Dynamic Hedging of Foreign Exchange Risk using Stochastic Model Predictive Control

Farzad Noorian, Philip Leong

Computer Engineering Laboratory
School of Electrical and Information Engineering
University of Sydney
Introduction to hedging

What is the best way to reduce FX risk/transaction cost?
- From a dealer’s view → Obligated to absorb incoming flow from clients
- Intraday time-scale

The Clients ↔ Trades ↔ FX Broker ↔ Risk ↔ T-Cost ↔ Other Brokers ↔ Exchange Rate
Outline

Formalizing the Problem → Formulating Risk & Cost → Forming a Control Problem → Solving the Problem → Benchmarks & Backtesting
Objective and Available Control Mechanism

› Minimize risk and cost:
  - **Risk**: Exposure of positions to exchange rate volatility.
  - Transaction **costs**: incurred by interbank trade.

› Method: keeping or reduce positions

- Reducing Positions
  - High Cost
  - Low Risk

- Keeping Positions
  - Low Cost
  - High Risk
At each time time-step:

- The positions are updated with Client Flow and Hedging actions:
  \[ X_{t+1} = X_t + H_t + F_t \]

- Constraints on maximum hedging size:
  \[ 0 \leq |H_t| \leq h_{max} \]

- Constraint on closing all open position at the end of trading period:
  \[ X_{N+1} = 0 \rightarrow H_N = -(X_N + F_N) \]

- Cost for each hedging action:
  \[ C = \sum (H_i \delta_i)^2 \]

- P&L at the end of trading period:
  \[ L = \Sigma X_i R_i \]

- Goal: Reduce Cost & Risk
Model Predictive Control

The problem:

- **State space model:**
  \[ x_{t+1} = g(x_t, u_t) \]

- **Constraints:**
  \[ h(x_t, u_t) < 0 \]

- **Performance objective:**
  \[ J^* = \min p(X_N) + \sum q(x_t, u_t) \]

The Solution:

1. **Update Dynamic Model**
   - Approximate \( x_t \)

2. **Optimize control for model**
   - Find \( u_i \) \( i \in [t, N] \) that Minimizes \( J^* \)

3. **Apply control**
   - Only \( u_t \)

\[ t = t + 1 \]
MPC doesn’t account for uncertainty (Gaussian noise only).

Stochastic models + MPC → Stochastic MPC

\[
\min \mathbb{E}[\sum q(x_t, u_t)] + \lambda \text{VAR}[p(x_N)]
\]

or other forms.

Usual approach:
1. Use Monte Carlo methods to generate “scenarios” from available distribution
2. Solve numerically using Stochastic Programming
Flow and exchange Rate are stochastic variables.

Optimization objective:

$$\textbf{argmin}_H E[C] + \lambda \text{VAR}(L)$$

$$\text{s.t. } 0 \leq |H_t| \leq h_{max}$$

- $\lambda$: Risk preference parameter

Current problem simplifies to a quadratic optimization:

$$\textbf{argmin}_H H^T A H + 2H^T B + C$$
› Solving a *Receding horizon problem*

› Only the *current* action is applied at each step.

\[ t = t + 1 \]
Generating scenarios

› Need models for:
  - Exchange rate (volatility)
  - Client Flow

› Older approach: Model Driven
  - simplicity

› New approach: Data Driven
  - flexibility
Generating scenarios

› Exchange Rate model:
  - Random walk diffusion \( \sim N(0, \sigma^2) \)
  - Covariance matrix for multivariate model
  - Announcement based jumps

› Example:
  - Different stochastic scenarios
  - One event at 15:00 PM
Generating scenarios

Client Flow model:

- Observation: Magnitude with daily seasonality
- Model: Histogram analyses of hourly trades
Strategies

• SMPC Hedging
  - With different levels of prediction

• Prescient strategy:
  - MPC with flow and exchange rate known in advance (peeking into future)

• Naïve strategies:
  - Hard Limits: Limit maximum positions to $x_{max}$
  - Installments: Close all position at maximum of next $n$ trading time-steps.
Example of Hedging – What happens?

- The position accumulates while the exchange rate fluctuates.
  - Black: Observed
  - Gray: Stochastic paths
Example of Hedging – Benchmark 1: Hard Limits

- Controlled Risk, but high cost
Example of Hedging – Benchmark 2: Installments

› Better cost, but no control over risk
Example of Hedging – SMPC Hedging

› Reducing risk before the 15:00 announcement

![Graph showing hourly hedging changes](image-url)
Example of Hedging – SMPC Hedging

› Closing all positions at the end of the day
50 trading session – Each 32 time-steps
5 Currencies, synthetic announcements
Cost-Risk curve (similar to Markowitz’s) for different parameters
Effect of prediction – Synthetic Test

![Graph showing the relationship between average cost and risk (SD of P&L) for different hedging strategies.](image)

- Prescient Flow and Volatility
- SMPC Hedging
- SMPC Hedging + %30 Prediction
- SMPC Hedging + %60 Prediction
Test with Real World Data – Cost-Risk Curve

› Using data from Westpac Banking Corporation
› 2 month of data for model fitting, 4 months for testing
› 5 Currencies (AUD, USD, EUR, NZD, JPY) + Announcements from dailyfx.com
Effect of prediction – Test with Real World Data

![Graph showing normalized cost vs. normalized risk with different prediction methods]
Conclusions:

- SMPC hedging outperforms naïve strategies.
- By being data-driven, hedging result will improve by better estimations of the flow and volatility.

Future work:

- Using volatility directly → Simplifies scenario generation
- Time-varying spreads
- Non-quadratic transaction cost → No longer quadratic optimization
- Better client and volatility modeling
Thanks for listening.

Code available at

Your comments are highly appreciated:
- farzad.noorian@sydney.edu.au