exponentially increasing time constant is created by applying a voltage difference across a resistive polysilicon line. This sets up a linear voltage gradient along the resistive line which results in exponential changes in bias currents when using MOSFETs operating in the weak-inversion regime. A similar method of biasing was used to set the quality factors for each of the cascade of second-order sections. As the input signal travels through the cascade of second-order
sections, the high-frequency components of the signal are selectively filtered out, with the higher frequency components only present at the start of the cascade and only the lower frequency components remaining toward the end of the cascade. The frequency-gain curves of a filter section in the cascade display the characteristic steep cut-off slope above the section’s characteristic frequency due to the successive removal of the high-frequency components by the filters before it. This is not the case for the first few sections, as the suppression of high-frequency components has not yet accumulated. Measurements of the output gain for two output taps of the Lyon and Mead cochlea are shown for a number of frequencies in Figure 3.6. In this figure, the measurement data are shown as small dots, while the continuous line shows the theoretical gain curve.

Figure 3.6 Frequency response data from Lyon and Mead’s Silicon cochlea (from (Lyon and Mead 1988)).

Following on from the success of the original silicon cochlea, Carver Mead’s PhD student John Lazzaro created a comprehensive silicon inner ear model (Lazzaro and Mead 1989) which, in addition to the silicon cochlea, included circuits modelling the inner hair cell (IHC) and spiral ganglion neuron cell (SC). Figure 3.7 shows a block diagram of his IC where the circuits for the inner hair cell, the spiral ganglion neuron cell, the outer hair cell, and the second-order section are labelled IH, SG, OH, and SO,
respectively. The neuronal circuits provided a spike encoding of the output of the silicon cochlea. This and other work has helped spawn a new era of research in modelling neuronal cells in silicon.

The Lyon and Mead silicon cochlea has been successful in modelling various characteristics of the real cochlea. It highlighted some of the challenges in modelling the cochlea and has provided an excellent starting point for investigations into silicon models of the cochlea and associated neuronal circuits.

![Figure 3.7 A block diagram of the Lazzaro and Mead inner ear chip.](image)

### 3.2.2 Further Improvements to Lyon and Mead’s Model

The Lyon and Mead cochlea is still employed today in various forms. Figure 3.8 shows the historical tree diagram from Figure 1 with the direct descendants of the Lyon and Mead cochlea highlighted in red.
In 1992, Lloyd Watts along with Carver Mead and Richard Lyon published an improved version of the original silicon cochlea (Watts, Kerns et al. 1992). In this work they demonstrated various circuit techniques that improved the linear range and large signal stability of the individual transconductance amplifiers and improved the matching within second-order sections and between second-order sections on a single chip. One source of instability in the second-order sections is the theoretical limit on the quality factor (Q) which approaches infinity when $\alpha = \frac{G_3}{2G} = 1$. However, the original Lyon and Mead cochlea exhibited a large signal stability limit, where rail-to-rail oscillations would occur at much lower $Q > 2.63$. The results of the improvements were better matching of Q between the stages and better exponential spacing of the characteristic frequencies increased linear range and the removal of the large signal instability. Figure 3.9 shows the output frequency response at several output taps along the silicon cochlea before and after the improvements to the circuits. These measurements indicate that there is a definite improvement in the uniformity of the frequency response at the various
output taps of the filter cascade. While Watts’ et al. did achieve a slightly wider linear operational range for individual transconductance amplifiers, improving the dynamic range and stability of the entire cochlea remain unresolved issues awaiting further research.

Lazzaro et al. used the Watts’ silicon cochlea in 1994 as part of a speech recognition system (Lazzaro, Wawrzynek et al. 1994). In this system, spiking neurons were used to encode the output of the silicon cochlea. The spikes were then communicated to the rest of the system using the address-event representation (AER) protocol. With this type of communication protocol, the address of an event or spike is put on a bus. In this case, the address indicates which output tap the spike corresponds to in the filter cascade. A computer or another chip then decodes the AER information locating the place where the event occurred. AER has now become a standard protocol for the efficient and compact communication of spikes from silicon neurons. It is used extensively in many neuromorphic chips today.

In 1995, van Schaik and colleagues from the Swiss Federal Institute of Technology (EFPL) improved upon the Watts’ silicon cochlea (van Schaik, Fragniere et al. 1995). They found that they could further improve the uniformity of the frequency response at the various output taps along the filter cascade by connecting the resistive polysilicon
line(s) to the base of compatible lateral bipolar transistors (CBLTs), rather than MOSFETs which in turn controlled the characteristic frequencies of the sections. This scheme avoids the main component of mismatch for MOS transistors in weak inversion, which is due to the variance in the threshold voltage, since bipolar transistors do not have such a threshold voltage. Figure 3.10 shows a comparison between the Watts silicon cochlea (see left figure) and the van Schaik silicon cochlea (see right figure). Unlike Figure 3.9, the output signals in Figure 3.10 have been differentiated to represent the BM velocity signal, rather than the pressure wave signal. The uniformity of the frequency response and Q values for the cascade of second-order sections of the van Schaik silicon cochlea are impressive and many silicon cochleae since this time have used CLBTs for biasing the cochlear filter elements.

![Figure 3.10 Comparison between the Watts et al. silicon cochlea (left) and the van Schaik et al. silicon cochlea (right).](image)

The improved matching of the cochlear filter elements is significant and means that we can now explore binaural sound processing using silicon cochleae. For example, a pair of matched silicon cochleas by van Schaik et al. were used by Chan et al. in sound localisation experiments (Chan, Liu et al. 2007).

The linear range of the transconductance amplifiers used in the cochlear filter elements was significantly extended by Sarpeshkar et al. (Sarpeshkar, Lyon et al. 1996). Sarpeshkar introduced a wide-linear-range transconductance amplifier with a linear range of up to 1.5 V compared to the maximum linear range of 260 mV for the transconductance amplifiers used by Watts et al. Figure 3.11 shows a schematic of the
second-order section. In this figure, WLR, represents the wide-linear-range transconductance amplifiers used in the circuit while the Fuse amplifier is discussed in more detail in section 3.4.

![Figure 3.11 The Sarpeshkar et al. second-order section.](image)

Sarpeshkar et al. also presented a wide-dynamic-range cochlea (Sarpeshkar, Lyon et al. 1998) which included the wide-linear-range amplifier discussed previously, along with automatic gain control, off-set compensation, IHCs, noise and distortion analysis, a digital implementation of a silicon cochlea, low power circuits, and a discussion on the potential use of silicon cochleae with cochlear implants. However, there is little published data on the operation of this 1D silicon cochlea as a whole.

The improvements to the Lyon and Mead cochlea have mainly resulted from improved circuit design along with the addition of neuron circuits and enhanced programmability. Nonetheless, the basic design of the 1D silicon cochlea with cascaded cochlear filter elements has remained the same and is still used today in many auditory signal processing applications.

### 3.2.3 Alternative Designs

#### 3.2.3.1 1D cascade/parallel Designs

One of the issues with the 1D cascade type of silicon cochlea is that once one of the second-order sections in the cascade fails, all of the subsequent second order stages
become unusable. Another problem with the 1D cascaded filter structure is that there is a limit to the number of stages that can be employed in practical real-time systems. This limit arises because as the number of filter stages increases, the delay through the cascade also increases, and at a certain point the latency of the output signal from the filter stages becomes too large. Additional issues with the 1D silicon cochlea include the limited dynamic range and large signal stability of the transconductance amplifiers and the accumulation of noise in the cascade. Silicon cochleae that consist of one-dimensional parallel sections or a two-dimensional structure (see section 3.3) circumvent some of the major drawbacks of the cascaded filter sections.

![Figure 3.12 Filterbank for the Liu et al. silicon cochlea.](image)

The 1D silicon cochlea design has not been restricted to the filter cascade design proposed by Lyon and Mead. Although many of the alternative designs solve the delay problem associated with the filter cascade, it is often at the expense of the biological
accuracy of the model. In 1991 Weimin Liu, Andreas Andreou and Moise Goldstein Jr. proposed a speech processing front end that was based on the biological auditory periphery (Liu, Andreou et al. 1991). They describe a silicon cochlea with 30 sections comprised of a first order low-pass filter and a second order band-pass filter. Their auditory periphery model also included inner hair cell and synapse models. The model was expanded further in 1992 (Liu, Andreou et al. 1992). Figure 3.12 shows a block diagram of the filterbank used in the Liu et al. cochlea. This filter configuration is less affected by the delay problem found in 1D cascaded cochleae because the resonant elements (the band-pass filters) are in parallel while the cascaded elements (the low-pass filters) are either first-order or damped second-order filters whose delay can be varied independently of the characteristic frequencies of the resonators.

Figure 3.13 Measured results from the Liu et al. cochlea showing (a) the frequency response at 10 outputs and (b) a comparison between peak frequency and output channel.

Measurement results from this silicon cochlea were well matched to theoretical results, however, the frequency response of the cochlear sections did not display, to the same extent, the steep roll-off after the characteristic frequency that is demonstrated in the Lyon and Mead cochlea. Figure 3.13 shows the frequency response of ten outputs of the Liu et al. filterbank and also a comparison of the best frequency versus the output channel number.

While the frequency response shown in Figure 3.13 (a) differs somewhat from that of the biological cochlea, the silicon cochlea by Liu et al. proved to be a useful research tool in speech recognition systems. (Furth and Andreou 1995) used a similar cochlear
filterbank design to create a low power version of the model of Liu et al. using MOSFETs operating in the weak-inversion regime.

In (Jyhfong, Wing-Hung et al. 1994) a silicon cochlea using a different kind of filter structure, referred to as switched-capacitor, is proposed. This model uses a parallel filterbank in which the outputs of three biquad filters are amplified and summed using a sum-gain amplifier (SGA) to create an output channel. Figure 3.14 illustrates the architecture of what these authors refer to as the *shared-component parallel filterbank*.

![Figure 3.14 Architecture of the Jyhfong et al. parallel filterbank.](image)

Results from the parallel filterbank without the preemphasis filter are shown in Figure 3.15. Here we see that at frequencies below the best frequency the frequency response curves match quite well with those of the biological cochlea. However, at frequencies above the best frequency, the roll-off is greater than second order, but not as steep as those from the biological cochlea.
The model proposed in (Jenn-Chyou and Chung-Yu 1996) also employs switched-capacitor circuits to implement a silicon model of the cochlea. However, unlike the model of Jyhfon et al. it was based upon a transmission-line model of the cochlea (Allen 1985). In this model, sections of the basilar membrane transmission-line model (see Figure 3.16) were cascaded together.

![Diagram of transmission-line model for the basilar membrane](image)

**Figure 3.16** A section of the transmission-line model for the basilar membrane.
In theory, transmission-line models of cochlear wave motion can be bidirectional, however, this implementation was unidirectional due to the circuits that were used to realise the model. Measurement results of the frequency response at several outputs are shown in Figure 3.17. The measured curves are similar to the response of the biological cochlea. A dynamic range of 67 dB was achieved, however, only four cochlea sections per chip could be realised because of the large area required for the switched-capacitor circuits. The results shown in Figure 3.17 are derived from eight chips connected together to create a cascade of 32 sections.

![Figure 3.17 Frequency response of 8 stages of the Jenn-Chyou and Chung-Yu cochlea.](image)

3.2.3.2 Digital 1D Models

Silicon cochleae have not been restricted to the analogue domain and there have been several impressive implementations in the digital domain. The first digital implementation by Summerfield et. al. which included some of the active features found in the biological cochlea will be discussed in further detail in section 3.4.

The digital cochlea in (Jones, Meddis et al. 2000) used a field programmable gate array (FPGA) to implement the cochlea as a filterbank of second order band-pass filters. This filterbank was specifically built to extract pitch from complex sounds and, in addition to the simplified cochlea, included detailed models of the inner hair cells,
auditory nerve fibres, stellate cells and coincidence cells. Another FPGA design was used in (Leong, Jin et al. 2001) to create a parameterised version of the Lyon cochlear model. Figure 3.18 shows the output from equally spaced taps from the Leong et al. FPGA cochlea. When compared to the software implementation of Figure 3.3 it matches very closely, as one would expect.

![Figure 3.18 Frequency response from evenly spaced output taps of the Leong et al. FPGA-based cochlea.](image)

The question of digital over analogue or vice versa is open in silicon cochlea modelling. Clearly both methods have their advantages and disadvantages. Proponents of analogue claim that issues of noise and device mismatch prevalent in analogue better represent the challenges that the body must overcome to function correctly. Digital implementations are not immune to noise, however, but they are more easily parametrised than analogue designs. This improves the tuneability of the design allowing testing of a number of circuit configurations.

3.3 2D Cochlear Models

In the 2D silicon cochlea the number of cochlear filtering elements is not limited by the accumulated delay through the filters since the pressure signal is coupled through a
model of the cochlear fluid rather than through a cascade of filters. This results in a silicon cochlea that is capable of being used in real-time applications for the entire frequency range of the cochlea. Secondly, errors in a single resonator/filter have little effect on the operation of the remainder of the cochlea. Figure 3.19 highlights in red the 2D silicon cochleae.

\begin{align*}
- 2 \rho \frac{\partial^2 \phi}{\partial t^2} &= S(x) \frac{\partial \phi}{\partial t} + \beta(x) \frac{\partial^2 \phi}{\partial y \partial t} + M(x) \frac{\partial^3 \phi}{\partial y^2 \partial t^2} \quad \text{at } y = h \tag{3.2}
\end{align*}

3.3.1 The Resistive Network

To date all of the 2D silicon cochleae that have been fabricated use a resistive network to model the cochlear fluid. This idea was first put forward in (Watts 1992) where it was shown that Laplace’s equation governs both the velocity potential, $\phi$, in an ideal incompressible and inviscid fluid and the voltage, $V$, in a resistive network.

Figure 3.19 2D silicon cochleae (highlighted in red).
where $\beta(x)$ is the viscous loss term, $S(x)$ is the membrane stiffness, $M(x)$ is the membrane mass, and $\rho$ is the mass density of the fluid. Equation (3.2) was determined using the 2D biological model boundary conditions of the membrane at the height, $h$, of the cochlear duct, i.e. $y = h$, and hard walls on all sides. Substitution of voltage for velocity potential and resistance, $R$, for the mass fluid density ($\rho$) gives a second-order equation which can be modelled in silicon by a filter/resonator circuit. This can be written in the Laplacian domain as:

$$\frac{V(x)}{I(x)} dx = \frac{S(x)}{s^2} dx + \frac{\beta(x)}{s} dx + M(x) dx = Z_m(x) dx$$

(3.3),

where $I(x) = -\frac{dx}{R} \frac{\partial V}{\partial y}$ is the current drawn by a resonator at position $x$ and $Z_m(x)$ is the impedance looking into that resonator. A block diagram of the 2D cochlear model from (Watts 1992) is shown in Figure 3.20. In this figure, the “B” in the boxes represents the resonators that model the basilar membrane response.

![Figure 3.20 The 2D cochlear model from (Watts 1992).](image)

**3.3.2 Voltage Domain Techniques**

The 2D silicon cochlea implementation described in (Watts 1992) was designed using circuits that operate in the voltage domain. It follows (Watts, Lyon et al. 1991)
which describes a transmission-line cochlea model similar to that proposed in (Allen 1985). The velocity potential, $\phi$, in the cochlear fluid is analogous to voltage, and hence the pressure wave, $p$, which is what a microphone records must be differentiated to obtain velocity. The Watts 2D silicon cochlea model is shown in Figure 3.20 and was fabricated using a 61-by-5 resistor array and 61 resonator circuits. The output frequency response of 11 equally spaced resonators is shown Figure 3.21.

![Figure 3.21 Frequency response of the output of 11 equally spaced resonators for the 2D silicon cochlear of (Watts 1992).](image)

The cochlear resonators were implemented using transconductance amplifiers similar to those of (Lyon and Mead 1988) and (Watts, Kerns et al. 1992). One of the drawbacks to this design is that 7 transistors are required to implement each resistor. More recent implementations of the 2D cochlear model avoid this problem by operating in the current domain.

### 3.3.3 Current domain Techniques

(Fragniere 1998) describes a 2D cochlear model in which pressure and voltage are mathematical analogues as are BM acceleration and current. A simple schematic of this model is shown in Figure 3.22 where the height of the cochlear duct has been modelled by decreasing the height of the resistive network from base to apex. Detailed and
systematic simulations using this model demonstrate that the increased height of the network at the basal end makes little difference in the simulation outputs and that a constant height of two elements provides a reasonable approximation.

The results from a software implementation (Shiraishi 2004) of this model are shown in Figure 3.23.

![Figure 3.22 Electrical model of the 2D cochlea from (Fragniere 1998).](image)

The (Fragniere 1998) 2D silicon cochlea avoids using resistors by relying instead on pseudo-conductances implemented using single transistors operating in the weak-inversion regime. As the pseudo-conductance transistors operate in the current domain, the resonators in the 2D cochlear model were also designed to operate in the current domain. More recent implementations of this model in the current domain are described in (van Schaik and Fragniere 2001), (Shiraishi 2004) and (Hamilton, Jin et al. 2008). The last of these references forms the basis for the rest of this thesis and will be discussed in greater detail in Chapters 5, 6, 7 and 8.
3.3.4 Bidirectional Coupling

The 2D cochlea design described in (Wen and Boahen 2005) uses a novel approach referred to as Active Bidirectional Coupling (ABC) to model the micromechanics of the biological cochlea. It was fabricated with 360 fully-differential BM sections and includes two 360 by 13 fluid grids. The resonator circuits operate in the current domain and the resistance of the cochlear fluid is modelled using single transistors operating in weak inversion. The model’s frequency response is shown for six resonator sections in Figure 3.24. Figure 3.24 highlights the non-uniformity of the response curves suffered by many current domain implementations of silicon cochleae. The inaccuracies arise because of mismatch between nano- and pico-amp currents which can be close to the noise floor in the circuit. While silicon cochleae implemented in the current domain are more convenient, power efficient and area efficient, there still remain unresolved issues in improving the matching and dealing with noise.
3.4 Active Silicon Cochleae

The silicon cochleae presented thus far have only modelled the passive behaviour of the biological cochlea. One of the most interesting properties of the biological cochlea is its ability to adapt to the intensity of its input. There have been few silicon cochleae that have attempted to reproduce this active behaviour.

In 1992, Summerfield and Lyon implemented Lyon’s cochlear model using a digital application specific integrated circuit (ASIC) (Summerfield and Lyon 1992). Their cochlear model included 71 cascaded filter sections. The output of each filter section was half-wave rectified in order to simulate the nonlinearity of the biological inner hair cells. The half-wave rectified signals were then passed as inputs to cross-coupled automatic gain control (AGC) circuits in which the additional inputs originate from adjacent AGC blocks. Thus, unlike the analogue implementation of the same model, the digital model includes active gain control. A block diagram of the Summerfield and Lyon digital cochlea is shown in Figure 3.25.
Along with the extended linear range, (Sarpeshkar, Lyon et al. 1996) replaced the feedback transconductance amplifier with a nonlinear and positive feedback Fuse circuit to implement level-dependent adaptation. The transfer function of this circuit is shown in Figure 3.26.

The fuse circuit models the action of biological outer hair cells (OHCs). That is to say, it alters the quality factor (Q) and gain of the second-order section based on the magnitude of the input signal. The silicon cochlea by Sarpeshkar et al. is one of the first active 1D silicon cochlea implementations. Figure 3.27 shows the changes in the quality factor, Q, of a single second-order section with different input amplitudes.
Figure 3.27 Frequency response of the Sarpeshkar et al. second-order section demonstrating nonlinear gain.

The 2D silicon cochlea presented in (Wen and Boahen 2005) includes possible physical mechanisms by which the active, nonlinear behaviour of the cochlea may arise. Figure 3.28 illustrates how the electronic BM velocity amplitudes vary when the intensity of the input amplitude is varied. Small input signals result in higher selectivity and relative gain, while large input signals result in a more damped response.

Figure 3.28 The basilar membrane velocity amplitudes from various input intensities from (Wen and Boahen 2005).

The remainder of this thesis discusses the approach that we took in modelling the active cochlea.
3.5 Summary

In this chapter we have described a number of different silicon cochlea designs. As our understanding of the biological cochlea has improved these designs have utilised models that have gradually included more and more of the cochlea’s impressive signal processing characteristics. Silicon cochleae are used for a variety of applications; from a tool for researchers to validate mathematical models of the biological cochlea in real-time to audio front-ends in speech recognition systems. The complexity of a model used to design a silicon cochlea will depend on its end application, however, the spirit of silicon cochlea design remains the same no matter what the application: to leverage biological functionality in modern technology.

In our quest to build better and more realistic silicon cochleae we are being confronted with many of the challenges that biology has had to overcome. While there has not been a silicon cochlea built to-date which rivals the biological cochlea, the situation is improving, with better models and better technology. In 20 years we have seen a move from simple 1-dimensional filter cascades to more biologically plausible 2-dimensional structures which include outer hair cell functionality. There have also been improvements in circuit design and as miniaturisation of CMOS fabrication technology continues, we can include more circuits (filters, cell structures etc.) on a single integrated circuit.

There are still many open issues in silicon cochlea design today: the use of analogue over digital or vice versa, the operating domain: current or voltage, and the choice of biological model. Several of these issues are engineering issues which, with time and experimentation, will be resolved. The choice of biological model is dependent on our understanding of the cochlea which continues to change as our observational capabilities improve.

In the two decades since Lyon and Mead’s silicon cochlea we are still faced with many of the same design challenges: noise, dynamic range and mismatch to name but a few. We have shown in this chapter, however, that silicon cochlea designs have improved significantly in 20 years and by overcoming the design challenges that still face us, we are improving our understanding of how the cochlea evolved into what it is today.
3.6 References


Chapter 4. An Analysis of the tau cell

4.1 Introduction

This chapter focuses on the log-domain circuit known as the tau cell (van Schaik and Jin 2003). The tau cell is programmable and can be configured in a number of ways without the state-space analysis required in a number of other classes of log-domain filters (Frey 1998). The tau cell was used by Shiraishi in her design of a 2-D cochlea (Shiraishi 2004), however, mismatch and noise issues prevented her from obtaining satisfactory results.

Here we explore the causes of mismatch in tau cell circuits as well as ways to improve dynamic range and distortion performance so that they can be used to implement the filterbank in a 2-D cochlea.

![Image of the tau cell]

Figure 4.1 The tau cell.

4.2 The tau cell

The tau cell (van Schaik and Jin 2003) is a basic building block representing a class of log-domain filters. A schematic of the tau cell is shown in Figure 4.1. The tau cell is designed for complete programmability via the time constant, $\tau$, and the current feedback, $A_i$. It can be used as a building block to create a number of more complex, higher order filters. The transfer function for a single tau cell is given by:
\[ T_i = \frac{i_i}{i_{i-1}} = \frac{1}{\tau_i s + 1} \quad \text{(4.1)} \]

where \( \tau = \frac{C U_T}{I_0} \) is the time constant, \( \tau_i \) is the time constant of the \( i \)th stage, \( i_{i-1} \) is the input current, \( i_i \) is the output current of the \( i \)th stage and \( T_{i+1} \) is the transfer function for the next filter stage. If there is no next stage then \( T_{i+1} = 0 \) and \( A_i = 0 \).

![Multiplier Circuit](image)

**Figure 4.2** A multiplier circuit based on a translinear loop.

The tau cell is based on the principle of translinear loops. The translinear loop is a fundamental concept in log-domain circuit design (Gilbert 1975). It is described in (Gilbert 1990) as follows: In a closed loop containing an even number of forward biased junctions, arranged so that there are an equal number of clockwise-facing and counter-clockwise-facing polarities, the product of the current-densities in the clockwise direction is equal to the product of the current densities in the counter-clockwise direction. This concept is based on the relationship:

\[ e^A \times e^B = e^{A+B} \quad \text{(4.2)} \]

and, as such, is only valid when current is described by an exponential relationship as is the case for BJTs and MOSFETs operating in the subthreshold region. This concept can be best described when looking at the circuit in Figure 4.2.
In Figure 4.2, $M_1$ and $M_3$ comprise the counter-clockwise junctions while $M_2$ and $M_4$ comprise the clockwise junctions. Transistor $M_5$ is necessary to correctly bias the source voltage of both $M_2$ and $M_3$. Hence, given the definition above we can say:

$$I_1 I_3 = I_2 I_4 \quad (4.3).$$

We can also show that if,

$$I_2 + I_3 = I_{bias} \quad (4.4),$$

where $I_{bias}$ is the current through $M_5$ set by $V_{bias}$, then

$$I_2 = \frac{I_1 I_{bias}}{I_1 + I_4} \quad (4.5),$$

and hence we can define an output current, $I_2$, based on input currents, $I_1$, $I_4$ and $I_{bias}$, when $I_3$ is constant.

Equation (4.3) demonstrates the ease with which a multiplier circuit can be constructed when operating in the log-domain. Understanding the principle of translinear loops can also be an important tool when analysing the function of other log-domain circuits.

In second-order filters implemented using the tau cell, the current-controlled current source in Figure 4.1 can be implemented using a multiplier circuit similar to that in Figure 4.2. In Figure 4.1 the closed loop of gate-source junctions necessary to form a translinear loop is created by transistors $M_1$ to $M_4$ and hence,

$$I_{i-1} I_0 = I_2 I_i \quad (4.6).$$

### 4.3 An Analysis of Matching

In our analysis of matching in the tau cell we focused on two things: layout and circuit configuration. In analogue design, in particular those designs which utilise circuits operating in the subthreshold region and/or designs where close matching is essential for correct circuit performance, the use of common-centroid layout (Sackinger and Fornera 1990) techniques is widespread. Dummy cells are also common in these situations where they can be used to equalize the impact of parasitics on two or more active devices and maintain local symmetry by keeping processes like etching and implanting as identical as possible for all active cells. Thus, in our analysis we compared tau-cell filters that were laid out using careful, common-centroid and dummy-transistor
techniques with those which were laid out to minimise the amount of area they took up on chip.

In order to evaluate the impact of circuit configuration on matching in the tau cell we utilised the fact that it is programmable. Specifically, we designed a second-order low-pass filter that could be configured in 3 different ways based on the selection of $I_0$ for each tau cell stage.

![Figure 4.3 Second-order tau-cell filter structure.](image)

A second-order low-pass filter can be realised by connecting two tau cells as illustrated in Figure 4.3. Here we see that the first cell has current feedback, $A_1$, while the second tau cell has no feedback. The schematic view of the second-order low-pass filter is given in Figure 4.4. In our test integrated circuit the capacitors $C_A$ and $C_B$ were connected off-chip. Also note that the inclusion of transistor $M_4$ is for demonstrative purposes only i.e. it does not impact on circuit operation. The general equation for the second-order low-pass filter in Figure 4.4 is given by:

$$T(s) = \frac{I_{\text{out}2}}{I_{\text{in}}} = \frac{1}{\tau^2 s^2 + \frac{\tau}{Q} s + 1}$$

(4.7),

where $\tau$ is the time constant and $Q$ is the quality factor. The derivation of these values depends on the chosen circuit configuration. Three circuit configurations were implemented using the circuit in Figure 4.4. It should be noted, however, that there are many more circuit configurations that result in the transfer function given in (4.7). The three that we present here were selected based on the ease with which they can be realised in circuit form and the equation that governs the quality factor ($Q$) in each case.
A summary of the three circuit configurations explored here is given in Table 4.1 where \( \tau_1 \) is the time constant for the first tau-cell section and \( \tau_2 \) is the time constant for the second tau-cell section.

![Figure 4.4 Schematic of the second-order low-pass tau-cell filter.](image)

<table>
<thead>
<tr>
<th>Type</th>
<th>Time-constant relationship</th>
<th>Bias-current Relationship</th>
<th>Current Multiplier, ( A_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \tau = \tau_1 = \tau_2 )</td>
<td>( I_0 = I_A = I_B )</td>
<td>( 2 - 1/Q )</td>
</tr>
<tr>
<td>2</td>
<td>( \tau = \tau_1/Q = Q \tau_2 )</td>
<td>( I_0 = I_A, I_B = A_1 I_A = A_I_0 )</td>
<td>( Q^2 )</td>
</tr>
<tr>
<td>3</td>
<td>( \tau = Q \tau_1 = \tau_2/Q )</td>
<td>( I_0 = I_A, I_B = I_A/Q^2 = I_A/Q^2 )</td>
<td>1</td>
</tr>
</tbody>
</table>

In the type 1 filter the time constant of each tau-cell section is equal and hence, \( I_A = I_B = I_0 \). The Q-value is determined by the value of \( A_1 \) which can be varied so as to determine the effects of different Q-values on matching. In the type 2 filters the value of the time constants of each section are not equal and are dependent on \( A_1 \). In order to configure this filter type \( I_A = I_0 \) and \( I_B = A_1 I_A \). The type 3 filter has a constant multiplier so that the Q-value is determined by the ratio of \( I_B \) and \( I_A \). Here we set \( I_B = I_A/Q^2 \) and \( I_A = I_0 \). For all the filter types the capacitors are set so that \( C_A = C_B \). The second-order low-pass filter is configured to be Type 1, 2, or 3 using simple multiplexing circuits and digital switches. The current feedback, shown in Figure 4.4 to be a current-controlled current source (CCCS) represented by the diamond box (the circular current source...
represents a constant current source), was implemented using a log-domain multiplier shown in Figure 4.5. Further details of the operation of the multiplier circuit are given in section 4.4. The CCCS is necessary to create controllable Q-values.

In order to create a common-centroid layout of the second-order low-pass filter, each transistor was divided into three unit-sized transistors of 4 \( \mu \text{m} \) width (W) and 12 \( \mu \text{m} \) length (L). The transistors were laid out in a checker-board arrangement, with a grid of metal 1 and metal 2 creating the interconnections and a border of dummy transistors. Unit-sized NMOS transistors of \( W/L = 4 \ \mu \text{m}/12 \ \mu \text{m} \) and a checker-board layout arrangement were also used in the common-centroid layout of \( M_8 \) and \( M_9 \) from the multiplier circuit (Figure 4.5). The matching of \( M_{10} \) is not critical and hence common-centroid layout was not used. To further save area \( M_{11} \) and \( M_{12} \) were laid out close together rather than using common-centroid layout. The PMOS devices were very large (\( W/L = 28.8/12 \)) and due to area constraints were not laid out using common-centroid layout. Figure 4.6 shows the arrangement of the transistors in (a) the second-order low-pass tau-cell filter and (b) the multiplier circuit. The filters that were laid out without using common-centroid layout techniques were placed so as to take up minimum area on the chip. The NMOS devices had dimensions \( W/L = 12/12 \) while the PMOS devices had \( W/L = 28.8/12 \).

![Figure 4.5 The multiplier circuit.](image)
An integrated circuit with four filters laid out using common-centroid techniques and two laid out to minimize area was fabricated using MOSIS AMI 1.6 µm technology. This chip also contained the circuitry necessary to implement the three types of filter configurations outlined in Table 4.1. The currents representing $I_0$ and $A_1$ were made programmable via a current splitter circuit (Delbrück and van Schaik 2005) and four selection bits (each). The top-level layout of the chip is shown in Figure 4.7. From this we can clearly see that the filters laid out using common-centroid are much larger than those that were not. Specifically, the layout using common-centroid had dimensions 375 µm by 265 µm versus the layout without which had dimensions 270 µm by 140 µm.

Testing of the chip concentrated on the matching between filters rather than matching to theoretical results. This is because cochlear filterbanks need to be well matched to one another rather than the theoretical equations for the filter.

The chip was tested by measuring the frequency response for each filter for different filter types, Q-values and -3 dB cut-offs. The frequency response was measured by feeding a constant amplitude current into a filter and varying the frequency. The output gain was recorded with the corresponding input frequency. The frequency response was reconstructed using MATLAB.
Our testing revealed surprisingly that for low Q-values the filters that were laid out to minimise area (i.e. the filters that were laid out without utilising common-centroid techniques) were better matched to one another (on the same chip and between chips) than were the common-centroid layout filters.
Our population consisted of 5 chips each with 2 filters that were laid out for minimum area and 4 filters which utilised common-centroid layout techniques. Our results showed that the variation in the output for the filters that used common-centroid was consistently greater than the variation in the output of the smaller filters. The results for low-Q settings (Q < 1) are shown in Table 4.2. Here the filters laid out using common-centroid and dummy transistors are referred to as matched while the filters laid out to minimise area are referred to as non-matched.

![Diagram](image)

**Figure 4.8** Example data from the analysis in Table 4.2.

In Table 4.2 it can be seen that the variation of the output of the matched filters was always greater than the non-matched filters. The variation in the outputs was taken as the average of the standard deviation of output magnitude in decibels measured every 20 Hz between 20 Hz and 2 kHz (data points in Figure 4.8) as well as the standard deviation in the -3 dB cut-off point for each of the filters (-3 dB freq in Figure 4.8).
Table 4.2 Comparison between Matched and Non-Matched Filters for Q < 1

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Matched Standard Deviation</th>
<th>Non-matched Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variation in Data Points (dB)</td>
<td>3dB Cut-off (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>1.92</td>
<td>7.1</td>
</tr>
<tr>
<td>2</td>
<td>1.43</td>
<td>12.7</td>
</tr>
<tr>
<td>3</td>
<td>1.61</td>
<td>10.3</td>
</tr>
</tbody>
</table>

As well as showing overall matching differences between matched and non-matched filters Table 4.2 shows that there are significant differences in matching between filter types. Specifically, type 2 filters appear to be better matched than type 1 and type 3 filters. Table 4.3 shows a comparison between Type 2 and Type 3 filters with Q > 1.
Type 1 filters were not included in this comparison because the current splitter which was used to set the value of \( A_1 \) did not allow multiples less than 2 and the Q value of the type 1 filters is set by choosing a number less than 2 according to equation (4.8) which is derived from Table 4.1.

\[
Q = \frac{1}{2 - A_1}
\]  

(4.8).

From Table 4.3 we can see that a standard deviation of almost 100 Hz in resonant frequency (labelled in Figure 4.9) was measured for the type 2 filters while a standard deviation of less than half of this was measured for type 3 filters.

<table>
<thead>
<tr>
<th>Type</th>
<th>Matched Standard Deviation</th>
<th>Non-Matched Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variation in Data Points (dB)</td>
<td>3dB cut-off (Hz)</td>
</tr>
<tr>
<td>2</td>
<td>3.35</td>
<td>102.9</td>
</tr>
<tr>
<td>3</td>
<td>1.79</td>
<td>30.2</td>
</tr>
</tbody>
</table>

Note that the Q value in Table 4.3 is given as the resonant frequency divided by the difference between the two frequencies -3 dB below the magnitude of the output at the resonant frequency.

Figure 4.10 shows the transfer functions for both matched and non-matched type 3 filters. From this we can see that the variation in the non-matched filters is predominately in the gain rather than resonant frequency, while the matched filters show significant spread in both gain and resonant frequency. The matched filters appear to have higher Q values than the non-matched filters, however, both are well below the theoretical value of 5.6. The deviation from theoretical Q values can also be found in simulation and is due to the presence of parasitic capacitance not included in the ideal theoretical model.
The results from our test chip indicate that laying out tau cells using the common-centroid method does not improve the matching performance of the filters and can even result in degraded performance. We also observed that type 2 filters have better matching for $Q < 1$ while type 3 filters have better matching for $Q > 1$.

There could be a number of reasons why matching was not improved by the common-centroid and dummy transistor layout techniques that we employed. By increasing the number of connections required, the common-centroid layout possessed a larger number of parasitic capacitors which may have increased the variability of the cut-off and resonant frequencies. By taking up a larger area on the die, the matched filters may have been more exposed to variations in doping concentrations which may not have been equalized by the common-centroid layout, possibly because the variations were not uniform as assumed by the method of common-centroid layout. Another issue is that by dividing our 12 $\mu$m/12 $\mu$m transistors into unit-sized transistors of 4 $\mu$m/12 $\mu$m we may be more susceptible to fabrication errors given the smaller transistor size. Smaller transistors would be more sensitive to leakage currents which may lead to further mismatch errors. Sensitivity analysis of transistors in subthreshold shows that better

---

**Figure 4.10** Non-matched (top) and matched (bottom) filter transfer function for type 3 filters.
matching is achieved for transistors with bigger area (Hastings 2001). Specifically, longer transistors result in better matching and wider transistors promote operation in the sub-threshold region. By using transistors that are long but have minimum width our unit size transistors may not be operating optimally in the subthreshold region and could even be operating in the moderate region or strong inversion. In these regions of operation the equations that govern the tau cell are no longer valid and hence this could be a large source of variability.

We believe the differences in matching between different filter types is related to the time constant of each tau cell that makes up the second order filter, $\tau_1$ and $\tau_2$, respectively. From Table 4.1 we see that for $Q > 1$ the time constant of the first section of the type 2 second order filter is greater than the time constant for the second section, i.e. $\tau_1 > \tau_2$. For type 3 filters this is reversed and $\tau_1 < \tau_2$. This means that the bandwidth of the first tau-cell section of the type 2 filters is smaller than the bandwidth of the second stage. As a result, the input signal is attenuated in the first stage and consequently is more susceptible to noise and mismatch errors that would further reduce the bandwidth. This situation is reversed for $Q < 1$. In this case the type 3 filters have a first stage bandwidth less than the second stage. Thus, despite its more complicated implementation of $Q$, type 1 filters would appear to have the best configuration to improve matching for both high and low values of $Q$. Here the bandwidths of both tau cell stages are the same, resulting in a better balance between signals.

Across the board, our testing showed that mismatch was more prevalent in gain and $Q$ value rather than the cut-off or resonant frequency. To understand this better we undertook an analysis of the tau cell that did not assume matching bias currents. Figure 4.11 shows the same schematic of the second-order low-pass filter shown in Figure 4.4 except that the current that flows into and out of transistors M3 and M6 is not assumed to be matched. The transfer function for this filter is:

\[
\frac{I_{\text{out}2}}{I_{\text{in}}} = \frac{I_{A1}I_{B1}}{s^2C^2U^2_i + sCU_i[(2I_{B2} - I_{B1}) + (2I_{A2} - A_iI_{I1} + I_{A1})] + [(2I_{B2} - I_{B1})(2I_{A2} - A_iI_{I1} + I_{A1}) + A_iI_{I2}I_{B1}]} \tag{4.9}
\]
Figure 4.11 The second order low-pass filter assuming current mismatch.

From (4.9) we see that the resonant frequency is determined by:

$$\omega_0 = \frac{\sqrt{I_{A1}I_{B1}}}{CU_T}$$  \hspace{1cm} (4.10)$$

Since $I_{A1}$ and $I_{B1}$ are both $p$-sourced currents they should be fairly well matched to one another and to the optimum bias current, $I_0$, when the filter is configured as type 1. This is because the current mirrors that source them will have identical gate and source voltages and their drain voltages should be similar. When the filter is configured as type 2 or type 3 $I_{A1}$ and $I_{B1}$ are not equal but proportional to one another. The greatest source of mismatch, therefore, would be the ratio between these two currents. Table 4.2 confirms that there is greater mismatch in -3dB frequency for the type 2 and type 3 filters compared to type 1. The gain, $k$, from (4.9) is given as:

$$k = \frac{I_{A1}I_{B1}}{(2I_{B2} - I_{B1})(2I_{A2} - A_iI_{01} - I_{A1}) + A_iI_{02}I_{B1}}$$ \hspace{1cm} (4.11)

Here we configure the filters to have unity gain, i.e. $k = 1$, hence,

$$I_{A1}I_{B1} = (2I_{B2} - I_{B1})(2I_{A2} - A_iI_{01} - I_{A1}) + A_iI_{02}I_{B1}$$ \hspace{1cm} (4.12)

From (4.12) we see that to obtain unity gain, matching is required between a large number of currents. Some of these currents are sourced from $p$-type devices and others originate from $n$-type devices. Since matching between $n$-type and $p$-type devices is difficult, this explains why there is a greater spread in gain than frequency in our results.
The results from testing the second-order tau cell indicate that there is greater mismatch in gain than in -3 dB frequency. An analysis of the equations that govern the second-order tau cell indicate that this mismatch may be explained by variations in the bias currents that are used to set the time constant, \( \tau \), quality factor, \( Q \), and gain, \( k \), of the filter. Hence, matching of these currents is particularly important when trying to obtain close matching between tau-cell filters.

### 4.4 A New Multiplier Cell

This section looks at a derivation of a multiplier cell for the type 1 second order filters which, based on our results in section 4.3, are the most robust for both high and low values of \( Q \). To improve matching, the number of current mirrors, in particular those which mirror current between \( n \)-type and \( p \)-type devices, should be kept to a minimum.

The multiplier cell for the tau cell can be implemented using the circuit in Figure 4.12. Here the most straightforward implementation of the feedback term copies the currents, \( I_{out1}, I_{out2} \) and \( A_1I_0 \) into a translinear multiplier circuit. In Figure 4.12 the input currents \( I_{out1} \) and \( I_{out2} \) represent the output currents from the first and second stage of the second order tau cell, respectively. These are shown in Figure 4.11. In Figure 4.12 \( I_{out} \) represents the output from the multiplier.

![Figure 4.12 The original multiplier cell.](image-url)
In Figure 4.12 the four transistors, M7, M8, M9 and M4, create a translinear loop. Therefore:

\[ I_{\text{out}1} I_{\text{out}1} = A_1 I_0 I_{\text{out}2} \]  \hspace{1cm} (4.13)

We can assume without loss of generality that the DC components of \( I_{\text{out}1} \) and \( I_{\text{out}2} \) are identical and we can simplify (4.13) and solve for \( I_{\text{out}} \) to get:

\[ I_{\text{out}} = \frac{A_1 I_0 i_{\text{out}2}}{i_{\text{out}1}} \]  \hspace{1cm} (4.14)

where \( i_{\text{out}1} \) and \( i_{\text{out}2} \) are the AC components of \( I_{\text{out}1} \) and \( I_{\text{out}2} \), respectively.

A first simplification can be made to this circuit by the realisation that the voltage on nodes \( V_1 \) and \( V_2 \) in Figure 4.12 are actually identical to the voltages on the nodes \( V_1 \) and \( V_2 \) in Figure 4.4, respectively. Therefore we can connect these nodes directly and remove transistors M7, M4, M21, M22, M23, and M24 from Figure 4.12. This avoids copying the currents \( I_{\text{out}1} \) and \( I_{\text{out}2} \) and improves matching.

Transistors M11, M12, M13 and M14, in Figure 4.12 and Figure 4.5 are necessary to mirror the output current of the multiplier, \( I_{\text{out}} \), because node \( V_\text{out} \) cannot be directly connected to \( V_A \) (see Figure 4.4). Both \( V_A \) and \( V_s \) will be close to \( V_{\text{ref}} \) in normal operation and transistor M8 would not be guaranteed to stay in saturation, as is needed for correct operation of the translinear loop.

The transfer function for the second-order low-pass filter (4.7) is obtained under the assumption that \( C_A \) and \( C_B \) (from Figure 4.4) are the dominant capacitances in the circuit and thus much larger than the gate capacitance seen at nodes \( V_1 \) and \( V_2 \), for instance. Under this assumption, inspection of Figure 4.4 reveals that the voltages at \( V_1 \) and \( V_2 \) are increased by an identical amount from the voltages at \( V_A \) and \( V_B \), since both M3 and M6 have a constant bias current \( I_0 \) flowing through them, as will be the case when we have configured the second order low-pass filter to be of type 1. Furthermore, from Figure 4.5 it is clear that M8 and M9 form a differential structure and as such will be insensitive to a common DC level shift on \( V_1 \) and \( V_2 \) as long as enough voltage headroom remains for M10 to operate. This can be ensured by choosing an appropriate level for \( V_{\text{ref}} \). In Figure 4.5, \( V_1 \) and \( V_2 \) can thus be connected to \( V_A \) and \( V_B \) in Figure 4.4 instead. Because of the downward level shift, this now allows the output node \( V_{\text{out}} \) (in Figure 4.5) to be connected
directly to $V_A$ (in Figure 4.4) as well. This obviates the need for the current mirrors M11-M12 and M13-M14, further simplifying the circuit and improving matching. This final implementation is shown in Figure 4.13. It can be seen that transistor M8 has its drain connected to $V_A$ and its gate connected to $V_B$. Because the voltage swing on these nodes is small in this type of log-domain filter, saturation of M8 is nonetheless guaranteed.

![Figure 4.13 The new multiplier cell for Type 1 filter configurations.](image)

### 4.5 An Analysis of DC Input Level

It has been demonstrated in the previous sections that subthreshold currents are susceptible to both noise and mismatch and that this can adversely affect circuit operation. Restriction of the DC input level to a low current level can also restrict the allowable signal swing in the circuit and hence the gain of the circuit. This is of particular concern when dealing with second-order filters with high Q values. Harmonic distortion analysis shows that a large ratio between signal level and DC level can improve total harmonic distortion (THD) performance markedly (although at the expense of greater power consumption) and this is of particular importance as we intend to use the tau-cell filters in audio applications.

A small-signal analysis can be performed in order to understand the role of the DC input level in the operation of the tau cell without assuming that the transistors are operating in a particular region (i.e. we perform the analysis without assuming strong inversion or subthreshold operation of the transistors). Small-signal analysis is a valid
method of analysing log-domain filters since the voltage signal swing, independent of the region of operation (strong or weak inversion), will always be small when compared with the voltage rails for the tau cell. When the small-signal condition is not met the analysis is not valid.

First we will analyse a first-order tau cell assuming all the transistors are in weak inversion. We will then perform the same analysis without assuming weak inversion for the input and output transistors. Note that DC voltages and currents are given in capital letters and AC voltages and currents are given in lower-case letters.

The small-signal model of the circuit from Figure 4.1 is shown in Figure 4.14. From this we see,

$$i_{i-1} = v_{in} g_{m1},$$  \hfill (4.15)

where (Enz, Krummenacher et al. 1995),

$$g_{m1} = \frac{I_{DC}}{U_T}$$ \hfill (4.16).

In (4.16), $I_{DC}$ is the DC offset current of the input current. Now by Kirchhoff’s Current Law,

$$sCv_a - g_{m2}v_{gs2} - g_{m3}v_{gs3} = 0 \hfill (4.17).$$

Inspecting Figure 4.1 we see that,

$$v_{gs2} = v_{in} - v_A \hfill (4.18),$$

and,
\[ v_{gr3} = v_o - v_A \] (4.19).

Also, by the fact that there is only a DC level shift between \( v_o \) and \( v_A \), we can say that the AC voltages \( v_o \) and \( v_A \) are equal. Therefore, in (4.17) we can say that \( v_{gr3} = 0 \) and replace \( v_A \) with \( v_o \). Rewriting (4.17) given (4.18), and (4.19),

\[ sCv_o - g_{m2}(v_o - v_{in}) = 0 \] (4.20).

Given that the bias current, \( I_0 \), flows in transistor 2 and 3, the transconductance of these two transistors is given by,

\[ g_{m2} = g_{m3} = \frac{I_0}{U_T} \] (4.21).

Figure 4.14 also shows that,

\[ i_i = g_{m4}v_{gs4} \] (4.22),

where, given the DC bias current, \( I_{DC} \), flows through M4,

\[ g_{m4} = \frac{I_{DC}}{U_T} \] (4.23),

and, by inspection,

\[ v_{gs4} = v_o \] (4.24).

Combining (4.15) and (4.16) and solving for \( v_{in} \) we see,

\[ v_{in} = \frac{i_{\text{in}}U_T}{I_{DC}} \] (4.25).

Similarly, if we combine (4.22), (4.23) and (4.24) and solve for \( v_o \) we get,

\[ v_o = \frac{i_oU_T}{I_{DC}} \] (4.26).

Substituting (4.20), (4.21) and (4.26) into (4.20) we get,

\[ \frac{sCi_U_T}{I_{DC}} + \frac{i_{i1}}{I_{DC}} - \frac{i_o}{I_{DC}} = 0 \] (4.27).

Hence, solving for the transfer function,

\[ T(s) = \frac{i_i}{i_{i-1}} = \frac{1}{sCU_T + 1} \] (4.28),

and substituting for the time constant, \( \tau \), given in (4.1) we get,
\[ T(s) = \frac{1}{s \tau + 1} \]  

Equation (4.29) is the same as the equation for a first order tau cell given in (4.7) when \( A_t = 0 \) which is true for first order tau-cell filters. Hence, the small-signal analysis has given us the same result as a translinear loop analysis.

The analysis above was based on the assumption that the drain currents for the input and output transistors were subthreshold currents. If we now assume that the input and output transistors, M1 and M4 respectively, are in strong inversion we must use the strong inversion equation for drain current (Enz, Krummenacher et al. 1995),

\[ I_D = \frac{1}{2} n \mu_c c_{ox} \frac{W}{L} \left( V_G - V_{th} - V_S \right)^2 \]  

where, \( n \) is the slope factor, \( \mu_c \) is the carrier mobility, \( c_{ox} \) is the gate oxide capacitance, \( W \) is the width of the transistor, \( L \) is the length of the transistor, \( V_G \) is the gate voltage, \( V_S \) is the source voltage and \( V_{th} \) is the threshold voltage. Note that (4.30) is an approximation that does not take into account the channel length modulation of the device, however, it is sufficient for our analysis.

The condition for strong inversion is that \( V_{GS} - V_{TH} > 0 \) and \( V_{DS} > 0 \), where \( V_{DS} \) is the drain to source voltage. The transconductance of a transistor in strong inversion is given by:

\[ g_m = \sqrt{2 n \beta I_D} \]  

where,

\[ \beta = \mu_c c_{ox} \frac{W}{L} \]  

Now returning to the small-signal analysis we replace (4.16) and (4.23) with,

\[ g_{m1} = \sqrt{2 n \beta I_{DC}} \]  

and,

\[ g_{m4} = \sqrt{2 \beta I_{DC}} \]  

respectively. Now combining (4.15) and (4.33) we get,

\[ V_m = \frac{i_{i-1}}{\sqrt{2 n \beta I_{DC}}} \]  

and, combining (4.22), (4.24) and (4.34) we get,
\[ v_o = \frac{i_i}{\sqrt{2n\beta_{DC}}} \]  

(4.36).

Now substituting (4.21), (4.35) and (4.36) into (4.20) we get,

\[ \frac{sC_i}{\sqrt{2n\beta_{DC}}} + \frac{I_i i_i}{U_T \sqrt{2n\beta_{DC}}} - \frac{I_i i_{i-1}}{U_T \sqrt{2n\beta_{DC}}} = 0 \]  

(4.37).

The transfer function is therefore given by,

\[ T(s) = \frac{i_i}{i_{i-1}} = \frac{1}{sC U_T + 1} \]  

(4.38),

and hence,

\[ T(s) = \frac{1}{s\tau + 1} \]  

(4.39).

Equations (4.38) and (4.39) are identical to (4.28) and (4.29) respectively and hence the operation of the tau cell is independent of the region of operation of the input and output transistors.

The analysis presented above is simplified; however, when we change the DC level on our chip we see that the tau cell (in this case a second-order tau cell with low-Q value) shows little change in -3dB frequency for DC values that span the subthreshold and strong inversion regions (see Figure 4.15).

![Figure 4.15 Second-order tau-cell filter gain for varying DC offsets.](image-url)
In Figure 4.15 we see that there is a difference of approximately 2 dB between the lowest DC offset (47nA) and the highest DC offset (6.42µA). This difference can be explained when we look at the transfer function of the tau cell without substituting the transconductance, $g_m$, based on region of operation.

$$T(s) = \frac{g_m^4}{g_m^1} \frac{1}{1 + \frac{sC}{g_m^2}}$$  \hspace{1cm} (4.40).

In (4.40) we see that the gain is dependent on the ratio between the transconductances of M1 and M4 (see Figure 4.1). Ideally, these two values will be equal although there will be some mismatch. When the drain of M4 is connected to a PMOS current mirror and as the DC offset increases, the voltage at the drain of M4 can vary downwards, decreasing the value of the offset current through M4 and creating mismatch between $g_m^1$ and $g_m^4$ so that $g_m^4 < g_m^1$. As the DC offset current is increased there comes a point where M4 moves out of the saturation region and into the cut-off region. This limits the amount by which we can increase the DC offset in the tau cell.

![Diagram](image)

**Figure 4.16 Output configuration of the tau cell for preventing the desaturation of M4.**
This limitation can be remedied, somewhat, by connecting the output of the tau cell directly to the read-out amplifier rather than mirroring the output signal using PMOS devices. This is shown in Figure 4.16 where $V_{\text{min}}$ is a bias voltage and $I_{\text{DC}}$ is the DC offset current.

When the tau cell is used to create a second order filter the maximum DC offset is limited by the multiplier circuit. Specifically, the drain voltages on transistors M8 and M9 in Figure 4.13 can drop relative to $V_s$, resulting in one or both of these transistors moving out of the saturation region and into the cut-off region. This problem can be solved by reducing the value of $V_{\text{ref}}$. This leads to an increase in the value of $V_{\text{DS}}$ and moves the transistor(s) out of the cut-off region and back into the saturation region of operation. This solution is only effective, however, as long as M10 remains saturated. Smaller values of $V_s$ push M10 into the cut-off region and hence there is a limit to the size of the DC offset current.

![Graph](image)

Figure 4.17 Output of second order low-pass tau-cell filter varying the DC offset and $V_{\text{ref}}$. 

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Figure 4.17 shows the output of a second-order low-pass tau-cell filter for 4 separate cases. For all of the curves the Q-value is set to 4 and the filter type is 2 (since the Q value for our preferred filter type, type 1, could not be varied in our chip implementation). The red curve shows the output of the filter when the DC offset is 120 nA and the reference voltage is 650 mV. In this case the input and output transistors are in the subthreshold region and the resonant peak is clearly visible. The black curve shows the output of the filter when the DC offset level is increased by several orders of magnitude, to 15 µA. Here we do not see the resonant peak nor do we see the filter rolling off at the correct frequency. The cyan and orange curves show the effects of decreasing $V_{ref}$. Here we see that as $V_{ref}$ is decreased the filter output matches closely with the subthreshold case. The orange curve has similar Q (1 versus 1.2) and resonant frequency (800 Hz versus 900 Hz) characteristics as the red curve.

Based on our simplified small-signal analysis and the test chip results shown in Figure 4.15 and Figure 4.17 we can see that we are able to use large DC offset currents in the tau cell and still obtain correct results.

### 4.6 An Analysis of THD

Operating at high DC offset currents can lead to significant improvements in total harmonic distortion (THD) in the tau cell. In (Sansen 1999) THD is calculated for a simple MOSFET amplifier operating in strong inversion. This is given in (4.41) where $I_D$ is the DC offset of the transistor and $\hat{i}_d$ is the peak relative current swing.

$$\text{THD} = \frac{i_p}{8} = \frac{1}{8} \frac{\hat{i}_d}{I_D} \quad (4.41).$$

Here we see that the THD is improved by reducing the signal swing relative to the DC offset. (Sansen 1999) did not include an analysis of a MOSFET operating in weak inversion, however, by following his methodology (see Appendix A) we are able to obtain the following relationship for THD:

$$\text{THD} = \frac{i_p}{4} = \frac{1}{4} \frac{\hat{i}_d}{I_D} \quad (4.42).$$

Equation (4.42) shows that distortion is even more prevalent in subthreshold devices than those operating in strong inversion.
Both equations (4.41) and (4.42) demonstrate the advantage of using DC offset currents significantly larger than the peak signal swing. Thus, we are able to substantially improve the distortion performance of the tau cell by utilising bias currents that operate in strong inversion.

4.7 Conclusions

In this chapter we have explored various methods to improve the performance of the tau cell log-domain filter. We discovered that matching between tau-cell filters was not significantly improved by employing the conventional technique of common-centroid layout and the use of dummy transistors. We also saw that thoughtful circuit configuration can improve both the matching and noise performance of a circuit. By ascertaining that the best circuit configuration for our purpose was type 1 and identifying the greatest sources of mismatch in this circuit we were able to reduce the number of resistors in our multiplier cell from 13 to 3. Our analysis of the relationship of the DC offset current and the operation of the tau cell has meant that the tau-cell filters are not restricted to a particular gain as the DC offset can be increased to accommodate the desired maximum level. The fact that the DC offset can be large also means that the distortion performance of the tau cell is greatly improved albeit at the expense of greater power consumption.

4.8 References


Chapter 5. The 2D Active Cochlea Model

5.1 Introduction

In this section we present our model of the active 2D cochlea. The models we briefly discussed in (Chapter 2 Section 2.5) were based on both 1D and 2D models of the cochlea. Here we build upon a passive model of the 2D cochlea (Fragniere 1998; van Schaik and Fragniere 2001) and add to it active elements.

5.2 The 2D Passive Cochlea Model

An analogue circuit model of the fluid dynamics within the cochlea may be achieved using a resistive network to simulate the cochlear fluid and with a number of resonator circuits to simulate the BM. The resonator circuits are attached to the resistive network and have an exponentially decreasing resonant frequency that is similar to the decreasing frequency from base to apex in the real BM (van Schaik and Fragniere 2000). A simplified circuit diagram of the model is shown in Figure 5.1. This model may be described as 2D since it models wave propagation horizontally along the BM and vertically in the fluid around it. A simplified equation describing the fluid motion within the cochlea can be written as follows:
\[ \{ p_{SM}(x,0) - p_{ST}(x,0)\} w(x) dx = a_{BM}(x) \left\{ m(x) dx + \frac{h(x)}{s} dx + \frac{k(x)}{s^2} dx \right\} \tag{5.1}, \]

where \( p_{SM}(x,0) \) is the pressure difference across the scala media, \( p_{ST}(x,0) \) is the pressure difference across the scala tympani, \( w(x) \) is the width of the BM, \( a_{BM}(x) \) is the acceleration of the BM, \( m(x) \) is the mass of the BM, \( h(x) \) is the viscosity associated with the BM, and \( k(x) \) is the stiffness of the BM. The circuit model is similarly described by an equation, (5.2), that is mathematically equivalent (5.1).

\[ V_{SM}(x,0) - V_{ST}(x,0) = Z_{BM}(x) J_{BM}(x) = I_{BM}(x) \left( \frac{1}{G_{BM}(x)} + \frac{1}{sC_{BM}(x)} + \frac{1}{s^2 S_{BM}(x)} \right) \tag{5.2}, \]

where \( V_{SM}(x,0) \) is the voltage analogue of \( p_{SM}(x,0) \), \( V_{ST}(x,0) \) is the voltage analogue of \( p_{ST}(x,0) \), \( Z_{BM}(x) \) is the equivalent electrical impedance of the BM, \( I_{BM}(x) \) is the electrical current density, \( G_{BM}(x) \) is the conductance of the BM, \( C_{BM}(x) \) is the capacitance of the BM, and \( S_{BM}(x) \) is the super-capacitance (frequency dependent negative resistance) of the BM. In the passive model the width, \( w \), mass, \( m \), and viscosity, \( h \), of the BM are assumed to be constant for the entire length of the BM. Thus we can see that in this model the analogue for pressure is voltage, the analogue for BM acceleration is current, the analogue for BM mass is the inverse of conductance, the analogue for BM viscosity is the inverse of capacitance and the analogue for BM stiffness is the inverse of super-capacitance. This model represents the passive 2D cochlea and further details can be found in (Fragniere 1998) and (Shiraishi 2004).

The resonators in Figure 5.1 comprise a conductance \( (G_i) \), capacitor \( (C_i) \) and super-capacitor \( (S_i) \). Here the “\( i \)” is equivalent to a section, \( dx \), of the BM after spatial quantisation. While the conductance and capacitor are basic electrical elements, the super-capacitor is not. It has an electrical characteristic in which the voltage, \( V_{sc} \) across its terminals is proportional to the double integral of the current, \( i_{sc} \), which flows through it. In the frequency domain this relationship is given by:
\[
V_{sc} \propto \frac{i_{sc}}{(j\omega)^2} = \frac{-i_{sc}}{\omega^2} \tag{5.3}
\]
Given the negative relationship between voltage and current, the super-capacitor is also referred to as a frequency dependent negative resistance (FDNR).

Following from (5.3) the frequency domain relationship between voltage, \(V_{BMi}\), and current, \(I_{BMi}\), and hence the input impedance, \(Z_{BMi}\), looking into the \(i\)-th resonator along the silicon BM, is given by:

\[
I_{BMi} = \frac{s^2 S_i}{s^2 \frac{G_i}{C_i} + s \frac{S_i}{C_i} + 1} \cdot V_{BMi} = \frac{V_{BMi}}{Z_{BMi}} \tag{5.4}
\]

The sensing cells in the cochlea, the inner hair cells, transduce the BM velocity into a neural signal. BM velocity is thus taken as the output for each resonator. Since the current, \(I_{BMi}\), represents the acceleration of the BM, it must be integrated to obtain a representation of BM velocity. Integrating (5.4) we get:

\[
I_{BMi} = \frac{s S_i}{s^2 \frac{G_i}{C_i} + s \frac{S_i}{C_i} + 1} \cdot V_{BMi} \tag{5.5}
\]

The response of the circuit model described above to a given input signal closely matches the response of a passive biological cochlea in which the OHCs have been inhibited (Fragniere 1998), (van Schaik and Fragniere 2000).

5.3 The 2D Active Cochlea Model

In this work, we have extended the afore-mentioned circuit model by adding an Automatic Quality-factor Control (AQC) circuit to the resonator circuit or by replacing the resonator circuits with Hopf oscillators. The AQC circuit or the Hopf oscillators are used to represent the action of the OHCs governed by the dynamics of a Hopf bifurcation. Since both of these approaches are governed dynamical systems that display a Hopf bifurcation, they are both mathematically equivalent. We chose to implement AQC because it has an intuitive feedback loop that adds or subtracts energy from the system based on the intensity of the input signal. AQC was proposed in (Lyon 1990) and we will
show two ways of implementing such a loop. In the Hopf oscillator implementation there is no visible control loop because the dynamics of Hopf amplification are implicit to the oscillator. Later in this chapter we will show that the AQC control loop is equivalent to Hopf amplification and subsequently parametric amplification.

We will begin in 5.3.1 by showing the equivalence between capacitance, quality factor (Q) and effective viscosity in our 2D model. This is achieved by showing that the equation for a second-order band-pass filter is of the same form as BM velocity in the 2D model. In 5.3.2 we discuss the theory behind the Hopf bifurcation and parametric amplification. We use this theory to shown that the AQC implementation is both stable and equivalent to Hopf amplification in 5.3.3. The Hopf oscillator implementation is discussed in 5.3.4.

5.3.1 Equivalence of Capacitance, Quality Factor and Effective Viscosity

In the 2D active cochlea model we no longer consider the effective viscosity constant but rather dependent on BM velocity. It can be seen that equation (5.5) is proportional to the typical band-pass filter response given in (5.6), where $\tau$ is the time constant of the filter and $Q$ is the quality factor.

$$T(s) = \frac{Out(s)}{In(s)} = \frac{s\tau}{s^2\tau^2 + s\frac{\tau}{Q} + 1}$$

Comparing (5.5) and (5.6) we see that capacitance, $C_i$, and hence the inverse of viscosity, $h$, is proportional to $Q$ (5.7). By varying $Q$ in response to BM velocity the OHCs compensate for the viscosity so that the effective viscosity is reduced. Increasing $Q$ leads to un-damping while decreasing $Q$ leads to positive damping of the system.

$$Q \propto C_i \propto \frac{1}{h}$$

5.3.2 The Hopf Bifurcation and Parametric Amplification

A Hopf bifurcation is a critical point in a nonlinear system where there is a transition between a stable equilibrium point and a limit cycle. A feature of the Hopf bifurcation is that there is a smooth transition between this stable equilibrium and the
limit cycle and back again without hysteresis. This is an important feature for a biological system where there is sure to be mismatch and noise that may otherwise push it into an unstable region without the ability to re-establish its correct operating point.

The Hopf differential equation for a single variable is given by:

\[ \dot{y} = (\mu + j\omega_0) y - y^3 + x \tag{5.8} \]

where \( y \) is the output signal, \( \mu \) is the control parameter or set-point, \( \omega_0 \) is the resonant frequency and \( x \) is a forcing function provided by some external force. The control parameter, \( \mu \), is varied to obtain sub-critical, critical and unstable (limit cycle) solutions to (5.8). The regions of operation for a dynamical system governed by (5.8) are given in Table 5.1. A graphical interpretation of these operating regions is given in Figure 5.2.

When operating at the critical, bifurcation point the system is said to be supercritically
The cubic term in (5.8) is representative of a compressive nonlinearity at resonance. This compressive nonlinearity results in a damped response for large inputs and a highly tuned response for small inputs. Hence, the Hopf equation possesses similar dynamics to those observed in the mammalian cochlea.

![Figure 5.3 A generic resonant system with positive feedback.](image)

In (Tapson, Hamilton et al. 2008) we showed that a feedback loop that adds the weighted energy of its output signal to its input possesses the dynamical properties of a Hopf bifurcation. Figure 5.3 shows a generic resonant system with positive feedback that can be described by the following equation:

\[
 x + Ay = M \frac{d^2y}{dt^2} + \xi \frac{dy}{dt} + ky 
\]  

(5.9),

where \(A\) is the feedback factor, and \(M, \xi,\) and \(k\) are parameters of the resonant system corresponding to energy storage, dissipation and restoring force respectively. If we select \(A\) such that it is given by:

\[
 A = g(\mu - |y|^2) 
\]  

(5.10),

where \(g\) is a constant gain factor and \(\mu\) is the system set-point, we can re-write (5.9) in (5.11) so that it is in the form of a second-order system that displays the dynamical properties of the Hopf bifurcation (Strogatz 1994).

\[
 M \frac{d^2y}{dt^2} + \xi \frac{dy}{dt} + (k - g\mu)y + gy|y|^2 - x = 0 
\]  

(5.11).

In (5.11) we are on the critical point, and hence, supercritically stable when \(k = g\mu\).

In (Tapson, Hamilton et al. 2008) we also showed that the second-order resonant system governed by (5.11) can also be thought of as a parametric amplifier. A parametric
amplifier is an amplifier whose parameters are periodically changed via a *pumping oscillator*. The pumping oscillator is tuned to a different frequency (pump frequency) to the operating frequency of the amplifier. In (5.11), if \( y \) is in the form of a sinusoid (\( y = Y \sin \omega t \)) then its square is given by:

\[
|Y \sin \omega t|^2 = Y^2 \sin^2 \omega t = \frac{1}{2} Y^2 (1 + \cos 2\omega t)
\]

which results in a signal with DC bias, quadrature shift and a frequency twice the input frequency. This frequency is an ideal *pump* frequency and hence we have parametric amplification.

![Figure 5.4 Phase shift versus linear \( (G_L) \), parametric \( (G_P) \) and total \( (G_L+G_P) \) gain for a parametric amplifier.](image)

A feature of parametric amplification is its robustness to noise and a wide range of phase shifts. We can re-write (5.10) to obtain expressions for linear (\( G_L \)) and parametric (\( G_P \)) gain:

\[
A = g(\mu - |y|^2) = g(\mu - Y^2 \sin^2 \omega t) \\
= g\left( \mu - \frac{Y^2}{2} (1 + \cos 2\omega t) \right) \\
= \left( g\mu - \frac{Y^2}{2} \right) - \left( gY^2 \cos 2\omega t \right) \\
= G_L + G_P
\]

(5.13).
In Figure 5.4 we see the linear ($G_L$), parametric ($G_P$) and total gain ($G_L+G_P$) of a parametric amplifier over the full range of possible phase shifts. Here we see that there is only a very narrow range of phase shifts around $|\pi/2|$ which lead to a reduction in overall gain. Hence, in implementing the Hopf equation using feedback (as we propose in sections 5.3.3 and 5.3.4) we do not require phase locking. This is an important point as it suggests that in the biological cochlea the OHCs need not phase lock perfectly with the movement of the BM (as suggested in Chapter 2 section 2.3.3) if their dynamics are governed by the Hopf equation (and thus by parametric amplification).

**5.3.3 Automatic Q Control (AQC)**

Our implementation of the active 2D cochlea model utilising AQC is shown in Figure 5.5. Here the circuit elements $G_i$, $C_i$ and $S_i$ have been replaced with a resonator circuit with the same transfer function as given in (5.6) and a control loop that varies its Q-value. The impedance matching circuit is necessary to ensure that the impedance of the resonators in the model ($Z_{BM}$) is maintained when using the resonator circuits with AQC.
The control loop (shown in Figure 5.6) consists of the resonator, a peak detection (PD) circuit, and a control circuit. The output signal of each resonator, $I_{OUT}$, is equivalent to BM velocity which is equivalent to the voltage across the capacitor, $C_i$, in Figure 5.5. The Set-point determines the amount of gain and compression in the system. The higher the Set-point the greater the obtainable Q-value.

To implement (5.6) we cascade two tau cells and implement the feedback, $A_1$ (see Chapter 4 Section 4.2), using a current multiplier (the details of the resonator circuit will be discussed in Chapter 6 Section 6.3.2). Hence, there are two feedback loops in our system.

Figure 5.6 The AQC loop.

Figure 5.7 Illustrates the two feedback loops in the active resonator. The instantaneous feedback loop (black) and the AQC loop (red).
circuit implementation of AQC. The first is the instantaneous loop that sets the quality factor by way of the multiplier circuit and the second is our control loop that varies the current used to set the Q-value based on the intensity of the output from the resonator. These two loops are illustrated in Figure 5.7. In this figure the instantaneous loop is in black while the AQC loop is in red. Analysis of these feedback loops is very complicated. We wish to solve them in a similar fashion to that shown in (Tapson 2006). The block diagram of Figure 5.7 can be simplified to that shown in Figure 5.8 where we have two passive first-order tau cells and a gain factor of $A$ in the feedback loop which is added (and as such is positive feedback) to the input.

![Figure 5.8 Simplified block diagram of the feedback loops in the active resonator.](image)

We now show the equivalence between the block diagrams in Figure 5.7 and Figure 5.8. In Figure 5.7 we obtain expressions for $I_{OUT1}$, $I_{OUT2}$, $I_Q$ and $I_{OUT}$ as follows:

$$I_{OUT1} = \frac{I_{IN}}{s\tau + 1} - A + I_Q$$  \hspace{1cm} (5.14),

$$I_{OUT2} = \frac{I_{OUT1}}{s\tau + 1}$$  \hspace{1cm} (5.15),

$$I_Q = A \frac{I_{OUT2}}{I_{OUT1}}$$  \hspace{1cm} (5.16),

$$I_{OUT} = I_{OUT1} - I_{OUT2}$$  \hspace{1cm} (5.17).
Rearranging (5.15) we get:

\[
\frac{I_{OUT2}}{I_{OUT1}} = \frac{1}{s\tau + 1}
\]  
(5.18),

which can be substituted into (5.16) to get:

\[
I_Q = A \frac{1}{s\tau + 1}
\]  
(5.19).

Substituting (5.19) into (5.14) we get:

\[
I_{OUT1} = \frac{(s\tau + 1) \cdot I_{IN}}{(s\tau + 1)^2 - A \tau}
\]  
(5.20).

Substituting (5.20) into (5.15) we get:

\[
I_{OUT2} = \frac{I_{IN}}{(s\tau + 1)^2 - A \tau}
\]  
(5.21),

and hence we can obtain \( I_{OUT} \) by substituting (5.20) and (5.21) into (5.17):

\[
I_{OUT} = \frac{s\tau \cdot I_{IN}}{(s\tau + 1)^2 - A \tau}
\]  
(5.22).

Rearranging (5.22) we find:

\[
(s\tau + 1)^2 \cdot I_{OUT} - A \tau \cdot I_{OUT} = s\tau \cdot I_{IN}
\]  
(5.23),

\[
(I_{IN} + A \cdot I_{OUT}) = \frac{(s\tau + 1)^2 \cdot I_{OUT}}{s\tau}
\]  
(5.24),

\[
I_{OUT} = \frac{s\tau}{(s\tau + 1)^2} \cdot (I_{IN} + A \cdot I_{OUT})
\]  
(5.25),

\[
I_{OUT} = \left[ \frac{s\tau + 1}{(s\tau + 1)^2 - \frac{1}{(s\tau + 1)^2}} \right] \cdot (I_{IN} + A \cdot I_{OUT})
\]  
(5.26).

The final expression, (5.26), is equivalent to the block diagram in Figure 5.8 where the output is multiplied by a factor \( A \) and is added to the input. This value subsequently passes through two cascaded tau cells and the final output current is obtained by subtracting the outputs of these two tau cells.

Figure 5.8 shows a system with positive feedback similar to the generic resonant system in Figure 5.3. We showed that the generic resonant system in Figure 5.3 is governed by the dynamics of the Hopf equation and parametric amplification. We can therefore consider the system that describes AQC as being governed by the same
dynamics. Rather than replicating the analysis given in section 5.3.2 we will use several methods to prove the stability of our system. These methods can be used to find the location of the Hopf bifurcation in this system.

The two feedback paths in Figure 5.8 can be considered separately as their bandwidths are several orders of magnitude apart. The instantaneous feedback loop has the following transfer function:

$$\frac{I_{OUT}}{I_{IN}} = \frac{s \tau}{s^2 \tau^2 + (2 - A)s \tau + 1}$$  \hspace{1cm} (5.27),

which is equivalent to the equation for the band-pass filter given in (5.6) when the Q-value is given by:

$$Q = \frac{1}{2 - A}$$  \hspace{1cm} (5.28),

and the resonant frequency, $\omega_0$, is given by:

$$\omega_0 = \frac{1}{\tau}$$  \hspace{1cm} (5.29).

In Figure 5.9 we have redrawn the instantaneous feedback loop to facilitate an analysis of its stability. Equation (5.28) indicates that an infinite Q-value will be obtained when $A = 2$ and, hence, this is the point of infinite gain and selectivity. This value is also predicted by the Barkhausen criterion. The Barkhausen criterion states that a feedback system produces infinite gain when the real part of the loop gain at the resonant frequency,
$L(j \omega_0)$ is equal to 1 and the imaginary part is equal to 0. In Figure 5.9 the loop gain is obtained by multiplying the open-loop gain,

\[ \frac{s \tau}{s^2 \tau^2 + 2s \tau + 1} \]  

(5.30), with the feedback factor, $-A$. Hence, the loop gain is given by:

\[ L(s) = \frac{-A s \tau}{s^2 \tau^2 + 2s \tau + 1} \]  

(5.31).

In the $j\omega$-domain (5.31) can be rewritten as:

\[ L(j \omega) = \frac{-j \omega \tau A}{1 - \omega^2 \tau^2 + j2\omega \tau} = \frac{2A \omega^2 \tau^2 + jA \omega \tau(1 - \omega^2 \tau^2)}{(1 - \omega^2 \tau^2)^2 + 4\omega^2 \tau^2} \]  

(5.32).

Solving the real part of (5.32) when $\omega = \omega_0 = 1/\tau$ we get,

\[ Re(L(j \omega_0)) = 1 = \frac{2A}{4} = \frac{1}{2} A \]  

(5.33), and hence, $A = 2$. Inspection of (5.32) shows that its imaginary part is equal to 0 for all values of $A$.

---

**Figure 5.10** The Nyquist diagram (polar plot) of the loop gain of the instantaneous feedback loop.
The Barkhausen criterion, while being valid for this feedback system, has been disproved for a number of feedback systems. It is more common to analyse the stability of a feedback system using Nyquist’s stability criterion which can be expressed as:

\[ Z = N + P \]  

(5.34),

where \( Z \) is the number of zeros of \([1 + L(s)]\) in the right-half of the s-plane, \( N \) is the number of clockwise encirclements of the -1+j0 point, and \( P \) is the number of poles of \( L(s) \) in the right-half of the s-plane. Inspection of (5.31) gives \( Z = 0 \) and \( P = 0 \), hence, for stability we want \( N = 0 \). To find \( N \) we must draw the polar-plot of \( L(j\omega) \). Figure 5.10 shows the polar-plot of \( L(j\omega) \) when \( \omega = \omega_0 \) and \( A < 2 \), \( A = 2 \) and \( A > 2 \). When \( A < 2 \) and \( A = 2 \), \( N = 0 \) since there are no clockwise encirclements of the point -1+j0. The system is unstable, however, when \( A > 2 \). Figure 5.10 also shows that when \( A = 2 \) and the phase is equal to -180° the loop gain is equal to -1. When the loop gain is equal to -1 feedback theory predicts that the system will oscillate. Hence, when \( A = 2 \) the system will have maximum gain and it will oscillate. The system is unstable for \( A > 2 \) and stable for \( A < 2 \).

It was shown in (Ueta and Kawakami 2004) that the Barkhausen criterion, Hopf bifurcation and the solution to a technique called the Virtual Source Method are equivalent for LCR oscillators. Using the Virtual Source Method given in (Ueta and Kawakami 2004) we can show that the Barkhausen criterion is equivalent to the Hopf bifurcation for our log-domain implementation of a resonator. The Virtual Source Method is based on the idea that an oscillator creates an output without the presence of an input signal. First we must determine the impedance of our circuit which is simply the transfer function given in (5.27). Re-writing this in the \( j\omega \)-domain we get:

\[ T(j\omega) = \frac{j\omega\tau}{1 - \omega^2\tau^2 + j\omega\tau(2-A)} \]  

(5.35),

and hence,

\[ I_{OUT} = T(j\omega)I_{IN} \]  

(5.36).

In the Virtual Source Method we replace the input, \( I_{IN} \), with a virtual source that is equal to 0+j0. Thus, in order for the circuit to continue to produce an output the virtual source must cancel with the denominator of the impedance, i.e.
\[ I_{\text{out}} = \frac{j\omega \tau}{0 + j0} \cdot 0 + j0 \] (5.37).

Setting the denominator of (5.35) to \(0 + j0\) and separating the real and imaginary parts we get:

\[
\text{Re}(\text{denominator}) = 1 - \omega^2 \tau^2 = 0
\]
\[
\text{Im}(\text{denominator}) = \omega \tau (2 - A = 0)
\] (5.38).

Solving these equations we find:

\[
\frac{\omega}{\tau} = \frac{1}{\omega_0}
\]
\[ A = 2 \] (5.39).

In the Virtual Source Method the solution to the real part of the denominator gives the resonant frequency while the solution to the imaginary part of the denominator gives the location of the Hopf bifurcation. Thus, from (5.39) we can see that the resonant frequency has been confirmed and a Hopf bifurcation occurs at \(A = 2\) which is on the cusp of limit cycling and at the point of infinite \(Q\).

We have shown that our AQC-loop is stable for values of \(A\) less-than and equal to 2. We have also shown that we are able to tune the resonator to a Hopf bifurcation when \(A = 2\) and that this is the point of maximum gain. This analysis has also shown that infinite \(Q\) is equivalent to a Hopf Bifurcation.

### 5.3.4 Hopf oscillators

Our implementation of the active 2D cochlea model using Hopf oscillators is similar to the implementation of AQC where the circuit elements \(G_i\), \(C_i\) and \(S_i\) are replaced with a Hopf oscillator circuit and an impedance matching circuit.

In defining the Hopf oscillators we manipulate the equation defining a band-pass filter (5.6) to be in the same form as the Hopf differential equation (5.8). Equation (5.6) has been re-written in (5.40) where \(\text{Out}(s)\) and \(\text{In}(s)\) are termed \(Y\) and \(X\) respectively.

\[
\frac{Y}{X} = \frac{s \tau}{s^2 \tau^2 + s \frac{\tau}{Q} + 1}
\] (5.40).

Equation (5.40) can be expanded as follows:
\[
\left( s^2 \tau^2 + s \tau \frac{\tau}{Q} + 1 \right) Y = s \tau \cdot X \tag{5.41}.
\]

After integrating equation (5.31) (i.e. dividing by \( s \)) and dividing by \( \tau \):
\[
\left( s \tau Y + \frac{Y}{Q} \frac{Y}{s \tau} \right) = X \tag{5.42}.
\]

Equation (5.42) can be re-written in differential form, replacing \( \tau \) with \( 1/\omega_0 \) to give:
\[
\dot{y} = \left( -\frac{1}{Q} + j \frac{\omega_0}{\omega} \right) y + x \tag{5.43}.
\]

By selecting \( Q \) to be of the form,
\[
Q = \frac{1}{y^2 - \mu} \tag{5.44},
\]
we can create resonators that are governed by the Hopf differential equation given in (5.8) which we call Hopf oscillators.

In equation (5.40) and (5.44) the value of \( Q \) determines the quality factor and hence the gain and selectivity of the resonator. One of the features of the biological cochlea is that as the input signal gets smaller, the gain and frequency selectivity increase, i.e., the Q-value of the cochlea increases, and vice versa when the input signal gets larger. In (44) we see, assuming \( \mu \) is zero, that as the energy in the output signal, \( y^2 \), decreases \( Q \to \infty \). As \( y^2 \to \infty \), \( Q \to 0 \). Thus, the Hopf oscillators exhibit dynamic gain control similar to that observed in the biological cochlea.

Looking at equation (5.44), tinnitus (ringing in the ears) and hence, otoacoustic emissions, might be explained by very low input signals and/or \( \mu \geq 0 \) pushing the Hopf differential equation into the limit cycle region. Other phenomena that define the nonlinear behaviour of the cochlea (e.g. combinational tones) can be fitted to this model as they tend to be features of most nonlinear systems.

### 5.4 Comparison with the Lyon and Zweig active models

Our active model with AQC is similar to the AGC model proposed by Lyon (see Chapter 2 section 2.5). Here we include both the instantaneous feedback and feedback which can be mediated by the efferent fibres (via the Set-point). Unlike Lyon we have not cross-coupled the AQC loops for each resonator. We did not do this since the Lyon
model was based on a 1D cochlea and therefore the resonators were coupled in only 1 direction. In our 2D model all the resonators are coupled through the conductive network which represents the cochlea fluid. The physical distance between resonators means that their influence on one another is stronger for neighbouring resonators and diminishes as distance increases.

The active model with Hopf oscillators is similar to Zweig’s model (see Chapter 2 section 2.5) which uses double-pole oscillators and negative damping to model the nonlinear action of the OHCs. Unlike Zweig we have chosen our oscillators so that their integral is equivalent to the Hopf equation and we have not found it necessary to include time-delay feedback for stability. It is unclear whether these differences are critical as the model that Zweig proposed does not discuss whether the OHCs are coupled together and if so, how this is to be implemented.

5.5 Conclusions

In this chapter we have described our 2D active cochlea model by discussing firstly the passive 2D model and then the active model which has taken two forms. One includes a control loop in the BM resonators that changes the Q-value based on the input signal. We have shown that our system has the dynamical properties of the Hopf bifurcation when certain conditions in the AQC loop are met. The other active model replaces the resonators with an oscillator that is governed by the Hopf equation. Both of these approaches are equivalent as we have shown that both have the dynamics of the Hopf equation. We have shown that systems which are governed by the Hopf equation are also examples of parametric amplification. Hence, our 2D active cochlea model possesses the nonlinear characteristics of the Hopf equation, which includes large-signal compression, as well as robustness to noise and a wide range of phase shifts.

5.6 References

Chapter 6. A 2D Silicon Cochlea with an Automatic Quality Factor Control Loop

6.1 Introduction

In this chapter we present a silicon cochlea with a local, instantaneous control loop that models the instantaneous action of the biological outer hair cells (OHCs). Importantly, we focus only on the instantaneous action of the OHCs and not on the slower feedback control exerted by the auditory brainstem via efferent fibres. This silicon cochlea was built using the active 2D cochlea model outlined in Chapter 5 Section 5.3 and uses automatic Q control (AQC) discussed in Section 5.3.3.

We will first discuss the implementation issues of the model in silicon followed by a detailed presentation of the circuits used to build the model. The results from a fabricated integrated circuit will be presented along with a discussion of the outcomes following the testing of this chip.

This chapter demonstrates that the silicon cochlea model with automatic Q control effectively simulates the dynamical response of biological cochlear resonances.

Figure 6.1 The 2D active cochlea model with AQC.
6.2 AQC Implementation

Our implementation of the 2D cochlea is shown in Figure 6.1. It operates in the current domain utilizing both pseudo-voltage (van Schaik and Fragniere 2000) and log-domain circuits. With this mode of operation current becomes an analogue for BM acceleration and voltage becomes an analogue for pressure, as required by equations (5.1) and (5.2) in Chapter 5.

A top-level block diagram of the AQC based on the control loop given in Figure 5.6 in Chapter 5 is shown in Figure 6.2. The control loop consists of the BM resonator (a second-order band-pass filter), a simple peak detector (Edwards and Cauwenberghs 1999), a decision module, a ramp generator, and a wide-linear-range (WLR) transconductance amplifier (Sarpeshkar, Lyon et al. 1997). In the integrated circuit implementation, the control loop can be externally disabled allowing the Q-value to be set manually. The output signal of each resonator is equivalent to BM velocity which is equivalent to the voltage across the capacitor, $C_i$, in Figure 6.1.

The peak detector continuously measures the peak level of the output signal from the BM resonator. The peak current forms an input to the decision module which employs two current comparators – one comparator is used to set the ceiling level for the signal amplitude and the other sets the threshold level. In other words, the decision module sets the target signal amplitude between the threshold and ceiling level as shown.
in Figure 6.3. The decision module employs hysteresis to prevent oscillations. The state logic describing the operation of the decision module is shown in Table 6.1.

![Figure 6.3 Decision Circuit Operation.](image)

<table>
<thead>
<tr>
<th>Condition</th>
<th>Q-Value</th>
<th>Control Signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal &gt; Ceiling</td>
<td>Q ↓</td>
<td>Vctrl = 0, Vctrl2 = 0</td>
</tr>
<tr>
<td>Threshold1 &lt; Signal &lt; Ceiling</td>
<td>Q constant</td>
<td>Vctrl = 0, Vctrl2 = 1</td>
</tr>
<tr>
<td>Signal &lt; Threshold2</td>
<td>Q ↑</td>
<td>Vctrl = 1, Vctrl2 = 1</td>
</tr>
</tbody>
</table>

The Q-value of the BM resonator is controlled by the output current from the WLR transconductance amplifier. The magnitude of this current is controlled by the output voltage, \( V_{ramp} \), from the ramp generator. Larger voltages generate larger currents and hence a larger Q-value. The bias current of the WLR transconductance amplifier is set to give the maximum Q-value when \( V_{ramp} \) is equal to the supply voltage \( V_{DD} \).

### 6.3 Circuits

#### 6.3.1 Resistive Network

The conductive network in Figure 6.1 was created using PMOS transistors operating as pseudo-conductances. The principle of pseudo-voltage and pseudo-conductance is
described relative to Figure 6.4. The current though the depicted transistor when operating in the sub-threshold regime is given by:

\[ I = I_s e^{\frac{V_G - V_h}{nU_T}} \left( \frac{V_S}{e^{U_T} - e^{U_T}} \right) \]  

(6.1),

where \( I_s \) is the specific current, \( n \) is the slope factor, \( V_{th} \) is the threshold voltage, \( V_G, V_S, \) and \( V_D \) are the terminal voltages referred to the local substrate (\( V_{DD} \) in most cases), and \( U_T = kT/q \) is the thermal voltage where \( k \) is Bolzmann’s constant, \( T \) is temperature and \( q \) is the charge of a single electron. We may define a pseudo-voltage as:

\[ V^* = V_0 e^{U_T} \]  

(6.2),

where \( V_0 \) is an arbitrary positive scaling constant. Combining equations (6.1) and (6.2) we obtain the following relationship:

\[ I = \frac{I_s}{V_0} e^{\frac{V_G - V_h}{nU_T}} \left( V_S^* - V_D^* \right) \]  

(6.3).

Here it is seen that using pseudo-voltages yields a relationship between current and voltage in the form of Ohm’s law, with the pseudo-conductance given by:

\[ G^* = \frac{I_s}{V_0} e^{\frac{V_G - V_h}{nU_T}} \]  

(6.4).

Figure 6.4 A pseudo-conductor.
As shown in equation (6.4), the pseudo-conductance $G_x$ (horizontal) and $G_y$ (vertical) in the conductive network (see Figure 6.1) may be controlled by varying the gate voltage, $V_G$, of the transistors in the network.

### 6.3.2 Basilar Membrane Resonators

The Basilar Membrane Resonator (BMR) was implemented using a log-domain and second-order band-pass filter, with an embedded AQC loop that sets the Q-value. The band-pass filter was implemented using two tau-cell log-domain filters (see Chapter 4). Figure 6.5 shows the circuit schematic for the band-pass filter. The transfer function for this filter is given by:

$$
\frac{I_{OUT}}{I_{IN}} = \frac{I_1 - I_2}{I_{IN}} = \frac{s\tau + 1}{s^2\tau^2 + \frac{5\tau}{Q} + 1} - \frac{1}{s^2\tau^2 + \frac{5\tau}{Q} + 1}
$$

where, $\tau$ is the time constant that determines the resonant frequency. The time constant is given by $\tau = \frac{CU_T}{I_0}$, where $C$ is the capacitance, $U_T$ the thermal voltage and $I_0$ the bias current. By using the tau cell for the band-pass filter design, the resonant frequency and Q-control can be configured in a variety of ways as described in Chapter 4. The objective of AQC is to maximize Q for low signal levels and have little to no Q for high signal levels. The tau cell band-pass filter is configured to have a Q-value which is governed by the following equation:

$$
Q = \frac{I_0}{2I_0 - I_q} = \frac{I_0}{2I_0 - A_1I_0} = \frac{1}{2 - A_1}
$$

where $A_1 \rightarrow 2$ when the input signal level is low and $A_1 \rightarrow 0$ when the input signal level is high. When using the AQC loop, the maximum Q-value is obtained by setting the bias current of the WLR amplifier to be approximately $2I_0$. This bias current can be set to its optimum value by turning off the automatic Q-control loop and varying the bias current until the output amplitude of the resonator is at its maximum when stimulating at the...
centre frequency. Errors in bias current due to noise or mismatch between different AQC circuits result in a reduction in Q as each WLR amplifier cannot be individually tuned. This is because we cannot have a Q-value that leads to an unstable output in any one resonator. Thus, the lowest value of WLR bias current that results in an unstable output will set the maximum bias current, and resonators that may have tolerated a higher bias current cannot be tuned to their optimum values.

![Figure 6.5 Schematic of the second order band-pass filter.](image)

The current-controlled current source shown in the first tau cell in Figure 6.5 is implemented using the log-domain multiplier described in Section 4.4 of Chapter 4.

![Figure 6.6 The peak-detector circuit.](image)
6.3.3 Automatic Q-Control Circuits

Figure 6.6 shows the peak-detector circuit for the AQC. This circuit was adapted from (Edwards and Cauwenberghs 1999). It rectifies the input current, $I_{\text{in}}$, using the capacitor, $C$, to hold the charge. A peak in the input current results in a peak voltage held across the capacitor. A small leakage current, $I_{\text{leak}}$, is included so that the circuit can track changes in the input.

![Block diagram of the decision circuit](image)

![Current comparator](image)

Figure 6.7 (a) Block diagram of the decision circuit (b) The current comparator.
The decision circuit (Figure 6.7a) comprises two current comparators (Figure 6.7b) and digital logic that implements the operation described in Figure 6.3. The digital logic adjusts the current comparator reference current, $I_{\text{ref}}$, so that the threshold level is set between $\text{Threshold1}$ and $\text{Threshold2}$ (see Figure 6.3). The output current from the peak detector, $I_{\text{peak}}$, is mirrored and fed into the current comparators as is the reference current, $I_{\text{ref}}$. The current, $I_{\text{peak}}$, is sourced into voltage node $V_X$, while the reference current, $I_{\text{ref}}$, provides a current sink (see Figure 6.7b). Thus, $V_X$ increases when $I_{\text{peak}}$ exceeds $I_{\text{ref}}$. Assuming that the transistor M3 is turned on, a copy of $I_{\text{peak}}$ flows into voltage node $V_Y$ and subsequently back into voltage node $V_X$ via transistor M1, forming a positive feedback loop. Hence, the increase in $V_X$ is reinforced and the output voltage, $V_{\text{comp}}$, goes low. The voltage $V_X$ decreases when $I_{\text{ref}}$ is greater than $I_{\text{peak}}$. In this case the decrease in $V_X$ is reinforced through the action of transistors M2 and M4 and the voltage $V_{\text{comp}}$ goes high. In summary, the decision circuit responds quickly to changes in $I_{\text{peak}}$, and the digital logic results in hysteresis, so that the decision circuit is not sensitive to small fluctuations in $I_{\text{peak}}$.

The circuit diagram for the ramp generator circuit is shown in Figure 6.8. Based on the voltage control signals, $V_{\text{ctrl}}$ and $V_{\text{ctrl2}}$, the capacitor, $C$, is either charged, discharged or held constant. As the capacitor is charged the output voltage, $V_{\text{ramp}}$, increases linearly.
$V_{\text{ramp}}$ decreases linearly when the capacitor is discharged. The rate at which $V_{\text{ramp}}$ rises and falls is determined by voltages $V_{\text{chg}}$ and $V_{\text{dischg}}$, respectively.

![Figure 6.9 The WLR transconductance amplifier.](image)

The WLR transconductance amplifier (Sarpeshkar, Lyon et al. 1997) is shown in Figure 6.9. It utilises a technique called *bump linearization* which extends the linear range of a transconductance amplifier by linearizing the hyperbolic-tan function that describes its input and output relationship when operating in the subthreshold region. It also uses well inputs and source degeneration to increase its linearity over a wide voltage range. In the AQC loop, the WLR transconductance amplifier is used as a voltage controlled current source. As the input, $V_{\text{ramp}}$, increases, the output current increases and vice versa. Although the current, $I_{\text{out}}$, from the WLR transconductance amplifier is bidirectional, the voltage $V_{\text{ramp}}$ is always greater than $V_{\text{in}}$ in normal operation and hence the circuit always sources current.

A linear change in $V_{\text{ramp}}$ will result in a linear change in $I_{\text{out}}$ and subsequently an exponential change in $Q$. As $V_{\text{ramp}}$ increases beyond its linear range (approximately 1.5 V) the output of the WLR transconductance amplifier saturates and $I_{\text{out}}$ approaches $AI_0$. 
approximately logarithmically. This logarithmic change in output current results in an almost linear change in Q value which improves the robustness of the AQC loop preventing the Q value from increasing too quickly.

6.3.4 The Terminator Circuit

The conductive network is terminated using a circuit that models the biological helicotrema to prevent low-frequency signal reflections. Low frequency signals that otherwise have not been given a low impedance path via a BM resonator circuit could create standing waves in the conductive network. The circuit diagram for the terminator is shown in Figure 6.10. It is a first order tau cell designed to model a conductor and capacitor in series. However, due to a layout error in the fabricated chip, it has the following characteristic:

\[ I_{BM} = \frac{s\tau + 1 - 2A}{s\tau + 1 - A} I_{in} \]  (6.7)

The value of A is determined from a biasing circuit described in subsection 6.3.8. From Figure 6.10 we see that the transistor implementing \( G^* \) is saturated, i.e., \( V_D \ll 0 \), so that \( V_D^* = 0 \). From this it follows that:

![Figure 6.10 The Terminator Circuit.](image)
\[ I_{in} = G^* V_{BM}^* \]  
and hence, from (6.6) we have,

\[ V_{BM}^* = \frac{1}{G^*}\left(1 + \frac{A}{s\tau + 1 - 2A}\right)I_{BM} \]  

The layout error does not greatly influence the operation of the silicon cochlea apart from some low frequency reflections in the resistive network and standing waves which may interfere with phase accumulation. This error was fixed in subsequent versions of the silicon cochlea (see Chapter 7 and 8).

**Figure 6.11 The Input Generator.**

**6.3.5 The Input Generator**

Pressure is represented by pseudo-voltage in the conductive network modelling the cochlear fluid. Since sound input from a microphone or sound card represents sound pressure as a voltage, we need to convert this voltage to a pseudo-voltage. We do this by converting the voltage linearly to a current using a WLR amplifier and then converting this current to a pseudo-voltage by passing it through a transistor operated as a pseudo-conductance. This yields the input generator circuit shown in Figure 6.11.
In Figure 6.11, $I_1$ is a current representation of the input voltage, $V_{in}$, plus a DC bias current, $I_{Bias}$. $I_1$ can be written in terms of the current $I_0$, and voltages $V_s$ and $V_{pv}$ as follows:

$$I_1 = I_0 e^{\frac{V_s - V_p}{U_T}}$$

(6.10),

where $U_T$ is the thermal voltage. This equation can then be re-written using the pseudo-voltage, $V_{pv}^*$, as follows:

$$V_{pv}^* = V_0 e^{\frac{V_p}{U_T}} = V_0 \frac{I_1}{I_0} e^{\frac{V_p}{U_T}} \propto I_1$$

(6.11),

where $V_0$ is some positive scaling constant. From this equation we see that $V_{pv}^*$ is a pseudo-voltage representation of the current $I_1$ and is, hence, proportional to $V_{in}$.

### 6.3.6 The Impedance Matching Circuit

The impedance matching circuit is necessary to ensure that the impedance of the BM resonators is maintained when using the resonator circuits with AQC. The input current to each resonator, $I_{INi}$, (as shown in the inset of Figure 6.1) must satisfy the following equation to ensure that equation (5.4) in Chapter 5 holds:

$$I_{BMi} = I_{INi} \cdot \frac{s^2 \tau^2}{s^2 \tau^2 + s \tau + 1}$$

(6.12).

The circuit required to implement this is shown in Figure 6.12. In Figure 6.12, $I_{out}$, $I_2$, and $I_1$ are all currents from the resonator (shown in Figure 6.5) and, $A_1I_2$ and $A_1I_1$ are obtained by multiplication using the translinear multiplier circuit of Figure 6.13. $G^*$ is a single transistor used as a pseudo-conductance which converts $V_{BMi}$ into the current $I_{INi}$ (van Schaik and Fragniere 2000) and $I_{DC}$ provides a DC offset to the resonator circuit.
6.3.7 The output circuit

The output current of the BM resonator, $I_{out}$, is fed off chip via a simple output circuit comprising a WLR amplifier and a simple buffer. The circuit is shown in Figure 6.14. In Figure 6.14, $V_+$ is a reference voltage and $I_{off}$ removes the DC offset from the output current, $I_{out}$. Thus, the relationship between $I_{out}$ and $V_{out}$ is:

$$I_{out} - I_{off} = G(V_+ - V_{out})$$  \hspace{1cm} (6.13),

where $G$ is the transconductance of the WLR amplifier.
6.3.8 The Resonator Biasing Circuit

The bias current for each resonator, $I_0$, and the bias current setting the maximum value for the quality factor, $I_q = A_1 I_0$, are distributed exponentially. In this way, the resonant frequencies of the BM resonators are also distributed exponentially. This is achieved by first applying a voltage difference across a resistive line that is realised using high resistance polysilicon and then linearly tapping a voltage off the line and feeding it into the base of three compatible lateral bipolar junction transistors (CLBTs) (van Schaik, Fragniere et al. 1995). The CLBTs convert the linear voltage change on the resistive line into an exponential change in the bias currents.
Figure 6.15 shows the bias circuitry for a single BM resonator. $V_{\text{gate}}$ and $V_{\text{gatenc}}$ are the biasing voltages for the MOSFETs incorporated in the CLBT. The cascode is included to increase the output impedance of the CLBT. The voltage, $V_{\text{var}}$, is used to vary the value of the bias current for setting the maximum value of the quality factor. $\text{line\_in}$ and $\text{line\_out}$ are the ends of the short section of resistive line for the single resonator. For the first resonator $\text{line\_in}$ connects to a pad and for the terminator $\text{line\_out}$ connects to a pad.

Figure 6.16 The 2D Active Silicon Cochlea Chip.

6.4 The Active Silicon Cochlea Chip

The silicon cochlea was fabricated in the AMI 0.5µm process. The integrated circuit included an on-chip bias current generator (Delbrück and van Schaik 2005) to improve current matching as well as allow easy bias current tuning. A photo of the chip is shown in Figure 6.16. There are 83 active BM resonators on a single chip. The layout for the resonator minimises the width of the cell and hence the number of rows of resonators required to fit the entire silicon cochlea on a single die. Figure 6.16 shows that three rows
are used with only two bends, i.e., direction reversals, in the chain of BM resonators. Previous silicon cochleae have shown that each bend increases mismatch. All of the capacitors shown in the previous circuit schematics were implemented as MOS capacitors as their performance in this process was comparable with the Poly-Poly capacitors.

The chip allows access to the output of each active BM resonator via a scanner consisting of a shift register and pass-transistors to select which BM resonator will be observed. In addition to providing access to the output, the scanner allows access to several control voltages in the automatic Q-control loop. The AQC mechanism can be switched on and off as can the BM resistive network. Thus, each resonator can be tested and tuned individually.

![Figure 6.17 Creation of $I_{ref1}$ and $I_{ref2}$ using $V_{clip}$](image)

The ceiling and threshold levels in the decision circuit ($I_{ref1}$, and $I_{ref2}$ in Figure 6.7a.) can be manually set via a single control voltage, $V_{clip}$. The relationship between $V_{clip}$, $I_{ref1}$, $I_{ref2}$ and the ceiling and threshold levels is shown in Figure 6.17. In Figure 6.17 “M” is the multiplier of the transistor which specifies the number of unit sized transistors that are connected in parallel. The “M” is selected so that the ceiling current is always greater than both the threshold currents, and threshold 2 is always greater than threshold 1. Various other currents used in the active BM resonator can be manually set.

### 6.5 Results

The operation of the silicon cochlea had to be tested using extremely high bias currents because the on-chip bias generator had too much gain resulting in instability of
the generated bias currents. The instability is normally controlled with the addition of a capacitor, however, an oversight meant that the node to which the capacitor must be connected was not brought out to a pin. As some of the bias currents on the chip were not brought out to pads, we were unable to switch off the bias generator. We were, however, able to obtain a stable bias generator output by reducing an external resistor, but this meant that the generated bias currents were at least 40 times larger (according to simulation) than they were originally designed to be. A summary of the performance characteristics of the chip is given in Table 6.2.

<table>
<thead>
<tr>
<th>Technology</th>
<th>MOSIS AMI 0.5µm CMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die size</td>
<td>5 mm²</td>
</tr>
<tr>
<td>Dynamic Range</td>
<td>46 dB (5mVpp – 1Vpp)</td>
</tr>
<tr>
<td>Noise</td>
<td>-43 dB (7 mV RMS)</td>
</tr>
<tr>
<td>Power Consumption</td>
<td>12.8 mA @ 4.4 V (56.32 mW)</td>
</tr>
</tbody>
</table>

### 6.5.1 Operation of the individual active BM resonators

We first configured the chip to test the individual BM resonators. This was achieved by bypassing the conductive network and inputting a test signal into an individual resonator via a multiplexer and scanner. The voltage across the resistive line used to bias the resonators was set to give a frequency range of approximately 750 – 4200 Hz and the AQC was initially disabled. Figure 6.18 a.) shows the frequency response of 21 resonators approximately equally spaced out of the 83 resonators in the network. The gain decreases some 5 dB towards the lower frequency resonators, which means that the output of some of the resonators is almost twice the output of others. This is an undesirable effect in a silicon cochlea. Figure 6.18 b.) shows the frequency response of the same 21 resonators with the same resistive line settings but now with the AQC turned on. When we compare Figure 6.18 a.) and b.) we see that the inclusion of AQC has little effect in improving matching.
In order to have a properly functioning silicon cochlea, it is important that we can control the resonant frequencies of the BM resonators precisely. To test this, we biased all of the resonators to have the same resonant frequency by setting the ends of the resistive line to the same voltage. Figure 6.19 shows the frequency response of the 21 BM resonators in this case. We see in this plot that there is some variation in the resonant frequency of the BM resonators. Statistical analysis shows the average resonant frequency to be 4133 Hz with a standard deviation of 550 Hz.

---

1 Note: in the following plots Output is given in decibels relative to 1 V.
We set the frequency range of the resonators from 200 Hz to 6.6 kHz and plotted the resonator number versus the corresponding resonant frequency in Figure 6.20. From this we can see that the best frequencies deviate somewhat from being logarithmically distributed but are at least monotonically decreasing from base to apex and therefore should not impact greatly upon the operation of the cochlea.

Figure 6.20 Resonator number versus corresponding resonator frequency.

Figure 6.21 Highlights a source of mismatch in our circuit: the addition of a DC current and subsequent mirroring of the output current to the peak detector and output circuit.
We wish to understand the variations in output amplitudes shown in Figure 6.18 and Figure 6.19. After a thorough examination of the layout, circuits and test set-up we found that the main source of mismatch is in the mirroring of the output current, $I_{out}$, of each resonator circuit to the output circuit (highlighted in Figure 6.21). Here we see that a DC current is added to the output current from the second-order tau cell band-pass filter and mirrored so that it can be copied to both the peak-detector circuit and the output circuit in each BM resonator circuit. This mismatch can be improved by finding more efficient ways of copying the output current which we demonstrate in Chapter 7.

We found that as a result of this mismatch the value of the reference voltage, $V+$, in the output circuit (see Figure 6.21) needed to be varied to avoid clipping and distortion from resonator to resonator. In fact we discovered a variation between 0.69 V and 3.09 V in $V+$ was necessary to obtain clean frequency response curves from the majority of the resonators.

![Figure 6.22 Frequency response of all 83 resonators with identical tuning and $V+$ varied to give optimum output.](image-url)

Figure 6.22 Frequency response of all 83 resonators with identical tuning and $V+$ varied to give optimum output.
Figure 6.22 shows the output of the 83 resonators with identical tuning. The voltage which sets the Q-value is set to give a higher value than in Figure 6.19 and hence we see even greater variation in the output amplitude and the resonant frequency. An examination of the matching between resonators with similarly tuned values of $V^+$ shows that this variation falls somewhat. Figure 6.23 shows the frequency response curves for 9 resonators (2, 7, 14, 15, 16, 22, 47, 58, 75) with the same $V^+$ voltage. Figure 6.23 (a) shows the frequency response of these 9 resonators when the Q-value is set very low while Figure 6.23 (b) shows the frequency response when the Q-value is set higher, identical to Figure 6.22. Table 6.3 shows a statistical analysis of the variation in both the resonant frequency (freq.) and output amplitude (output) for the data from Figure 6.23 (a) and (b). Here we see that variation in both the resonant frequency and output amplitude is greater when the Q-value is higher. The change in resonant frequency is moderate while the variation in output amplitude is more significant. This variation is much less than when compared with resonators with different $V^+$ tuning, however, and hence reducing the mismatch which leads to the need for $V^+$ to be varied will greatly improve matching in this system. Note that max spread in Table 6.3 refers to the difference between the maximum and minimum frequencies.
Table 6.3

<table>
<thead>
<tr>
<th></th>
<th>Low Q</th>
<th></th>
<th>Higher Q</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. (Hz)</td>
<td>Output (dB)</td>
<td>Freq. (Hz)</td>
<td>Output (dB)</td>
</tr>
<tr>
<td>Max. spread</td>
<td>300</td>
<td>1.5</td>
<td>400</td>
<td>3</td>
</tr>
<tr>
<td>Mean</td>
<td>955.56</td>
<td>-6</td>
<td>775</td>
<td>-2.9</td>
</tr>
<tr>
<td>S.D.</td>
<td>113</td>
<td>0.475</td>
<td>116.5</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Figure 6.24 shows the resonators from Figure 6.23 (except 14 and 15 which have been removed to improve clarity) when line_in and line_out are set to give a frequency range from 200 Hz to 10 kHz. Here we see that there is now no attenuation at lower frequencies (in fact there is slightly more gain at lower frequencies) and the variation in the gain is only about 2 dB.

Figure 6.24 Frequency response of 7 resonators with same V+ tuning and with large bypass capacitor.
Consider now changes in the quality factor. With the AQC disabled, the quality factor in a single resonator was varied from low to high. The frequency response of this resonator is shown in Figure 6.25. Here we see that we can achieve gains of 20 dB with very little change in the resonant frequency because the time constant and Q-value are independently determined in the tau cell configuration used in the band-pass filter. In other configurations a change in the Q-value necessitates a change in the time constant (see Chapter 4). The legend in Figure 6.25 indicates the value of a control voltage used to set the Q-value on chip. We see that small changes in voltage can result in large changes in Q-value due to the nonlinear function shown in equation (6.6). This can be a problem as any drift in the power supply voltage can result in large changes in the Q value. Therefore, we used a voltage regulator to maintain the power supply voltage. In this case $V_{dd}$ was held at 5.227 V.

![Figure 6.25 Frequency response of a single resonator with varying Q.](image)

The effect of the ceiling and threshold levels of the decision circuit on the operation of the AQC is shown in Figure 6.26. In this figure, $V_{clip} = 3.0$ V corresponds to a high ceiling level and $V_{clip} = 4.0$ V corresponds to a low ceiling level. In Figure 6.26 the curve corresponding to $V_{clip} = 3.0$ V has several bumps at low frequencies and is rounded at its peak; these non-idealities arise because the output was close to limit cycling at this point and as a result had an increased number of distortion products.
Figure 6.26 Frequency response of a single BM resonator with varying ceiling level.

While the frequency response curves illustrate the gain and frequency selectivity of the BM resonators, they do not show the extent of the distortion in the output signal. Figure 6.27 shows the fast Fourier transforms (FFTs) for two curves. These were measured using a 2.6 kHz sine wave of amplitude 100 mV as input into the resonator with resonant frequency set at approximately 2.6 kHz. The output was sampled at 500 kHz using a Tektronix digital Oscilloscope (TDS 3014) and MATLAB software was used to obtain the FFTs. Figure 6.27 a.) shows the FFT when $v_{clip} = 4.0$ V. Here we see the second harmonic at 5.2 kHz, however, the third harmonic, at 7.8 kHz, is barely visible. The total harmonic distortion (THD) for this curve was calculated to be 2.5%. Figure 6.27 b.) shows the FFT when $v_{clip} = 3.0$ V. In this plot both the second and third harmonic are clearly visible and the THD was calculated to be 10%. It is reasonable that a larger Q-value results in more harmonic distortion, especially since the control loop attempts to hold the Q-value at the edge of limit-cycling.
Using $V_{\text{clip}}$ to appropriately set the ceiling of a single BM resonator, we explore the nonlinear compressive effects of the AQC. Figure 6.28 and Figure 6.29 show the nonlinear compressive effects of a single BM resonator with AQC enabled. For these measurements, the frequency of the input signal is held constant, at or near to the resonant frequency of the BM resonator, while the amplitude of the input signal is varied. It can be seen, in Figure 6.28, that as the input amplitude increases, the gain of the output signal flattens out. The point at which the gain begins to flatten out is controlled by the level at which the ceiling is set via $V_{\text{clip}}$. Figure 6.29 shows that for low input intensities the gain increases along with the selectivity of the response as is required by our cochlea model.

Figure 6.30 shows the transient response of the 5th BM resonator to a sinusoidal input of amplitude 100 mV and frequency 1.4 kHz with $V_{\text{clip}}$ set high so that only low Q-values could be achieved. In Figure 6.30 we reset the output of the resonator at time zero by resetting the output of the ramp generator. From this time we see the amplitude of the output signal grow, resulting in a final gain of approximately 6 dB. Figure 6.30 also demonstrates how the DC level of the output signal does not remain constant. This is probably due to mismatch in the impedance matching circuit and is demonstrated even more dramatically in Figure 6.31 where we have increased the maximum Q value.
Figure 6.28 Nonlinear compressive effects of a single BM resonator.

Figure 6.29 Frequency response of a single BM resonator with varying input amplitude.
6.5.2 Operation of the Active 2D cochlea

Following extensive testing of the individual BM resonators we configured the chip as a 2D active cochlea. This was achieved by re-connecting the conductive network and inputting the test signal via the input generator circuit. In this configuration the output from the first few resonators is discarded because they do not have resonators of higher resonant frequencies earlier in the resonator chain and do not display the characteristic steep high-frequency roll-off. In this section we show gain, phase, two-tone suppression, combination tones and transient response in the 2D silicon cochlea and relate these to biology.
When configured in a particular way the output of the basal resonators showed two resonant peaks in our 2D cochlea output. The second peak or bump was in the same place for all resonators. The output of the first resonator for two different settings of the voltage, *line_out*, is illustrated in Figure 6.32. The second peak is the peak on the left. By increasing *line_out* and hence lowering the minimum resonant frequency in the cochlea, the peak shifts towards a lower frequency and its intensity was reduced somewhat. Given that the peak shifted when the tuning of the resonators was changed suggests that the peak is a result of interfering waves that constructively add at a particular frequency. By lowering the minimum frequency in the cochlea the peak was less noticeable. It was not until work on our third silicon cochlea that we were able to understand this phenomenon fully (see Chapter 8), however, we postulated that it was related to tuning and termination in the cochlea. Importantly, the phenomenon was not dependent on AQC since it did not change based on whether AQC was enabled or disabled. To avoid the second peak we operated at frequencies lower than 2 kHz and set *line_out* high.

Figure 6.32 Illustration of the second peak or bump (left) in the first resonator when the chip is configured as a cochlea.
Figure 6.33 shows the frequency response for resonators 18, 38 and 54 when the maximum Q value is set to a medium level. Only three resonators are shown to improve clarity. In Figure 6.33 the sharpness of the frequency response peaks is more obvious at higher frequencies than at lower frequencies. This is also the case in biology where we see sharper tuning at high frequencies and flatter tuning for the low frequencies (von Bekesy 1960).

The effect of increasing the maximum Q-value to higher values is shown in Figure 6.34. In this plot we have not only increased the maximum Q-value, but also reduced the frequency spacing of the resonators by reducing the voltage difference across the resistive line. The plot shows the output from resonators 11, 19, 26, 33, 44 and 53. Fragnière (Fragniere 1998), pp. 85-86, pp. 93-96) showed that it is necessary to reduce the resonant frequency spacing of the resonators when there is the possibility of high Q values to avoid instabilities. This effect is illustrated in Figure 6.35. Here we can see that when the Q-value is high for two adjacent resonators there is a region between them where there is no low impedance path to ground and this can result in standing waves. By overlapping the centre frequencies of two adjacent resonators a low impedance path is
established for all frequencies thus eliminating standing waves that lead to instabilities in
the cochlea output.

We found that in addition to improved stability, steeper frequency response curves
were obtained with higher gain when the frequency spacing of the resonators was
reduced. We attribute the fall in gain between the high frequencies at the basal end and
the low frequencies at the apical end of the cochlear resonator chain to the closeness in
the resonator spacing. In this case, as the input signal travels from the base to the apex of
the resonator chain, the signal is lost to adjacent, basal resonators since their resonant
frequencies must overlap to maintain stability as described above.

![Figure 6.34 Frequency Response of the 2D cochlea with high Q.](image)

![Figure 6.35 Illustrating the potential of a high impedance path for frequencies between the centre frequencies of two highly tuned resonators (top) and how this can be remedied by overlapping the resonator responses (bottom).](image)
The frequency and phase response of resonators 7, 22, 47 and 58 is shown in Figure 6.36 (a) and (b) respectively. Here the resistive line has been set to give a frequency range of 100 Hz to 10 kHz. From this plot it can be seen that the energy in the output of resonator 7 is much greater than the energy in the output of the other three resonators. We believe that this may be due to the fact that the signal is attenuated through the conductive network due to leakage in the pseudo-conductors and mismatch in the impedance matching circuit. Since resonator 7 is at the basal end of the cochlea the signal would be less attenuated. This may also constitute natural pre-emphasis for high-frequency signals which require more gain as they represent softer sounds.

![Figure 6.36 (a) The frequency response and (b) the phase response of resonators 7, 22, 47 and 58.](image)

The gain of the 2D cochlea with varying input intensity is shown in Figure 6.37. In Figure 6.37 we see the output from the 30th resonator for seven different input amplitudes spanning 40 dB when the chip is configured as an active 2D cochlea. While this particular configuration does not exhibit huge gain we can see that lower amplitude signals have higher gain and more selective response when compared to high amplitude input signals. In this case the lowest amplitude signal was 100 times smaller than the highest signal. It also shows that the resonant frequency shifts to the left (i.e. becomes lower) when the strength of the input signal increases. Noise at low frequencies has meant that the lower amplitude signals appear to have more gain at lower frequencies. We attempted to reduce this problem through averaging, however, given that the -46 dB signal is very close to the noise floor we were unable to remove all of the noise.
While the results in Figure 6.37 appear to indicate that the chip is working well, its shortcomings are revealed when we compare this data with that of biology. Figure 6.38 shows a comparison between actual biological basilar membrane measurements (Ruggero 1992) and data from Figure 6.37 on the same scale. Here we can see that while the shape of our output appears to be consistent with what we expect from the cochlea, the gain that we are able to achieve is severely limited when compared with biology.

In Figure 6.39 we show a comparison between the gain of resonator 7 and resonator 22. Here we see that there is slightly more gain for the basal resonator and more well defined frequency response curves than for resonator 22. This is at least partially due to the fact that the output of the 22nd resonator is more attenuated as can be seen in Figure 6.36 (a). Most biological data (as shown in Figure 6.38) shows frequency response data for high frequencies as it is easier to probe the cochlea basally than apically and the basal region of the cochlea is more sensitive. Our data from the 7th resonator has a similar shape to that in Figure 6.38 (with much less gain) while the data from the 22nd resonator
isn’t as well matched. It is therefore reasonable to compare data with physiological data when it is measured at similar frequencies while it is yet to be established whether the gain available at lower frequencies is the same as that at higher frequencies for the biological cochlea. Our chip suggests that there may be less gain at lower frequencies.

Figure 6.38 Comparison of data from biology (left) and our chip (from Figure 6.37) (right).

Figure 6.39 A comparison between the frequency response for varying gain between the 7th resonator (left) and the 22nd resonator (right).

The phase data for the 7th resonator is shown in Figure 6.40 for two different signal intensities (left). Here we see that the shape is very similar to physiological data from (Ruggero, Rich et al. 1997) (right). At the centre frequency the phase lag is larger for the -6 dB input than for -24 dB. Specifically the phase accumulation at the centre frequency is -0.467π and -0.4π at -6 dB and -24 dB respectively. In the biological data we can also see that the higher intensity input signals have a great delay than the low intensity signals.
The slope of the phase curve increases after the centre frequency until it plateaus in both the measured data and the biological data. The phase accumulation is much greater for the biological data than for the chip data. We believe that this is due to the fact that our cochlea was incorrectly terminated and as a result there are standing waves which interfere with the propagation of the travelling wave. For frequencies below the centre frequency the waveforms were below the noise floor and as a result there was some error in the measurement.

Figure 6.40 Basilar membrane phase as a function of frequency for (left) two different input intensities and (right) for input intensities 10 dBSPL to 100 dBSPL (data from (Ruggero, Rich et al. 1997))

Figure 6.41 shows the transient response of the 30th BM resonator to a sinusoid of amplitude 100 mV and frequency 600 Hz with the chip configured as an active 2D cochlea (note that the resistive line is tuned differently to how it was tuned in Figure 6.37 and Figure 6.38). In this case, the maximum Q-value has been set to a medium value as in Figure 6.33. In this figure we see that after the resonator has been reset at time zero the output level adapts in approximately 10 ms and then settles. Changing the settings of the AQC to allow for a higher Q results in the transient response shown in Figure 6.42. Here the output signal adapts in approximately 45 ms. This transient signal is on the edge of stability with the control loop having set the Q-value to its highest value without limit cycling. The FFT of the transient response shown in Figure 6.42 is given in Figure 6.43. Here we see a number of distortion products resulting from the fact that the output is tuned to be at the cusp of limit cycling.
Figure 6.41 Transient response of the 2D cochlea with mid Q.

Figure 6.44 shows the output response of the 7th resonator to a 3.6 kHz test tone in the presence of a 5 kHz suppressor tone of varying intensity (left) and physiological data showing two-tone suppression (Ruggero, Robles et al. 1992) (right). The x-axis shows the intensity of the 3.6 kHz test tone and the y-axis shows the output response. In Figure 6.44 we see that our chip is very gain limited when compared to the physiological data, the effect of the suppressor tone is correct, however, with the chip data showing a decrease in response with an increase in the intensity of the suppressor tone. The physiological data has been truncated for test tones of lesser intensity than the suppressor. We have not done this and as a result our results deviate somewhat for small test tones. Figure 6.45 shows two-tone suppression when the suppressor tone is at a lower frequency to the test tone. The effect of a lower frequency test tone is the same as for a higher test tone. This is also the case in biology.

Figure 6.42 Transient response of the 2D cochlea with high Q.
Figure 6.46 shows the frequency spectra at resonator 7 when two frequencies of equal intensity are input to the silicon cochlea (left) and physiological data showing combinational tones (Robles, Ruggero et al. 1997) (right). In both of these plots the two test tones are selected to be close to the characteristic frequency. We see that the odd-ordered distortion products are prominent in both cases.

![Figure 6.43 FFT of transient response from Figure 6.42.](image)

### 6.6 Discussion

The biological cochlea is believed to include two control loops (Camalet, Duke et al. 2000): a local and virtually instantaneous control loop which is attributed to the action of the OHCs, and a slower control loop which influences the OHCs’ response and operates from higher levels of the auditory pathway via efferent fibres. In our silicon cochlea design, the AQC loop is analogous to the biological local control loop. The transient response shown in Figure 6.41 and Figure 6.42 indicates that adaptation occurs quickly. There is a limit, however, to how quickly the Q-value can change and still maintain stability in the circuit. The slow biological control loop is not modelled by our silicon cochlea, but its effect would be similar to that of the control voltage, $V_{clip}$, which sets the maximum value that the Q-value can reach.
Figure 6.44 Two-tone suppression demonstrated in the chip (left) and biology (Ruggero, Robles et al. 1992) (right).

Figure 6.45 Illustrating two-tone suppression with a suppressor tone frequency less than the test tone. Chip data (left) and biological data (Ruggero, Robles et al. 1992) (right).

It is important to maintain the Q-value at the point before the resonator starts limit cycling as this is the point of maximum gain. There is a small range of Q-values for which the resonator will oscillate at the resonant frequency. Subsequent increases in the Q-value will result in oscillations of much lower frequency as the tau cell moves out of the log-domain region and into the square-domain region. This strongly impacts upon the operation of the cochlea which relies on a sequential decrease in resonant frequency from the basal to apical end of the resonator chain. If the Q-value is set uniformly across the BM resonator chain, then a high degree of matching in the resonator circuits is desirable. Otherwise, variations among the resonators means that the Q-value must be tuned down.
to avoid limit cycling in each and every resonator and this effectively reduces the gain available across the entire cochlea. As was illustrated in Figure 6.38 we are currently not able to obtain gains comparable to biology and this is partially due to the fact that our resonators could not be individually tuned in this implementation. Ideally, each resonator should be tuned individually, however, this requires a lot of space on chip and a large number of pins. It is more reasonable that independent AQC is applied across various sections of the BM resonator chain.

The need to discard the output of the first few resonators because they do not display high-frequency roll-off is wasteful in terms of chip resources, area, and power consumption. In biology there is likely a similar redundancy, however, a solution where the conductive network is terminated for high frequencies at its basal end might remedy the situation somewhat. No matter what the solution, it is certainly clear that the first few resonators do not require AQC.

Termination at the apical end of the cochlea is also important. In Figure 6.34 the resonator output curves have a bump between 200 Hz and 300 Hz. This bump or second peak (see Figure 6.32) is possibly due to constructively interfering standing waves which may result due the lack of a low impedance path to signal ground at these frequencies. Interfering standing waves may also have contributed to a reduction in phase accumulation at resonance (see Figure 6.40). Particular care must be taken to ensure correct termination as standing waves will degrade the performance of the cochlea. It is

Figure 6.46 Frequency spectra at a place along the basilar membrane show odd-order distortion products from the silicon cochlea (left) and a chinchilla cochlea (Robles, Ruggero et al. 1997) (right).
believed, however, that biological phenomena such as otoacoustic emissions utilise the ability of the cochlea to facilitate bidirectional and standing waves (Kemp 1986).

6.7 Conclusions

In this chapter we have presented our first attempt at an active 2D silicon cochlea that incorporates some of the nonlinearities present in the biological cochlea. We have discussed the implementation of our active 2D model from Chapter 5 used to create our cochlea, the circuits used to realise this model and shown results from a fabricated integrated circuit that implements this model. Our results indicated that our 2D silicon cochlea exhibits many of the nonlinear behaviours of the biological cochlea including two-tone suppression, combinational tones, and nonlinear gain. By building this model we have also discovered a number of possible improvements to our model and the circuits used in its implementation. Future iterations of this design focus on improving termination and tunability of the AQC in individual or grouped resonators.

6.8 References


Chapter 7.  A 2D Silicon Cochlea with Improved AQC

7.1 Introduction

In this chapter we present an improved version of the silicon cochlea with AQC presented in Chapter 6. The first attempt at the active 2D cochlea with AQC exhibits many of the characteristics of a biological cochlea; however, there were a number of issues including matching, gain, and errors in circuit design and/or layout. One of the most important differences between the implementation presented in this chapter and that in Chapter 6 is that in this version we include only a single set point in the control-loop rather than the ceiling and threshold that were imposed in the first chip. This allows us to tune the BM resonators close to a Hopf bifurcation, allowing us to obtain, theoretically, higher gain and selectivity.

We will first discuss the reasoning behind our improvements to the AQC. This will be followed by a discussion of the circuits used in this implementation and changes to circuits from Chapter 6 used in this design. We will then present results from a fabricated integrated circuit that implements these circuits. We will conclude with a discussion on the performance of this chip.

7.2 Improved AQC

The AQC operation in Chapter 6 is similar to that found in cochlear implants and hearing aids where the loudness of the output signal is set to be within a comfortable range (Target Amplitude in Chapter 6 Figure 6.3) for the user. While this is acceptable for a hearing prosthesis it does not allow us to easily set the AQC loop at a Hopf bifurcation point as the amplitude of the output from the BM resonators does not increase once it is above the threshold level. Hence, we are unable to obtain, to the same extent, the gain and frequency selectivity that can be obtained by the biological cochlea. It also appears that this solution is not biologically plausible. The nonlinear, active effects of the OHCs do not have much influence on the operation of the cochlea until the pressure-wave input signal is below some set point. This would suggest that the mechanical response of the basilar membrane inhibits the action of the OHCs at amplitudes above the
set point (Kemp 1986). The set point can be altered by higher auditory centres by way of efferent fibres allowing hearing to adapt to both noisy and quiet listening environments (Camalet, Duke et al. 2000). We have improved our AQC model so that it matches biology more closely by removing the threshold and ceiling levels from AQC operation and replacing them with a single set-point level that is adjustable.

Figure 7.1 The improved automatic quality factor control-loop.

Figure 7.1 shows the block diagram of the improved AQC. The second-order band-pass filter (BM resonator), peak detector and wide-linear-range (WLR) amplifier are almost identical to the circuits presented in Chapter 6 (any differences are discussed in section 7.3). Here we have removed the decision circuit and ramp generator and replaced them with a voltage-controlled current source (VCCS). The arrows in Figure 7.1 point in the direction of current flow in the circuit.

The operation of this improved control-loop will now be described. A current (with DC bias and AC signal) is input to the second-order band-pass filter. The output current of the band-pass filter is fed into a peak detector whose output represents the peak current, $I_{PD}$ (see Figure 7.2). This current is compared with the current, $I_S$, set by the VCCS simply by connecting them at the voltage node, $V_{COMP}$. Differences in $I_{PD}$ and $I_S$ result in a change in the voltage, $V_{COMP}$, through the charging and discharging of the parasitic capacitor, $C_{PAR}$. This voltage is input to the WLR amplifier whose output is a current, $I_Q$, which is based on the value of $V_{COMP}$ compared to some reference voltage, $V_{REF}$. When
$V_{COMP}$ increases above $V_{REF}$ the output current of the WLR amplifier in Figure 7.1, $I_Q$, will also increase. At values of $V_{COMP}$ less than or equal to $V_{REF}$, $I_Q$ will simply be a leakage current. This is because the output of the WLR amplifier is made unidirectional by connecting a current mirror to its bidirectional output. Table 7.1 summarises the modes of operation of the AQC loop.

![Figure 7.2 Current Comparison in the Improved AQC.](image)

Table 7.1

<table>
<thead>
<tr>
<th>Condition</th>
<th>$V_{COMP}$</th>
<th>$I_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_S &lt; I_{PD}$</td>
<td>$\downarrow$</td>
<td>$\downarrow^{1}$</td>
</tr>
<tr>
<td>$I_S &gt; I_{PD}$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$I_S \approx I_{PD}$</td>
<td>Constant</td>
<td>Constant</td>
</tr>
</tbody>
</table>

The Q-value in the second order band-pass filter is governed by the equation,

$$Q = \frac{I_0}{2I_0 - I_Q}$$  \hspace{1cm} (7.1),

\footnote{$I_Q$ stays constant if $V_{COMP}$ falls below $V_{REF}$}
where $I_0$ is a constant current and $I_Q$ is the output current from the WLR amplifier in Figure 7.1. Ideally, the Q-value is infinite and therefore the output of the band-pass filter is at a Hopf bifurcation, when $I_Q = 2I_0$. Hence, we set the bias current of the WLR amplifier to be $2I_0$ (or slightly higher) so that as $V_{COMP}$ approaches its maximum, $V_{DD}$, the Q-value approaches infinity and the Hopf bifurcation.

This control-loop differs to that presented in Chapter 6 by the fact that the Q-value of the second-order band-pass filter will continue to increase as long as $I_S > I_{PD}$. Hence, the setting of the current, $I_S$, will determine if and when the circuit operates on a Hopf bifurcation.

In this implementation of AQC the voltage that controls the VCCS is set off chip and is therefore simply a bias current source. In future implementations, however, the signals from higher auditory centres will be included on-chip and these signals will control the set-point voltages of the VCCSs. The VCCS is constructed using a WLR amplifier similar to the way we create $I_Q$ in Figure 7.1. The output of the WLR amplifier must be unidirectional so that $C_{PAR}$ can only discharge via $I_{PD}$ (see Figure 7.2). Figure 7.3 shows our VCCS circuit. Here $V_S$ is our set-point voltage, $V_R$ is a reference voltage, $I_{WLR}$ is the bidirectional output current from the WLR amplifier and $I_S$ is our current source. The current, $I_{WLR}$, is made unidirectional by forcing it through a PMOS current mirror. Hence, the equation governing the VCCS is,

$$I_S = \max(I_{WLR}, 0) = \max(A(V_S - V_R), 0)$$

(7.2),

where $A$ is the transconductance of the WLR amplifier.
7.3 Circuit Implementation

In this section we discuss the circuits used in this implementation of the active 2D cochlea with AQC that differ from those presented in Chapter 6.

7.3.1 Second-order tau cell band-pass filter

In Chapter 6 Section 6.5.1 we showed that the addition of a DC offset and subsequent mirroring of the output current from the second-order tau cell band-pass filter was a major source of mismatch in our circuit. Here we have removed the need for adding a DC offset by copying the output stage of the band-pass filter, via transistors M8 and M17, so that one output can be connected to the output circuit and the other can connect to the peak detector. This is illustrated in Figure 7.4.

As well as improving matching in the circuit by removing the need to mirror the output current several times, it also has improved the operation of the peak detector as the output current from the band-pass filter that connects to this, ideally, has no offset and hence no subtraction of the offset is required.

7.3.2 Set-point Implementation

In the first implementation of AQC a single voltage, $V_{\text{clip}}$, was used to set both the ceiling and the threshold levels for all of the resonators on the chip (see Chapter 6 Section 6.4). We found that this resulted in an overall reduction in gain as some resonators were becoming unstable before others. In this implementation we have only 12 resonators and we were therefore able to give individual set-point control to the majority of these circuits. The circuit in Figure 7.3 (with cascodes) is used to allow individual set-point control. Specifically, resonators 1 and 2 share a single set-point control VCCS and resonators 10, 11 and 12 share a single set-point control VCCS, while resonators 3 to 9 all have individual set-point control VCCSs from separate pins allowing the set points to be set off-chip.
7.3.3 The resonator biasing circuit

In this implementation we were very limited in area and as a result made some changes in order to lower the amount of space each resonator took up on the chip. In the resonator biasing circuit in Chapter 6 we used three CLBTs to create the biasing for the second-order band-pass filters. Figure 7.5 shows the new version of the resonator bias circuit. Here we only use 2 CLBTs instead of 3 and the value of \( I_q = AI_0 \) is set while the value of \( I_0 \) is variable. \( I_0 \) is set by a voltage, \( V_{var} \), that is connected to the emitter of the CLBT that biases it.
Another change in this circuit is the value of the resistance in each section of the resistive line. In the implementation in Chapter 6 each section had a resistance of 50 kΩ. We felt that this was too big and simulations suggested that the leakage current in the base of the CLBTs was comparable to the current through the resistors and as a result there was some mismatch in the voltages that set the biasing currents for each resonator. In this implementation we have used smaller resistors of value 1kΩ for each section.

![Impedance Matching Circuit](image)

**Figure 7.6 The impedance matching circuit used in Chapter 6.**

### 7.3.4 Impedance matching circuit

One of the biggest issues with the impedance matching circuit that we used in Chapter 6 was cancelling the DC offset current. Figure 7.6 shows the impedance matching circuit from Chapter 6. It includes the DC offsets for each AC current to illustrate how they should cancel.

From Figure 7.6 we see that, neglecting mismatch, all the DC currents should cancel before the conductive network and the pseudo-conductance, $G^*$, at node $V_{BM}$. If the voltage at $V_{BM}$ is greater than $V_{IN}$, however, then an unknown DC current, $I_{DC2}$, will conduct through $G^*$. This current is not guaranteed to cancel, however, as changes in the voltages at nodes $V_{BM}$ and $V_{IN}$ will change the current drawn through $G^*$. The need to cancel both $I_{DC}$ and $I_{DC2}$ increases the potential for mismatch in the circuit. A simplified version of this circuit is shown in Figure 7.7.
In Figure 7.7 we see that there are no DC current sources. Instead the DC level that is input to the resonator is determined by the system itself, i.e. it is dependent on the value of $V_{BM}$ and $V_{IN}$. The DC current source in Figure 7.6 was included to provide an offset for the second-order band-pass filter. It can be removed because the DC current drawn through $G^*$ will provide the DC offset for the second-order band-pass filter. From our analysis in Chapter 4 Section 4.5 we know that both strong and weak inversion currents can be used as the DC offset current to the tau cell. By removing the fixed DC current source the matching in the circuit is greatly improved to that in Figure 7.6.

### 7.3.5 The terminator circuit

The terminator circuit used in Chapter 6 was incorrectly laid out on chip. Extensive simulations suggested that the value of the DC voltage at the termination node, $V_{term_{end}}$, should equal the DC voltage at the input to the conductive network which is approximately 3V (i.e. $V_{term_{end}}$ should be an AC ground). Unfortunately, in terminating the AC signal DC voltage is dropped across the pseudo-conductance, $G^*$, and as a result the voltage at node, $V_{term_{end}}$, tends to be much lower than the voltage at the input. Figure 7.8 shows our improved termination circuit.
In Figure 7.8 it can be seen that we use an averaging circuit to obtain the DC current that flows through $G^*$, $I_{DC}$, and add this current back to the node, $V_{term\_end}$. In an added precaution we connected $V_{term\_end}$ to a pad so that it can be probed and/or set off-chip.

### 7.4 Physical Implementation

An integrated circuit was designed and fabricated using MOSIS AMI 1.6µm on a 5 mm$^2$ Tiny Chip. This IC includes 12 BM resonators with improved AQC, a bias generator (Delbrück and van Schaik 2005), a scanner, and 9 VCCSs for set-point control. Specifically, we connected individual set-point control to BM resonators 3 – 9, BM resonators 1 and 2 were connected to the same set-point control and BM resonators 10 – 12 were connected to the same set-point control. Ideally, each BM resonator should have its own set-point control, however, we were constrained by the number of pins available to us on the Tiny Chip.
A summary of the performance characteristics of the chip is given in Table 7.2. Note that the noise is given at the output and not referred to the input as the gain of each resonator changes due to AQC. Each BM resonator with improved AQC takes up an area of 150 μm by 910 μm. The layout of the chip is given in Figure 7.9.

![Figure 7.9](image_url) The active 2D cochlea with improved AQC chip layout.

| Table 7.2 |
|-----------------|-----------------|
| Technology      | MOSIS AMI 1.6μm CMOS |
| Die size        | 5mm²             |
| Dynamic Range   | 50 dB (10mVpp – 3.2Vpp) |
| Noise           | -24 dB (22 mV RMS) |
| Power Consumption | 55 μW @ 5 V    |
7.5 Results

7.5.1 Operation of the individual BM Resonators

The operation of the individual BM resonators was tested by bypassing the conductive network modelling the cochlear fluid and injecting the input signal directly into the BM resonator selected via the scanner. To test the matching of the resonators the resistive, polysilicon line was set with $line_{in} = 4.44$ V and $line_{out} = 4.54$ V. The results from injecting a 400 mV amplitude signal, stepping the input frequency to the resonators (using a step size of 100 Hz), and measuring the output amplitude at each frequency step for each of the 12 resonators is shown in Figure 7.10.

![Figure 7.10 Frequency response of BM resonators 1 to 12 with line_in = 4.44 V and line_out = 4.54 V.](image)

Figure 7.10 shows that there is substantial variation in the output amplitude of the resonators. A statistical analysis of the output amplitude of the 12 resonators is given in Table 7.3. This analysis and inspection of Figure 7.10 indicates that the spread of amplitudes is dominated by 3 outliers while the remaining 9 resonators are within 4 dB of
each other. These 3 outliers (resonators 1, 6 and 10) appear to be more highly tuned indicating that they have a higher Q-value. An inspection of these resonators’ bandwidths supports this conclusion. Hence, we can conclude that the majority of the mismatch is a result of mismatch in the current which controls the Q-value, \( I_Q \).

Table 7.3

<table>
<thead>
<tr>
<th></th>
<th>Output Amplitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Difference</td>
<td>11.5</td>
</tr>
<tr>
<td>Average</td>
<td>-15.04</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Figure 7.11 shows resonator tap number versus frequency. Here we see that the resonant frequency varies with tap number approximately logarithmically. This is the desired result and as such we can say that our frequency tuning matches closely with ideal.

Figure 7.11 Tap number versus resonant frequency.
By decreasing the frequency spacing between each resonator and reducing the maximum frequency we are able to achieve better matching in the output amplitude of the resonators. Figure 7.12 shows the frequency response of 11 resonators (the 12th resonator’s response was very noisy and as such was removed from the plot) when the voltages on either end of the polysilicon line controlling the resonator bias currents are changed so that: $\text{line}_\text{in} = 4.49$ V and $\text{line}_\text{out} = 4.74$ V. Here we see that resonators 1 – 8 have output amplitudes matched within 4 dB while the output amplitudes of resonators 9 – 12 are significantly lower. We believe that this is a result of the resonator biasing circuit that we have implemented on this chip. This can be explained as follows: as $\text{line}_\text{out}$ increases the lowest resonant frequency on the chip decreases and the bias current that sets the resonant frequency, $I_0$, decreases (see Figure 7.5). The bias current falls since the voltage on the base of the PNP CLBT increases. An increase in the base voltage restricts the amount by which the emitter voltage, held at $V_{\text{var}}$, can fall before the
transistor moves into the cut-off region. This in turn reduces the maximum Q-value that
can be obtained since varying $V_{var}$ essentially changes the Q-value.

Figure 7.13 (a) and (b) shows the result of setting each end of the resistive,
polysilicon line to the same voltage with each resonator being set to have the same
resonant frequency. In both cases $line_{in} = line_{out} = 4.6$ V. In Figure 7.13 (a) we see
that the maximum difference in gain is 10 dB and there is a slight variation in resonant
frequency. Figure 7.13 (b) shows the result of increasing the Q-value by reducing the
value of $V_{var}$. Reducing $V_{var}$ also results in a downward shift in the resonant frequency
(as can be seen when the resonant frequencies of Figure 7.13 (a) and (b) are compared).
Here we see that the maximum difference in gain is just below 20 dB. We can also see
that there is somewhat greater variation in the resonant frequency. Table 7.4 presents a
statistical analysis of the data from Figure 7.13 (a) and (b).

![Figure 7.13 The frequency response of the BM resonators tuned to have the same resonant frequency (a) for low Q (b) for higher Q.](image)

<table>
<thead>
<tr>
<th></th>
<th>Data from Figure 7.13 (a) (low Q)</th>
<th>Data from Figure 7.13 (b) (higher Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output (dBV)</td>
<td>8.5</td>
<td>16.8</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>Average</td>
<td>-21.92</td>
<td>-10.6</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>3783</td>
<td>2892</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.42</td>
<td>5.57</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>259</td>
<td>297</td>
</tr>
</tbody>
</table>
The results thus far indicate that mismatch in gain and resonant frequency is strongly related to the setting of the Q-value. Clearly some resonators are more highly tuned than others and this results in large differences in output amplitude and bandwidth as the Q-value is varied. The voltage, $V_{\text{var}}$, exerts the greatest influence over the Q-value. Specifically (with reference to Figure 7.5) the current that sets the Q-value, $I_Q = A_1 I_0$ is given by:

$$ I_Q = I_s e^{V_{EB}/U_T} = I_s e^{(V_{DD} - V_{\text{line_out}})/U_T} \quad (7.3), $$

where $I_s$ is the saturation current and $U_T$ is the thermal voltage. The equation governing the filter bias current, $I_0$, is given by:

$$ I_0 = I_s e^{V_{EB}/U_T} = I_s e^{(V_{\text{var}} - V_{\text{line_out}})/U_T} \quad (7.4). $$
If we wish to tune the resonator to the cusp of instability (the Hopf bifurcation) we set $I_Q = 2I_0$. Dividing (7) by (8) and solving for $V_{var}$ we find:

$$2 = e^{(V_{DD} - V_{var})/U_T}$$

$$V_{DD} - V_{var} = U_T \log e 2$$

$$V_{var} = V_{DD} - U_T \log e 2$$

(7.5).

If we assume $V_{DD} = 5V$ and $U_T = 25 \text{ mV}$ (at room temperature) we find that $V_{var}$ must be tuned to 4.983 V. This is most likely the greatest source of mismatch in resonator tuning as very small differences in $V_{DD}$ and $V_{var}$ can result in large changes in the Q-value.

Another issue with this biasing method is that the resonant frequency changes as $V_{var}$ varies. This makes it difficult to appropriately tune all of the resonators. We also find that if $(V_{var} - V_{line\_out})$ is below $V_{EB\text{ (on)}}$ then the transistor is in the cut-off region which limits the Q-value especially at low frequencies.

Figure 7.15 Frequency response of the 3rd resonator when the set point, $V_s$, is changed.
We can avoid some of the mismatch problems resulting from the setting of $V_{var}$ by enabling the AQC. Figure 7.14 shows the output of the 12 BM resonators with AQC turned on and the set point set very low. From this we can see that we have a highly damped response, however, the matching in output amplitude between the resonators is very good; the maximum difference is only about 3 dB.

The effect of changing the set point, $V_s$, (see Figure 7.3) when AQC is enabled is shown in Figure 7.15. Here we see that low set points result in a damped response as the maximum attainable Q-value is very low. High set points allow for high Q-values and as a result we see highly selective output responses.

Figure 7.16 Frequency response of BM resonators 3 - 9 when the set point is increased to allow higher Q-values.

Figure 7.16 shows the frequency response of resonators 3 – 9 when we adjust the set point to allow for higher Q-values and reduce the input amplitude from 400 mV to 100 mV. We are only interested in resonators 3 – 9 since they have their own set-point control. The set points of 1, 2, 10 – 12 cannot be individually tuned, as we already know
that resonators 1 and 10 are more highly tuned, they can introduce mismatch which affects the operation of the 2D cochlea (see 7.5.2). Hence we set these resonators to have a damped response.

Figure 7.16 shows that there is good matching in the output amplitude of resonators 3, 4, 5, 8, and 9. The Q-values of resonators measured at 10 dB below the peak amplitude ($Q_{10\text{dB}}$) is shown in Table 7.5. Here we see that resonator 6 is more highly tuned and resonator 7 is highly damped. Despite being able to adjust the set point it is not capable of fine-tuning and as a result we are unable to improve the tuning for resonator 6 and 7. This is because the VCCS that controls the set point allows only coarse tuning since the comparison node, $V_{\text{COMP}}$ (see Figure 7.2), is a very-high impedance node whose voltage is dependent on the value of a parasitic capacitance. Thus, when tuning through the Hopf bifurcation (which is inherently unstable) we can either set the output to be highly tuned or strongly damped with very little variation in-between. Therefore, even with AQC enabled we are subject to problems arising from the mismatch introduced through the setting of $V_{\text{var}}$. Table 7.5 shows that apart from the outliers, resonators 6 and 7, the $Q_{10\text{dB}}$ values decrease with frequency. This reinforces our observation that higher Qs are difficult to obtain at low frequencies given the biasing of $I_Q$.

<table>
<thead>
<tr>
<th>Resonator</th>
<th>$Q_{10\text{dB}}$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
</tr>
<tr>
<td>6</td>
<td>1.08</td>
</tr>
<tr>
<td>7</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>0.67</td>
</tr>
<tr>
<td>9</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Figure 7.17 (a) Frequency versus gain for the 8th BM resonator with AQC enabled (b) Illustrates large signal compression when AQC is enabled.

Figure 7.18 Frequency response of the 3rd resonator with AQC enabled and varying input amplitude.

Figure 7.17 shows nonlinear amplification in the 8th BM resonator. Here we have kept the tuning the same as for the results in Figure 7.16 and measured the frequency response for varying input amplitudes. The results have been normalised at 0 dB for
clarity. Figure 7.17 (a) shows frequency versus gain. Here we see that the gain and frequency selectivity, hence the Q-value, is greatest with the smallest input and lowest with the largest input. Figure 7.17 (b) shows input versus output at the resonant frequency (approximately 320 Hz) for the curves in Figure 7.17 (a). This plot also shows that lower input signals result in larger gain. As the input signal gets larger the output signal compresses (see Figure 7.17 (b) between input values -24 dB and -12 dB) after this point the Q-value of the second-order tau cell band-pass filter cannot be reduced further and the output increases linearly with the input.

![Figure 7.19 Transient response of the 3rd resonator.](image)

In testing the nonlinear, active amplification of the individual BM resonators we found that the reconstruction of the frequency response curves was not always smooth. Consider Figure 7.18, here we see that the gain is attenuated/distorted at the resonant frequency for the curves representing inputs of -38 dB, -32 dB and -26 dB. This is due to coarse tuning of the AQC. In these cases the peak amplitude of the current from the output of the second-order band-pass filter, $I_{PD}$ (see Figure 7.2), is close to the set point current, $I_S$, and as a result the voltage on the comparison node, $V_{COMP}$, falls and the Q-value is reduced. At frequencies just above and below the resonant frequency, however,
the difference between the peak current and the set-point current is less due to attenuation through the filter and as a result the Q-value does not fall to the same extent as it does at the resonant frequency. To remedy this, the tuning of the AQC would need to be improved to allow for fine tuning. This can be accomplished by putting a larger capacitor (rather than rely on a parasitic capacitance) at the comparison node, \( V_{\text{COMP}} \) (see Figure 7.2). This would increase the area that each resonator takes up on chip.

![Graph](image)

Figure 7.20 Transient response of the 3\(^{rd}\) resonator for a high input signal.

Figure 7.19 shows the transient response for the 3\(^{rd}\) resonator. Here the output of the resonator is reset at time 0 by pulling the comparison node, \( V_{\text{COMP}} \) (see Figure 7.2), to ground. The input to the resonator is a 2.5 kHz sinusoid with an amplitude of 25 mV (peak-to-peak). In this plot we can see that it takes approximately 4 ms to reach a steady output (note that there is some noise in the output). In this case the Q-value has been set high and hence, 4 ms is close to the maximum response time of the AQC. Figure 7.20 shows the output of the 3\(^{rd}\) resonator when the input amplitude is increased to 2.5 V (peak-to-peak) and the input frequency is 2.5 kHz. Here we see there is initially some overshoot in the output, however, it settles in less than 1 ms which is consistent with what we expect given that larger input signals result in small Q-values and therefore shorter time constants.
7.5.2 Operation of the 2D Cochlea

Figure 7.21 shows the frequency response of resonators 3, 4, 5, 6, 7, 9, 11, and 12 (from right to left) when AQC is disabled (i.e. the passive cochlea response). Here we see that the resonant frequency decreases from the basal (high frequencies) end of the silicon cochlea to the apical (low frequencies) end of the cochlea. We can also see that the gain of the cochlea, which corresponds in our model to BM velocity, is fairly constant except for resonators 3 and 6 which appear to be more highly tuned. In Figure 7.21 we have discarded the output of the 1\textsuperscript{st} and 2\textsuperscript{nd} resonator as they do not exhibit the characteristic cochlea frequency responses.
Figure 7.22 Frequency response of resonators 3, 4, 5, 8 and 9 with AQC enabled.

Figure 7.22 shows the frequency response of resonators 3, 4, 5, 8 and 9 when AQC is enabled. The cochlea is tuned as it was in Figure 7.16 and as such resonator 6 was omitted from this figure as it was highly tuned/unstable and resonator 7 was omitted as it was highly damped. The effect of resonator 6 on the other resonators can be seen at ~600 Hz. Here we see a second peak or bump in the frequency response curves due to coupling through the conductive network from resonator 6 which is oscillating at ~600 Hz. Changing the conductivity of the conductive network can alleviate this effect somewhat, however, as there is only a global control for the conductances, the output amplitude is strongly effected. Figure 7.23 shows resonator 4 when the conductances are varied. Here we see that, contrary to intuition, coupling increases as conductance decreases i.e. the voltage that controls the conductance, $G^*$, increases. We can see that coupling has increased when $G^* = 2.03$ V as the slope of the frequency response curve at high frequencies has increased relative to the other two curves and the effect of the highly tuned resonator 6 can be seen more prominently as a second peak. Figure 7.23 also
indicates that varying $G^*$ alters the tuning in the silicon cochlea. Specifically, as $G^*$ increases the frequency tuning in the cochlea shifts to the right i.e. it increases.

Figure 7.23 Frequency response of resonator 4 varying the magnitude of the conductive network.

Since the majority of the resonators on the chip can be individually tuned we found that the shape, resonant frequency and gain of individual resonator circuits is highly dependent on the tuning of their neighbours. Figure 7.24 shows the frequency response for resonator 4 when the tuning of adjacent resonators is varied. Here we see that when only resonator 4 is tuned at (or close to) the Hopf bifurcation (blue curve), it does not have, to the same extent, the characteristic steep slope at high frequencies after resonance that is exhibited when resonator 3 is also tuned to the Hopf bifurcation (red curve). We can also see that the resonant frequency shifts upwards when basal resonators are tuned to the Hopf bifurcation. In Figure 7.24 the resonant frequency shifts from 1.2 kHz to 1.3 kHz when resonator 3 is tuned to the Hopf bifurcation. When we tune apical resonators to the Hopf bifurcation (in this case resonator 5) we see that the steepness of the frequency response after resonance increases (green curve). This curve shows a bump/second peak.
at ~700 Hz that is not present in the other two curves, however, which may indicate that an apical resonator(s) has become unstable and is oscillating.

![Graph showing frequency response of resonator 4 with different tuning for adjacent resonators.](image1)

**Figure 7.24** Frequency response of resonator 4 with different tuning for adjacent resonators.

![Graph showing frequency response of resonator 3 with varying input amplitude and the improved AQC-loop enabled left and Laser velocimetric data from the cochlea of a living chinchilla (Ruggero 1992) right.](image2)

**Figure 7.25** Frequency response of resonator 3 with varying input amplitude and the improved AQC-loop enabled (left) and Laser velocimetric data from the cochlea of a living chinchilla (Ruggero 1992) (right)

The improved AQC-loop was tested by varying the input voltage to the cochlea between -26 dB and -2 dB (relative to 1 V) and measuring the frequency response for each of these input amplitudes. Figure 7.25 (left) shows the frequency response of resonator 3
when the improved AQC-loop is enabled. Here we see that for smaller input amplitudes the resonator has better selectivity (higher Q-value) than for large input amplitudes. When we compare this with biological measurements we see that our model gives comparable results for low Q values. Specifically our cochlea compresses an input range of 24 dB into approximately 13 dB which is similar to the compression shown in Figure 7.25 (right) excluding small input signals. Figure 7.25 (right) shows laser velocimetric data from the cochlea of a living chinchilla (Ruggero 1992). In this figure each frequency response curve is shown with the value of the input amplitude (in dB) that it resulted from.

When we compare Figure 7.25 left and right we see that both plots show higher Q-values for lower input amplitudes and both show that the resonant frequency shifts towards higher frequencies when the Q-value increases.

![Figure 7.26](image) A comparison in gain between the chip (left) and biology (Ruggero 1992) (right).

The frequency response curve corresponding to the -2 dB input in Figure 7.25 (left) shows how, for high input amplitudes, the response from the cochlea becomes passive, similar to the response of a highly damped band-pass filter. We can also see large-signal compression in this figure where the amplitude of the frequency response curve corresponding to the -26 dB input signal is only 13 dB below the amplitude of the frequency response curve corresponding to the -2 dB input signal.
Figure 7.26 shows gain versus frequency for both the chip data and the biological data in Figure 7.25. Here we see that, as with the data in Chapter 6, the real cochlea has significantly more gain than our cochlea. However, we note that the large signal compression is similar over the input range of approximately 30 dB which corresponds to an output gain range of 20 dB. Hence, we are limited by our input dynamic range which does not currently allow for small input signals that would theoretically result in gains equivalent to those from biology. The minimum input signal that we can currently present to the silicon cochlea without being lost in noise is 25 mV (peak-to-peak). To obtain results which match better with biology we would need to decrease this by several orders of magnitude and hence reduce the noise floor by the same amount.

In Figure 7.27 (a) the gain data for resonator 5 is shown when it is tuned at the Hopf bifurcation. Here we see that the compression is much larger than biology. Specifically we have compressed a 36 dB range into 8 dB. By doing this, differences in phase data for different input amplitudes are made clear. Figure 7.27 (b) shows that the phase accumulation for -26 dB, -8 dB and 10 dB is $\pi/10$, $\pi/5$ and $\pi/3$ respectively. Thus, this is consistent with data from the biological cochlea which shows that the phase lag at the resonant frequency is greater for higher intensity input signals.

Figure 7.27 (a) Frequency response for varying input to resonator 5 (b) Corresponding phase data.
Figure 7.28 Two-tone suppression demonstrated by (a) the 5th resonator tuned to a resonant frequency of 1.3 kHz in the presence of a 1.8 kHz suppressor tone, and (b) biology (Ruggero, Robles et al. 1992)

Figure 7.29 Two-tone suppression demonstrated by (a) the 5th resonator tuned to a resonant frequency of 1.3 kHz in the presence of a 900 Hz suppressor tone, and (b) biology (Ruggero, Robles et al. 1992)

As with the silicon cochlea presented in Chapter 6 the improved silicon cochlea exhibits nonlinear phenomena such as two-tone suppression and the generation of combinational tones. Figure 7.28 (a) shows the output of the 5th resonator when varying the input amplitude at the resonant frequency (1.3 kHz) in the presence of a suppressor tone at 1.8 kHz. When compared with Figure 7.28 (b), which shows biological data, we see that our silicon cochlea has a comparable shape with less gain. It should be noted, however, that the biological response is taken at a higher frequency which may explain the higher gain. Figure 7.29 (a) shows two-tone suppression when using a suppressor
tone with a lower frequency (900 Hz) than the resonant frequency (1.3 kHz). The biological data in Figure 7.29 (b) shows less gain than Figure 7.28 (b). Thus, when compared with this data our results show higher gain. Given the biological data it appears that gain is dependent on frequency and as such it is more useful to compare the shape and behaviour of the output of our silicon cochlea with the biological data.

![Figure 7.30 FFT of the output of resonator 5 showing distortion products.](image)

**Figure 7.30** FFT of the output of resonator 5 showing distortion products.

Figure 7.30 shows an FFT of the output of resonator 5 when it is tuned at a resonant frequency of 2.5 kHz (CF) and frequencies of 2.9 kHz \( (f_1) \) and 3.3 kHz \( (f_2) \) are applied to the input of the cochlea. These two frequencies are selected such that \( 2f_1 - f_2 = CF \). In Figure 7.30 we see that in response to these two frequencies a number of distortion products are created at the output of the cochlea and hence, this cochlea generates combinational tones.
7.6 Discussion

An issue with the AQC loop solution is that the current comparison voltage node, $V_{COMP}$, is a high impedance node which makes it susceptible to noise and allows only coarse tuning of the resonators’ Q-value. Since the parasitic capacitance on this node, $C_{PAR}$, is small and process-dependent, tuning the AQC through the Hopf bifurcation is not always smooth as the parasitic capacitance charges and discharges almost instantaneously depending on the difference between the set-point current and the peak detector current. This problem can be alleviated by adding a much larger capacitance to this node. This, however, will increase the size of the BM resonators substantially. Whether this is feasible or not depends on the fabrication process used and the desired size of the IC.

![Frequency response of resonators 3, 4, 5, 8 and 9 showing coupling from defective resonators 6 and 7](image)

Figure 7.31 Frequency response of resonators 3, 4, 5, 8 and 9 showing coupling from defective resonators 6 and 7

As with the previous implementation in Chapter 6 this implementation suffered from a small input dynamic range. This was due to a number of reasons. Firstly the noise performance of the front-end was not optimised. Secondly, and perhaps more
importantly, we did not try to compensate for offsets in the resonator circuits. In particular the peak detection circuit is susceptible to offsets and its performance at low current levels is poor. Future implementations of the 2D silicon cochlea with AQC will need better noise performance and they will need to employ a current peak detector circuit which has a larger dynamic range.

One of the advantages of the 2D silicon cochlea over the 1D silicon cochlea is that the cochlea is still operational even if a resonator stage is broken or incorrectly tuned. We have found that this is true up to a point. Certainly we still obtain very good cochlea curves and nonlinear behaviour when one or more resonators are defective, however, these resonators appear to introduce distortion products and instabilities into the output of the operational resonators. This can be seen in Figure 7.31 where instabilities from resonators 6 and 7 can be seen coupled into resonators 3, 4, 5, 8 and 9. In Figure 7.23 it was shown that changing the conductance of the resistive network can reduce the coupling between resonators, however, it was also shown that this impacts upon tuning and cochlea performance. In future implementations of the 2D silicon cochlea it may be desirable to have control over the conductance, $G^*$, for each resonator or the ability to disconnect defective resonators to eliminate undesirable coupling without impacting upon desirable coupling.

![Figure 7.32 Innervation of afferent (to the brain) and efferent (from the brain) fibres in the human cochlea. From (van Schaik 1998) adapted from (Watts 1992).](image-url)
In Chapter 6 we did not have individual set-point control on the chip and as a result we were unable to assess the effects of different tuning on neighbouring resonators. In this improved version of the 2D silicon cochlea with AQC, however, we have been able to study the effects of coupling between adjacent resonators. In Figure 7.24 we show that cochlea curves with the characteristic steep slope after resonance are obtained when resonators basal to the resonator of interest are tuned at or close to the Hopf bifurcation. We also show that when resonators apical to the resonator of interest are tuned at or close to the Hopf bifurcation a number of distortion products and instabilities are introduced into the resonator’s output. This suggests that if a high quality factor is desired for a particular frequency (frequency band) then resonators (in biology the OHCs) basal to the resonator which represents this frequency should be tuned towards the Hopf bifurcation. The innervation of the afferent (towards the brain) fibres in the biological cochlea also suggest a dependence on the tuning of basal OHCs. In Figure 7.32 we see that after penetrating the spiral lamina (habenula perforata) the afferent fibres travel approximately 0.6 mm basal-ward before connecting to approximately ten OHCs.

Figure 7.33 Frequency response of resonator 5 when the tuning of nearby resonators is varied.
Thus, from Figure 7.32 we can see that the tuning of basal resonators is important in determining the tuning of more apical resonators. The architecture of the cochlea suggests that the tuning of 10-or-so OHCs is of particular importance. We were unable to determine the number of resonators basal to the resonator of interest that need to be tuned to the Hopf bifurcation given the fact that we only had five fully operational resonators (3, 4, 5, 8, 9). Figure 7.24 shows the response of resonator 4 and in Figure 7.33 we show the response of resonator 5. Here we have not shown the results of varying the tuning of apical resonators as the closest resonators, 6 and 7, were defective and as a result were tuned to have highly damped responses. Figure 7.33 also shows that tuning basal resonators to a Hopf bifurcation leads to steeper tuning, greater gain and a basal-ward shift in resonant frequency. When resonator 4 is tuned to the Hopf bifurcation (red curve) we see no significant change in gain or resonant frequency when compared with the case when only resonator 5 is tuned to the Hopf bifurcation (black curve). The slope after the resonant frequency, however, is steeper for the red curve when compared with the black curve. When we tune resonator 3 to the Hopf bifurcation (green curve) the gain increases by approximately 3 dB and the resonant frequency shifts from 1 kHz to 1.3 kHz. The green curve also exhibits an even greater slope at high frequencies after the resonant frequency. When resonator 4 is tuned away from the Hopf bifurcation, leaving only resonators 3 and 5 tuned to the Hopf bifurcation (blue curve), we see that there is even greater gain (up by a further 2 dB from the green curve) and the resonant frequency has shifted basal-ward from 1.3 kHz to 1.4 kHz. Thus, we can see that just like biology, the resonators immediately next to the resonator of interest need not be highly tuned. Given the small size of this silicon cochlea we are unable to determine the optimum distance between resonators although it is reasonable to suppose that this would depend upon the quantisation of the silicon cochlea i.e. how closely each resonator is tuned. In this implementation, due to the coarse tuning of the AQC, we were unable to ascertain whether tuning the basal resonators to a Hopf bifurcation is necessary or whether simply increasing their tuning relative to other resonators is sufficient.
7.7 Conclusions

In this chapter we have presented an improved AQC loop that models the system-level behaviour of the OHCs. We have built and tested an IC with BM resonators that implement the improved AQC loop and connected them together to create a 2D cochlea. Results from this IC show that our cochlea exhibits nonlinear, active quality-factor control suggesting that our model reasonably approximates biology.

Despite its small size (only 12 BM resonators with just 5 resonators operational with individual set-point control of AQC) we were able to explore coupling between resonators and see that the behaviour of this silicon cochlea model is similar to that of the biological cochlea.

7.8 References


Chapter 8. A 2D Silicon Cochlea with Hopf Oscillators

8.1 Introduction

In this chapter a 2D silicon cochlea with Hopf oscillators is presented. Chapters 6 and 7 explored the use of an automatic quality-factor control-loop to reproduce the active behaviour of the biological cochlea. We have shown in Chapter 5 Section 5.3.3 that a band-pass filter with infinite Q is equivalent to tuning the resonator to a Hopf bifurcation. We have also shown that our 2D silicon cochleae with AQC exhibit many of the expected nonlinear properties of the biological cochlea such as large signal compression, two-tone suppression and the generation of combinational tones along with demonstrated increases in gain and frequency selectivity with decreases in input signal level. The physical implementation of the cochlear resonators with AQC, however, has proven to be problematic. Specifically, the need for precise peak detectors and current comparators with large capacitors require a lot of space on chip which is not always feasible.

Figure 8.1 The 2D active cochlea model with Hopf oscillators.

Here we have used the equivalence between a second-order band-pass filter and the equation describing the Hopf bifurcation shown in Chapter 5 Section 5.3.4 to create an oscillator which is governed by the Hopf equation. In this circuit implementation of our 2D cochlea model the basilar membrane resonators with AQC have been replaced with Hopf oscillators. A schematic of our 2D active cochlea model with Hopf oscillators is given in Figure 8.1.
We will first discuss the circuits used to implement the Hopf oscillators and the 2D silicon cochlea. This will be followed by a presentation of the results from a fabricated integrated circuit which employs these circuits. We will end this chapter with a discussion of our results and the performance of this version of the active 2D cochlea.

8.2 Circuit Implementation

Apart from the Hopf oscillator, the CLBT bias circuit, and the set-point control, the circuits used in this implementation of the 2-D silicon cochlea are identical to those described in Chapter 6 or the improved version of these circuits presented in Chapter 7.

8.2.1 The Hopf Oscillator

We wish to implement the Hopf differential equation (8.1) as an analogue circuit. Here \( y \) is the output, \( x \) is the input (forcing function), \( \mu \) is the Hopf control parameter, and \( \omega_0 \) is the resonant/oscillator centre frequency.

\[
\dot{y} = (\mu + j\omega_0) y - y^3 + x \quad (8.1)
\]

In Chapter 5 we discussed the equivalence between the Hopf equation and the equation that describes a second-order band-pass filter. Specifically we found that when \( Q \) was set so that,

\[
Q = \frac{1}{I_{\text{out}}^2 - \mu} \quad (8.2),
\]
where \( I_{\text{OUT}} \) is the output of the second-order band-pass filter, we can configure our second-order band-pass filter to be a Hopf oscillator. The Q-value for the second-order band-pass filter that we have used thus far is given by (8.3). When we solve this simultaneously with (8.2) we find an expression for \( A \) (8.4).

\[
Q = \frac{1}{2 - A} \quad (8.3)
\]

\[
A = 2 + \mu - I_{\text{OUT}}^2 \quad (8.4)
\]

Thus, to create a Hopf oscillator we need to implement \( A \). The Hopf oscillator was constructed using a second-order band-pass filter, a multiplier, an absolute value circuit, and a squaring circuit. All the circuits operate in the log-domain. A top-level diagram of the Hopf oscillator is shown in Figure 8.2.

![The Tau Cell second-order band-pass filter.](image)

Figure 8.3 The Tau Cell second-order band-pass filter.

The second-order band-pass filter was created using two cascaded first-order Tau Cell filters and is shown in Figure 8.3. This is identical to the second-order band-pass filter that was used in Chapter 6 and Chapter 7. As with the previous implementations it has been designed so that each Tau Cell has the same time constant, \( \tau \). The current-controlled current source, \( I_{\text{ctrl}} \), shown in Figure 8.3, where,
\[ I_{crl} = \frac{Al_1I_2}{I_1} - AI_0 \] (8.5),

is implemented by the multiplier cell in Figure 8.4. This circuit provides the instantaneous feedback in the second-order band-pass filter that sets the \( Q \)-value of the circuit. Its operation was discussed in Chapter 4 Section 4.3.

![Multiplier cell diagram](image)

**Figure 8.4** The multiplier cell that implements the current-controlled current source, \( I_{crl} \), in Figure 8.3.

The output of the second-order band-pass filter, \( I_{out} \), is a bidirectional current. To avoid having to use a two-quadrant squarer or a fully-differential squarer the absolute value of the output current is taken. The absolute value circuit is shown in Figure 8.5 (a). In this circuit the current from the negative signal swing goes into \( M_{11} \) while the current from the positive signal swing goes through \( M_{13} \). The value of \( V_{bias} \) is varied to obtain a balance between the positive and negative signal swings.
The output of the absolute value circuit is then mirrored into the single quadrant squaring circuit (Gilbert 1990) shown in Figure 8.5 (b). This circuit is based on the translinear principle where:

\[ I_{\text{abs}}^2 = I_M I_{sq} \]  

(8.6),

and \( I_M \) is a constant scaling current and \( I_{sq} \) is the output of the squaring circuit.

The square of the output of the second-order band-pass filter is normalized so that the largest output signal, assuming we are on the critical point (i.e. \( \mu = 0 \)), results in \( A \approx 2 \) and the smallest output signal results in the largest possible \( Q \)-value that doesn’t cause limit-cycling. Thus, we choose the constant scaling current to be,

\[ I_M = \frac{I_{\text{MAX}}^2}{2I_0} \]  

(8.7),

where \( I_{\text{MAX}} \) is the maximum allowable output amplitude and \( I_0 \) is the bias current of the corresponding second-order band-pass filter. Equation (8.7) is implemented using a squaring circuit similar to Figure 8.5 (b). The value of \( I_{\text{MAX}} \) is tuneable and is dependent on the linear range of the second-order band-pass filter. By choosing \( I_M \) equal to equation (8.7) the output of the squaring circuit will be:

\[ I_{sq} = \frac{|I_{\text{out}}|^2}{I_{\text{MAX}}^2} \cdot 2I_0 \]  

(8.8).

The output of the squaring circuit is then subtracted from \( 2I_0 + \mu I_0 \) to get:
\[ A I_0 = 2I_0 + \mu I_0 - \frac{|I_{out}|^2}{I_{MAX}} \cdot 2I_0 \]  

as required by (8.4) when the maximum output is scaled to 2 at the critical point.

### 8.2.2 The CLBT Bias Circuit

The bias circuit in Chapter 7 has several drawbacks. Firstly, changing the Q-value necessitates changing the resonant frequency, making it difficult to assess the range of Q-values available and the effect that changes in Q-value have on the resonant frequency in the circuit. The second problem arises from the fact that small changes in control voltage, \( V_{var} \), result in large changes in the current through the CLBT. Subsequent mismatch in the BM resonators means that some resonators become unstable before others, necessitating a reduction in \( V_{var} \) which reduces the gain available in the silicon cochlea.

![Figure 8.6 The CLBT bias circuit.](image)

For this silicon cochlea the biggest concern was reducing the area of the bias circuitry. We were also concerned with improving the tuneability of the resonators especially given that this cochlea deliberately pushes the resonators into a region on the cusp of instability. To this end we used only a single CLBT per resonator. The CLBT bias circuit used in this silicon cochlea is shown in Figure 8.6. By using a single CLBT to create the current \( I_0 \) we need to create the currents \( 2I_0 \) and \( \mu I_0 \) by scaled MOSFET
current mirrors. This increases mismatch in the Q-value between adjacent resonators, however, our implementation of set-point control alleviates this somewhat.

### 8.2.3 Set-Point Control

The global control voltage $V_{var}$ in the silicon cochlea with improved AQC presented in Chapter 7 had a minimum value determined by the first resonator to become unstable. This meant that the maximum Q-value that each resonator could achieve before going unstable was not necessarily reached, reducing the overall gain and selectivity of the cochlea. Here we have avoided this by allowing individual control of the current $\mu I_0$.

![Figure 8.7 Implementation of $\mu I_0$.](image)

In the Hopf differential equation (8.1) the control parameter, $\mu$, is usually set to be either on the Hopf bifurcation, i.e. $\mu=0$, or below the Hopf bifurcation, i.e. $\mu<0$, and is rarely set to be in the unstable, limit-cycle region, i.e. $\mu>0$. In this physical implementation of the Hopf equation, however, it is reasonable to expect that the Hopf bifurcation will not occur at exactly $\mu=0$. In fact, based on the results from the previous implementations of the 2D silicon cochlea with AQC, the second-order Tau Cell band-pass filters require a current greater than $2I_0$ to reach the critical point. With this in mind we implemented the current source for $\mu I_0$ as a positive current source that can be tuned separately for individual resonators. Figure 8.7 shows the implementation of $\mu I_0$. Here the bias current of the WLR amplifier is set to $4I_0$ so that a bifurcation can be easily achieved. The output current from the WLR amplifier is bidirectional, $I_{WLR}$. By connecting it to a current mirror the resulting output current, $\mu I_0$, is single-ended. The voltage $V_\mu$ is controlled off-chip.
In (8.9) there are two variables, $\mu I_0$ and $I_{\text{MAX}}$, that we must define. In this equation the set-point (equivalent to $I_S$ in the AQC implementation in Chapter 7) is $I_{\text{MAX}}$. Setting $I_{\text{MAX}}$ high results in a reduction in large-signal compression while setting $I_{\text{MAX}}$ low increases large-signal compression. $I_{\text{MAX}}$ is a global current on this chip. It is set by varying a voltage, $V_{\text{MAX}}$, on the gate of an NMOS resistor. As such it is a source of mismatch between resonators, however, due to area constraints it was not feasible to make it tuneable for individual resonators.

### 8.3 Physical Implementation

An integrated circuit was designed and fabricated using MOSIS AMI 1.6$\mu$m on a 5 mm$^2$ Tiny Chip. This IC includes 12 BM resonators (Hopf oscillators), a bias generator (Delbrück and van Schaik 2005), a scanner, and 8 $\mu I_0$ control circuits (see Figure 8.7). Specifically, we connected individual $\mu I_0$ control to Hopf oscillators 3 – 8, Hopf oscillators 1 and 2 were connected to the same $\mu I_0$ control and Hopf oscillators 9 – 12 were connected to the same $\mu I_0$ control. Ideally, each oscillator should have its own $\mu I_0$ control, however, we were constrained by the number of pins available to us on the Tiny Chip.

| Table 8.1 |
| Technology | MOSIS AMI 1.6$\mu$m CMOS |
| Die size   | 5mm$^2$ |
| Dynamic Range | 40 dB (50 mVpp – 5 Vpp) |
| Noise     | -32 dB (26 mV RMS) |
| Power Consumption | 30 mW (including voltage regulator) |

A summary of the performance characteristics of the chip is given in Table 8.1. Note that the noise is given at the output and not referred to the input, as the gain of each resonator changes due to the Hopf oscillators. Each Hopf oscillator takes up an area of 140 $\mu$m by 910 $\mu$m. The layout of the chip is given in Figure 8.8.
8.4 Results

Testing the chip began with an investigation into the operation of the individual Hopf oscillator circuits. This was followed by testing the 2D silicon cochlea as a whole.

8.4.1 Operation of the individual Hopf oscillators

The operation of the individual Hopf oscillators was tested by bypassing the conductive network modelling the cochlear fluid and injecting the input signal directly into the Hopf oscillator selected via the scanner. To test the matching of the resonators
the resistive, polysilicon line was set with $\text{line}_{\text{in}} = 4.54 \, \text{V}$ and $\text{line}_{\text{out}} = 4.66 \, \text{V}$. The results from injecting a 100 mV amplitude signal, stepping the input frequency to the resonators (using a step size of 100 Hz), and measuring the output amplitude at each frequency step for resonators 1 to 12 is shown in Figure 8.9.

![Figure 8.9 The output of resonators 1 to 12 when the chip is configured to test individual resonators.](image)

In Figure 8.9 we have tuned the individual $\mu I_0$ values for each of the resonators to optimise the matching between the Q-values of the resonators. We see that those resonators which share their $\mu I_0$ control exhibit the worst Q-value matching. Specifically resonators 1, 9, 11 and 12 are not well matched to resonators 2 – 8, 10. From Figure 8.9 we see that resonators 9, 11 and 12 are well matched to one another, however, since they share their $\mu I_0$ control with resonator 10, increasing $\mu I_0$ would lead to resonator 10 becoming unstable and therefore $\mu I_0$ must be tuned down. This does not pose a significant problem, however, since these resonators were grouped together because they are not vital in testing the 2D cochlea configuration. Table 8.2 summarises the matching between the amplitude at the resonant frequency and $Q_{10\text{dB}}$ values when all of the
resonators are taken into account (including outliers) and when the outliers (1, 9, 11, 12) are excluded. Here we see that when the outliers are excluded from the statistical analysis the resonators are very well matched in both amplitude at the resonant frequency and $Q_{10\text{dB}}$ value. Inclusion of the outliers in the statistical analysis sees a reduction in the average amplitude and $Q_{10\text{dB}}$ value and shows an increase in the spread (max-min and standard deviation (std dev)).

Table 8.2

<table>
<thead>
<tr>
<th></th>
<th>Including Outliers</th>
<th>Excluding Outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude (dBV)</td>
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<td>2.18</td>
</tr>
<tr>
<td>$Q_{10\text{dB}}$</td>
<td>1.89</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Figure 8.10  Tap number versus frequency for the frequency response data from Figure 8.9.

Figure 8.10 shows the tap number versus resonant frequency for the BM resonator frequency response curves in Figure 8.9. Here we see that there is only a small deviation from a logarithmic (ideal) relationship between tap number and resonant frequency.
Figure 8.11  Output of resonator 3 (blue) when input signal (red) is varied from 100\(\mu\)V to 1V and back to 100\(\mu\)V.

In order to test the Hopf oscillator’s ability to adapt to signals of varying intensity we tuned resonator 3 to be on the cusp of a Hopf bifurcation and input a signal whose peak-to-peak voltage changed from 100\(\mu\)V to 1V and then back to 100\(\mu\)V. Figure 8.11 shows the output of resonator 3 in blue and the varying input in red. Here we see that the output is more compressed than the input. Specifically, in response to the input signal of 100\(\mu\)V-1V-100\(\mu\)V the output signal has a peak-to-peak profile of 43mV-217mV-43mV. Hence, the 1V input signal is attenuated by -13dB at the output and the 100\(\mu\)V input is enhanced by 50dB at the output. The area within the black box in Figure 8.11 has been expanded in Figure 8.12. Here we have zoomed-in on the transition between the high input signal (1V) to the low input signal (100\(\mu\)V). We can see that it takes the output approximately 4 cycles to settle. This is indicative of the fact that, in the case of the 100\(\mu\)V input signal, a very high Q-value is required to achieve a gain of 50dB.
Figure 8.12 A close-up (from Figure 8.11) of the transition between an input of 1V to 100µV (red) and the corresponding output (blue).

Figure 8.13 (a) shows the frequency response of resonator 4 when the input amplitude is varied from 0 dB (relative to 1 V) to -80 dBV in -20 dB steps. In this case resonator 4 has been tuned close to the critical point (Hopf bifurcation). Here we see that the 80 dB range has been compressed into 25 dB. We can also see that the output of resonator 4, when the input is equal to or below -60 dBV, is well below the noise floor except at the resonant frequency. Figure 8.13 (a) shows that the resonant frequency shifts slightly when the quality factor changes. For the -80 dB input the output is extremely selective and as such the resonant frequency is found by triggering on the output signal itself. At low input amplitudes the resonators simply become oscillators at the resonant frequency; hence the extremely high Q value and selectivity of their frequency response curves. Figure 8.13 (b) shows frequency versus gain for the same data. Note that in Figure 8.13 (b) the -80 dB signal is below the noise floor and the output oscillates independently of the input. Therefore we do not really have a gain of 50 dB. We have
left this in the figure, however, because it illustrates that the oscillator can reach supercritical stability.

Figure 8.13 (a) Frequency response of resonator 4 when it is tuned close to a Hopf bifurcation and the input is varied between 0 dB and -80 dB in -20 dB steps relative to 1V. (b) The gain of resonator 4 obtained from the data in Figure 8.13 (a).

Figure 8.14 Frequency response of BM resonators 2-8 and 10 when the chip is configured as a 2D silicon cochlea.
8.4.2 Operation of the 2D active cochlea

Testing of the individual resonators (Hopf oscillators) in section 8.4.1 revealed that, when tuned close to a Hopf bifurcation, the output smoothly transited through an oscillatory, infinite Q state to a band-pass filter-like non-oscillatory state when the input signal was varied between low and high amplitudes respectively. In Figure 8.13 we show a dynamic range of 60 dB, however, when the input of an oscillator is grounded an oscillatory output at the correct, resonant frequency is still observed.

![Figure 8.13](image)

Figure 8.13. Tap number versus resonant frequency for BM resonators in the 2D active cochlea.

In configuring the chip as a 2D cochlea we were confronted with a number of challenges. In our efforts to achieve high (infinite) Q-values we had not allowed the control parameter, \( \mu \), to be less than zero and as a result we were unable to achieve highly damped frequency responses. This also meant that tuning the cochlea was particularly difficult given that the 12, highly tuned oscillators were coupled together. Figure 8.14 shows the frequency response of BM resonators 2 – 8, and 10. In this test the cochlea was input with a 500mV (peak-to-peak) amplitude signal and the input frequency was varied in 100 Hz steps between 100 Hz and 10 kHz and 1000 Hz steps between 10 kHz and 20 kHz. The curves in Figure 8.14 have been cropped for clarity.
The ends of the resistive, polysilicon line that biases the CLBTs were unchanged from Figure 8.9 (with $\text{line}_{\text{in}} = 4.54$ V and $\text{line}_{\text{out}} = 4.66$ V). The tap number versus resonant frequency is plotted in Figure 8.15. Here we see that, as with Figure 8.10, the relationship between tap number and frequency on a logarithmic scale is approximately linear although there is greater deviation at resonators 5, 6 and 7. This deviation is also present in Figure 8.10 for the tuning of the individual BM resonators and is due to mismatch between the BM resonators. Both Figure 8.10 and Figure 8.15 show that resonator 5 and 6 have centre frequencies that are slightly too close to one another while resonator 7 has a centre frequency which is slightly too far from the centre frequency of resonator 6. In the 2D cochlea configuration this has the effect of increasing mismatch in the output amplitude and reducing the slope after resonance of resonator 7. This can be seen in Figure 8.14 where resonators 5 and 6 are seen to have the same resonant frequency. In Figure 8.16 (a) and (b) we show the effect of changing the Q values of resonator 5 and 6 to ascertain the effect of this on resonant frequency and output amplitude.

![Figure 8.16](image)

**Figure 8.16** Frequency response of BM resonators 2-8 and 10 when the chip is configured as a 2D silicon cochlea and the BM resonators are tuned slightly differently in (a) and (b).

In Figure 8.16 (a) both resonators 5 and 6 have low Qs while in Figure 8.16 (b) they have higher Qs. In Figure 8.16 (a) we can see that when resonator 6 has a lower Q it is more attenuated although it has a resonant frequency distinct to the resonant frequency of resonator 5. Figure 8.16 (b) shows that as the Q value of resonator 6 increases its resonant frequency gets close to that of resonator 5 while its output amplitude increases significantly. Figure 8.16 (a) and (b) also show that due to the fact that resonator 7
covers a much larger frequency range it is more susceptible to minor peaks and bumps through coupling from other BM resonators and its slope after the resonant frequency is not as steep as the other resonators. The mismatch in centre frequency and the subsequent coupling issues are not major problems as greater quantisation (i.e. an increase in the number of BM resonators) should offset the mismatch. In this chip we are covering a frequency range of over 4 octaves with only 8 resonators while there are at least 1500 IHCs and 4500 OHCs to cover this range in the human cochlea.

Another observation from Figure 8.14 is that the output of the resonators is attenuated by approximately 36 dB. This is based on a mean output amplitude of -36 dBV for the BM resonators at the resonant frequency and an input of 500 mV (peak-to-peak). We have seen attenuation in the conductive network in previous implementations, however, this is the greatest attenuation that we have seen and it is surprising given that the input generator circuit and the conductive network have not changed from the silicon cochlea in Chapter 7. Inspection of the layout for this chip, however, has brought us to the conclusion that the greater attenuation is due to the fact that we routed many signals using long lines of polysilicon to save area on the chip (AMI 1.6µm is only a 2 metal process). One of the lines which was routed with polysilicon was the line that sets $G_{xy}$ and $G^*$. We have found that we must use a much smaller voltage (1.7 V) on $G_{xy}/G^*$ to get a stable, observable output from the cochlea when compared to the voltage (2 V) used for the same signals in the silicon cochlea in Chapter 7. This is indicative of the input signal being attenuated through the conductive network and hence requiring more conductivity to pass between sections of the conductive network.

The greater attenuation has impacted the dynamic range given that the noise floor is reached at a high input amplitude (close to 100 mV). At the noise floor the BM resonators become oscillators. As an oscillator the output is independent of the input and as such the frequency response curves change very little between input amplitudes of 50 µV to 100 mV.

Figure 8.17 shows BM gain versus frequency for (a) BM resonator 5 and (b) biological data from (Ruggero 1992). In this plot we have put both the data from the chip and the biological data on the same scale. We see that the quality factors of all of the gain curves are much greater than those in the biological data. The high quality factors
are a consequence of the high noise floor in our system. In Figure 8.17 the -46 dB and -66 dB gain curves are below the noise floor and as such the resonator oscillates at the resonant frequency independent of the input frequency. This explains the shape of their gain curves at frequencies below the resonant frequency. We discuss this result in more detail in section 8.5.

![Figure 8.17 BM Gain versus frequency for (a) the 5th BM resonator and (b) biological data from (Ruggero 1992)](image)

The largest signal of our dynamic range in Figure 8.17 is 14 dB which corresponds to an input signal of 5V (peak-to-peak). We are able to use this high input signal as the input circuit contains a WLR amplifier which can handle a rail-to-rail input signal. The output of the WLR amplifier is a current and due to the attenuation in conductive network this signal does not saturate the BM resonators.
Since we have 8 operational and tuneable sections in our 2D silicon cochlea we are able to explore the coupling between BM resonator sections. In Chapter 7 we found that tuning basal resonators improved the gain and steepness of a BM resonator’s frequency response. We also found that immediately adjacent resonators were not as influential on the output response as resonators that were several sections away. Figure 8.18 shows the frequency response of the 4th (a) and 5th (b) BM resonators when the basal BM resonators are tuned low (blue curves) and when the neighbouring, basal BM resonator is tuned to a higher Q value (red curves). Here we see that when the basal BM resonator is tuned to a higher Q value both the gain and steepness after resonance of the BM resonator’s frequency response increases. We see that the frequency response does not drop off immediately after resonance, however, and there is a plateau (identified in Figure 8.19. 

Figure 8.18 Frequency response of (a) the 4th BM resonator and (b) the 5th resonator with basal resonators tuned low (blue) and the adjacent basal resonator tuned high (red).

Figure 8.19 Illustrates the effects of high (a) and low (b) Q values on the shape of the frequency response of BM resonator 5.
(a)) before the response becomes steep. This plateau occurs for several reasons. Firstly, the $Q$ values in this silicon cochlea are higher than those for the silicon cochlea in Chapter 7. Secondly, there are only 8 BM resonators covering a 4 octave range in this silicon cochlea while there is the same number of resonators covering a <3 octave range in the silicon cochlea in Chapter 7. Figure 8.19 (b) shows the plateau is removed when the $Q$ value of BM resonator 5 is lowered. Hence a BM resonator whose set-point has been tuned to achieve high $Q$ values requires a close basal BM resonator to be tuned high while a BM resonator with a lower maximum $Q$ value needs a basal BM resonator up to several sections away to be tuned high depending on quantisation in the silicon cochlea. This is consistent with our picture of the wiring of the efferent fibres in the mammalian cochlea (see Chapter 7 Figure 7.32) where connections are made basally to more than one OHC.

One question that we were unable to answer in Chapter 7 was how high the set-points (or $Q$ values) of the basal resonators need to be tuned. In this silicon cochlea, while all of the $Q$ settings are relatively high, we are able to change the $Q$ value smoothly over a much wider range than we could in Chapter 7. In Chapter 7 we could only change the $Q$ value coarsely between high values (close to the Hopf bifurcation) and low, highly damped values. In Figure 8.18 (a) we see that the adjacent, basal resonator (8.3) has a much higher $Q$ value, due to the sharpness of the peak at the plateau (~7 kHz), than the adjacent, basal resonator (8.4) in Figure 8.18 (b). In Figure 8.18 (a) this sharp, prominent peak is at the resonant frequency of basal resonator 3. This peak is clearly undesirable and since resonator 3 is not tuned to the Hopf bifurcation in Figure 8.18 (a) we can say that the basal resonators need not be tuned to the Hopf bifurcation. In fact, the basal resonators need only be tuned to a $Q$ value high enough to create a low impedance path for signals at their resonant frequencies. Since a signal will always chose the path with the lowest impedance, making the impedance lower for frequencies basal to the resonator of interest will result in very little energy at the basal frequencies travelling apically in the cochlea. Tuning the basal resonators to higher $Q$ values, however, results in undesirable secondary peaks and bumps in the apical resonators which can increase the effects of masking (see section 8.5) and create instabilities. Hence, the tuning of basal resonators is related to impedance matching in the cochlea. This also explains why the
Efferent fibres connect to more than one OHC. As the tuning changes at a particular place along the BM the tuning basally must also change in order to maintain the impedance matching along the BM.

![Figure 8.20](image)

**Figure 8.20** Two-tone suppression demonstrated by the 3rd resonator tuned to a resonant frequency of 7.4 kHz in the presence of (a) a 6.4 kHz suppressor tone, and (b) a 8.4 kHz suppressor tone.

![Figure 8.21](image)

**Figure 8.21** FFT of the output of resonator 5 showing distortion products.
As with the silicon cochleae in Chapter 6 and Chapter 7 we can demonstrate both two-tone suppression and the generation of combinational tones in this silicon cochlea. Figure 8.21 (a) and (b) shows two-tone suppression when the 3rd BM resonator is input with both lower and higher suppressor tones respectively. Figure 8.21 shows an FFT of the output of BM resonator 5 when it is tuned at a resonant frequency of 3.9 kHz (CF) and frequencies of 4.2 kHz ($f_1$) and 4.5 kHz ($f_2$) are applied to the input of the cochlea. The two frequencies, $f_1$ and $f_2$, that are selected such that $2f_1 - f_2 = CF$.

![Graph showing frequency response](image)

Figure 8.22   The frequency response of the 6th BM resonator when the input amplitude increases from 500mV peak-to-peak (blue) to 2.5 V peak-to-peak (red) shows a linear increase in amplitude for a masker tone at 1.4kHz and a compressed increase in amplitude at the resonant frequency.

### 8.5 Discussion

The results from this silicon cochlea compared to those in Chapter 6 and 7 indicate that (excluding attenuation due to layout constraints) we were able to achieve better amplitude and frequency matching for the entire cochlea. This is best demonstrated in Figure 8.14 where we showed the frequency response of resonators 2 – 8, 10 all of which
exhibited the characteristic steep slope after resonance. This improvement was due to our ability to set $\mu I_0$; the fine-control that we did not possess in Chapter 7.

Figure 8.16 (a) and (b) show the results when we change the tuning of several of the BM resonators. The minor peaks and bumps that are seen in the frequency response of the BM resonators are due to coupling through the conductive network. This effect causes masking in hearing. As a signal at the masker frequency increases the signal at the resonant frequency must increase by a larger amount in order to preserve the amplitude difference between the two signals and be detected. This is because at resonance a signal undergoes nonlinear compression whereas the masker will increase linearly. Figure 8.22 shows the frequency response of the 6th BM resonator when the input amplitude changes from 500mV (peak-to-peak) to 2.5 V (peak-to-peak). Here we see the output at resonance (4 kHz) has been compressed (14 dB into 9 dB) while the signal at the masker frequency has increased linearly by 14 dB.

In Figure 8.17 we showed the gain of the 5th BM resonator when its input amplitude is varied and compared this to biological data. This is not a valid comparison, however, given that the output of the BM resonator was oscillating when the input was -46 dBV and -66 dBV. It does show, however, the advantage of using Hopf oscillators given their ability to remain stable in reaching infinite Q. Even Ruggero’s data is somewhat artificial given that sound pressure levels of less that 20 dB are rarely (if at all) experienced in nature. In Table 8.3 we have calculated the $Q_{10\text{dB}}$ values for the data in Figure 8.17 (a) and (b). Here we see that the $Q_{10\text{dB}}$ values that we are achieving are comparable to the quality factors obtained by Ruggero for sound pressure levels below 20 dB especially when we consider that the $Q_{10\text{dB}}$ calculations from the data in Figure 8.17 (a) did not correct for the low and high frequency response which is not as steep as it should be because of the high noise floor. This explains why we were achieving a much smaller upwards shift in resonant frequency. It also illustrates why it is valid to use an oscillator to model the action of the OHCs as the oscillatory condition should never be met in reality.
Table 8.3

<table>
<thead>
<tr>
<th>Input (dBV)</th>
<th>$Q_{10\text{dB}}$</th>
<th>$f_0$ (kHz)</th>
<th>Input (dBV)</th>
<th>$Q_{10\text{dB}}$</th>
<th>$f_0$ (kHz)</th>
</tr>
</thead>
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<tr>
<td>14</td>
<td>3.2</td>
<td>2.9</td>
<td>80</td>
<td>1.4</td>
<td>7</td>
</tr>
<tr>
<td>-6</td>
<td>7.8</td>
<td>3.1</td>
<td>60</td>
<td>2</td>
<td>8</td>
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<tr>
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<td>10.7</td>
<td>3.2</td>
<td>40</td>
<td>3.6</td>
<td>9</td>
</tr>
<tr>
<td>-46</td>
<td>~320</td>
<td>3.2</td>
<td>20</td>
<td>4.5</td>
<td>9</td>
</tr>
<tr>
<td>-66</td>
<td>~320</td>
<td>3.2</td>
<td>3</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Figure 8.23 BM Gain versus frequency for (a) the 5th BM resonator and (b) biological data from (Ruggero 1992) on a more comparable scale.

Figure 8.23 shows the data from Figure 8.17 (a) and (b) replotted on a more comparable scale. Here we see that the Q values are now closer in value between the two data. The low and high frequency response in Figure 8.23 (a) deviates from that in Figure 8.23 (b) due to the high noise floor in our circuit. In Figure 8.23 (a) we can see that the low frequency slope for the -26 dB and -6 dB data begins at a similar steepness to that in Figure 8.23 (b) before it levels off.
8.6 Conclusions

In this chapter we have shown the circuits and results from a fabricated integrated circuit based on a model of the active 2D cochlea which utilises Hopf oscillators rather than AQC. This silicon cochlea demonstrates the same nonlinear and active characteristics that the previous implementations exhibited. We were also able to further explore coupling between BM resonator sections and we were able to demonstrate quality factors that are comparable with those measured in biology.

8.7 References


Chapter 9. Discussion, Future Work and Conclusions

We finish here with a discussion on some of the issues that have arisen from this work. This is followed by some recommendations for future work. We then make some concluding remarks and sum up the outcomes of this thesis.

9.1 Discussion

9.1.1 Construction of a larger 2D silicon cochlea

In Chapter 7 and 8 we were limited (by cost) to constructing small 2D silicon cochleae with only 12 stages. This gave us the opportunity to allow individual set-point access to the majority of resonators which in turn allowed us to explore coupling and tuning between BM resonators. It is clear from this work that in order to avoid matching problems and to be able to include efferent connections in the future, set-point access is required to all the BM resonators in a silicon cochlea (except perhaps the first resonator and the last few resonators). A complete 2D silicon cochlea that covers the frequency range of the human cochlea will require a much larger number of stages to avoid the stability problems and coupling issues that we sometimes encountered in our three silicon cochlea implementations. The more resonators that are added, however, the more impractical it is to allow set-point access via pins; we require an on-chip solution. This might be achieved by a local memory which holds the set-point value for the resonator. We might also use a programmable current splitter circuit. In Chapter 4 we used a current splitter circuit (Delbrück and van Schaik 2005) to change the Q values in our second-order band-pass filters. Using a current splitter makes sense because the set-point for all of our silicon cochlea implementations has been in the form of a current. Figure 9.1 shows an N-stage current splitter circuit. The current, $I_{CS}$, is divided by two at the first branch, which is divided by two at the second branch and so on until the smallest current, $I_{CS}/2^N$, is achieved. The set-point current, $I_S$, is created by adding the current from the branches of the current splitter in $M_2$. The current from the current splitter is selected via switches, $A_1$-$A_{N+1}$, used to trim the set-point current to the desired level. In Figure 9.1 $V_G$ is a bias voltage. Obviously the greater the number of stages the finer the
resolution of the set-point current, however, this comes at the expense of more selection bits, \(A_1 - A_{N+1}\), which need to be stored and set via some form of digital memory, such as a shift register.

Thus, construction of a larger 2D silicon cochlea is feasible although it will require greater area to allow set-point access. The majority of the circuitry required for set-point access is digital, however, and as such should not take up too much extra area. The current splitter need not be perfectly matched as long as there are enough bits to trim the set-point current to its desired value. Therefore we do not need large transistors for matching, further reducing the area required.

### 9.1.2 The Hopf Bifurcation

Our active 2D cochlea model is based on the idea that the OHCs are governed by the Hopf equation, however, it is still unclear whether the Hopf equation is the definitive equation which defines the cochlear amplifier. Certainly we have shown that some form of positive feedback is necessary to successfully model the nonlinear and active features of the cochlea, however, as we generally operated in the sub-critical region we believe that any nonlinear equation which adds energy back into the system will give similar results. The most appealing aspect of the Hopf bifurcation is that it can smoothly transit
through an unstable, limit cycle region of operation into a stable equilibrium region without hysteresis. This is an important feature if we consider ringing in the ears and tinnitus as examples of the cochlea operating in an unstable region. Clearly, the cochlea must have dynamics that allow its return to a stable operating point after going unstable, otherwise there would be a greater incidence of tinnitus and tinnitus-like afflictions.

Another appealing aspect of modelling the cochlear amplifier with the Hopf equation is that it is equivalent to parametric amplification. The benefits of parametric amplification include robustness to noise and a wide range of phase shifts. If the OHCs are governed by the Hopf equation the need for their movement to be phase-locked with the movement of the BM would be greatly relaxed.

In this work we have shown that when the OHCs are modelled using the Hopf equation we are able to approximate the dynamics of the human cochlea. We have not shown that this is the only way that the cochlea’s dynamics can be modelled.

### 9.1.3 Predictions about coupling

In this thesis we have made predictions about coupling in the cochlea which have not been tested or shown in biology. Specifically, through experimentations with our silicon cochlea, we have shown that the frequency response of a BM resonator is dependent on the tuning of other BM resonators which are located basally. In Chapter 7, with low Q values, we found that the tuning of resonators several sections away have the greatest impact. In Chapter 8, with higher Q values we found that basal resonators immediately adjacent were the most important. Thus, depending on the quality factor the tuning of the basal resonators will impact the shape of the frequency response. In Chapter 8 we found that the basal resonators do not need to be tuned to the Hopf bifurcation, however, they need to be tuned high enough to create a low impedance path for frequencies greater than the resonant frequency of the apical BM resonator. We found that the tuning of the resonators apical to the resonator under test had little impact on its tuning and high frequency response. Tuning the apical resonators to high quality factors, however, increases the likelihood of undesirable coupling; creating peaks and bumps (at the apical resonator’s resonant frequency) in the resonator under test’s frequency response. We have shown in Chapter 8 that this can cause masking.
9.2 Future Work

The results from our three 2D silicon cochleae indicate that our design is able to reproduce many of the nonlinear, active characteristics of the human cochlea. The next step is to create a much larger version of the silicon cochlea (in both software and silicon) and use it to understand other aspects of mammalian hearing. Given its ability of achieve high Q values and given the smaller area of its resonator circuits, this larger version will be based on the active 2D silicon cochlea with Hopf oscillators.

Of particular interest and relevance when using our active, 2D model, is the role of efferent fibres in binaural hearing. Our models can be extended to include efferent connections which would simply replace the manual set-point control. It has been shown in binaural hearing that a contralateral sound will influence efferent controlled suppression in the ipsilateral cochlea (Collet, Kemp et al. 1990), (Maison, Micheyl et al. 2000). Thus, the connectivity of the efferent fibres between two cochleae must play a role in sound localisation and sound source separation. By building a binaural, neuromorphic system based on our active, 2D model we may learn how, with only two sensors (our ears), we are able distinguish particular sounds in the presence of noise.

9.3 Conclusions

In this thesis we have presented a model of the 2D active cochlea. We have implemented this model in three separate silicon cochleae which are the first silicon cochleae to exhibit many of the nonlinear, active features of the human cochlea including large signal compression, two-tone suppression, combinational tones and masking. We have also been the first to explore coupling between cochlea sections and from this we have made predictions on the interactions between neighbouring cochlea sections in order to obtain the cochlea’s characteristic frequency response. Our silicon cochleae are the first to exhibit a similar level of compression as the mammalian cochlea and we have shown that we are able to successfully obtain high quality factors comparable to biology.

This thesis will hopefully form the basis for investigations into understanding higher auditory centres, including binaural hearing and the role of the efferent fibres. We believe that the results we have obtained and predictions we have make are evidence of the benefits of neuromorphic engineering and the utility that can be obtained from
building physical models of biological systems. Our hope is that our work encourages further research into human hearing from both a neuromorphic engineering perspective and a physiological one.

9.4 References

Appendix

A. Derivation of THD for a transistor operating in the sub-threshold region

The equation for the drain current, \( i_D \), for a transistor operating in the subthreshold region is given by (Enz, Krummenacher et al. 1995):

\[
\begin{align*}
    i_D &= i_s e^{\frac{v_G - v_n}{n U_T}} \left[ e^{\frac{v_S}{U_T}} - e^{\frac{v_D}{U_T}} \right] \\
    &= \left( i_d + I_D \right) \tag{A.1}
\end{align*}
\]

where \( i_s \) is the specific current, \( n \) is the slope factor, \( U_T = kT/q \) is the thermal voltage, \( v_G \) is the gate voltage, \( v_S \) is the source voltage, \( v_D \) is the drain voltage and \( v_{th} \) is the threshold voltage. The drain current comprises of an AC component and a DC component which can be denoted as follows:

\[
    i_D = i_d + I_D \tag{A.2}
\]

We can re-write (A.1) in terms of the AC and DC components of gate, source and drain voltage as follows:

\[
\begin{align*}
    i_d + I_D &= i_s e^{\frac{v_G - v_n}{n U_T}} \left[ e^{\frac{v_S}{U_T}} - e^{\frac{v_D}{U_T}} \right] \\
    &= \left( i_d + I_D \right) \tag{A.3}
\end{align*}
\]

Rearranging (A.3) and assuming that the effect of the forward current is dominant (i.e. removing the terms with \( v_D \)) we get:

\[
\begin{align*}
    i_d &= -I_D + i_s e^{\frac{v_G - v_n}{n U_T}} \left[ e^{\frac{v_S - n v_D}{U_T}} - e^{\frac{v_G - n v_D}{U_T}} \right] = -I_D + I_D \left( e^{\frac{v_s - n v_D}{U_T}} \right) \\
    &= -I_D + I_D \left( e^{\frac{v_s - n v_D}{U_T}} \right) \tag{A.4}
\end{align*}
\]

Dividing (A.4) by \( I_D \) we get:

\[
\frac{i_d}{I_D} = -1 + e^{\frac{v_s - n v_D}{U_T}} \tag{A.5}
\]

The Taylor expansion for an exponential, \( e^x \), is given by:

\[
\begin{align*}
    e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ... \tag{A.6}
\end{align*}
\]

Therefore substituting the Taylor expansion for \( e^x \) into (A.5) we get:
\[
\frac{i_d}{I_D} = \frac{v_g - n v_s}{n U_T} + \frac{1}{2} \left( \frac{v_g - n v_s}{n U_T} \right)^2 + \frac{1}{6} \left( \frac{v_g - n v_s}{n U_T} \right)^3 + \ldots \quad (A.7).
\]

From (Sansen 1999) we can approximate THD using the second harmonic (HD\textsubscript{2}) by:

\[
THD \approx HD_2 = 1 - 2 \frac{1 \left( \frac{v_g - n v_s}{n U_T} \right)^2}{4} = \frac{\hat{i}_d}{4 I_D} = \frac{i_p}{4} \quad (A.8),
\]

where,

\[
\frac{\hat{i}_d}{I_D} = \left( \frac{v_g - n v_s}{n U_T} \right) \quad (A.9).
\]

Thus, distortion is slightly higher for transistors operating in the subthreshold region compared with transistors operating in the strong-inversion region. Increasing the DC offset current, however, decreases harmonic distortion in both cases.

**References**
