Estimating Distances via Connectivity in Wireless Sensor Networks

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Abstract

Distance estimation is vital for localization and many other applications in wireless sensor networks. In this paper, we develop a method that employs a maximum-likelihood estimator (MLE) to estimate distances between a pair of neighboring nodes in static wireless sensor networks using their local connectivity information, namely the numbers of their common and non-common one-hop neighbors. We present the distance estimation method under a generic channel model, including the unit disk (communication) model and the more realistic log-normal (shadowing) model as special cases. Under the log-normal model, we numerically study the bias and standard deviation associated with our method and show that for long distances our method outperforms the method based on received signal strength (RSS); we investigate the impact of the log-normal model uncertainty; we provide a Cramér-Rao lower bound (CRLB) analysis for the problem of estimating distances via connectivity and derive helpful guidelines for implementing our method. Finally, on applying the proposed method based on realistic measurement data and also in connectivity-based sensor localization, the superiority of the proposed method is confirmed.

Index Terms

Distance estimation; Maximum-likelihood estimator; Cramér-rao lower bound; Generic channel model; Sensor localization

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I. INTRODUCTION

Wireless sensor networks, comprised of hundreds or thousands of small and inexpensive nodes with constrained computing power, limited memory and short battery lifetime, can be used to monitor and collect data in a region of interest. Accurate and low-cost node localization is important for various applications in wireless sensor networks, and thus great efforts have been devoted to developing localization algorithms, including distance-based algorithms and connectivity-based algorithms [1]. Distance-based localization algorithms rely on distance estimates and can achieve relatively good localization accuracy; if distance estimates are unavailable or suffer from huge errors, connectivity-based localization algorithms are applied but generally achieve coarse-grained localization accuracy. Besides, distance estimation is vital for sensor network management, such as topology control [2], [3], boundary detection [4] and so on.

In reality, distance estimation can be realized by using information such as received signal strength (RSS), time of arrival (TOA), time difference of arrival (TDOA), etc [1]. The RSS method (using RSS measurements) depends on low-cost hardware and only provides coarse-grained distance estimates; by contrast, the TOA and TDOA methods can provide distance estimates with higher accuracies at the cost of extra hardware. Due to cost constraints, it is impractical to equip all sensors in a large-scale sensor network with extra hardware to obtain accurate distance estimates and thus accurate location estimates. Although a number of connectivity-based localization algorithms have been proposed, see e.g. [5]–[8], achieving high localization accuracy usually demands a comparatively large number of anchor nodes, hereafter termed simply anchors, whose positions are known a priori (accordingly, we term other nodes whose positions are not known and need to be determined as sensors). Therefore, it is attractive to develop low-cost distance estimation methods with comparatively good accuracies. In this paper, we shall propose such a method which does not rely on extra hardware but provides comparatively accurate distance estimates.

In a static wireless sensor network, two nodes are termed one-hop neighbors as long as they can directly communicate with each other. An intuitive observation shows that with a higher probability two geographically close nodes share more common one-hop neighbors than two distant nodes. We quantify and exploit this observation to develop a maximum-likelihood estimator (MLE) for estimating the distance between a pair of neighboring nodes based on their
local connectivity information. Herein, local connectivity information refers to the numbers of common and non-common one-hop neighbors associated with this pair of nodes. Since only elementary computations and local connectivity information are involved on each node, the proposed method is energy efficient and totally distributed.

In this paper, we present the distance estimation method under a generic channel model, including the ideal unit disk (communication) model and the more realistic log-normal (shading) model as special cases. [See Section II for further details.] Then, we take the log-normal model as an example to demonstrate the proposed method: the impact of uncertainties in the log-normal model is examined; the bias and standard deviation of distance estimates produced by the method are numerically evaluated; the proposed method, though not comparable to the fine-grained distance estimation techniques like TOA and TDOA, outperforms the well-known RSS method for long distances; moreover, we analyze the influences of various factors on the problem of estimating distances using such local connectivity information based on the Cramér-Rao lower bound (CRLB), and derive useful guidelines for implementing the proposed method in reality; finally, on applying the proposed method based on realistic measurement data and also in connectivity-based sensor localization, the superiority of the proposed method is confirmed.

Prior to our work, [9], [10] came up with the methods of estimating distances based on the same idea as ours. The Neighborhood Intersection Distance Estimation Scheme (NIDES) presented in [9] heuristically relates the distance, e.g. from $A$ to $B$, to an easily observed ratio of two numbers, i.e. the number of their common one-hop neighbors and the number of one-hop neighbors of $A$, and then performs the distance estimation at node $A$ using this ratio and other a priori known information. The NIDES assumes the unit disk model and works in uniformly and randomly deployed wireless sensor networks. Its enhanced version presented in [10] adapted the ratio by taking into account the number of one-hop neighbors of $B$ and heuristically stated that the NIDES could be applied in arbitrary radio models. Although it turns out that the enhanced NIDES leads to the same solution as ours, their entire treatment rests on empirical observations and heuristic formulations rather than theoretical foundations. In comparison to their work, the contributions of this paper are: (1) a statistical model is formally established for the distance estimation problem and an MLE solution with mathematical proofs of the correctness is provided; (2) the problem is considered under a generic channel model widely used in the literature, including the more realistic log-normal model; (3) the performance of the proposed method is
comprehensively analyzed under the log-normal model in terms of bias, standard deviation and root mean square error (RMSE); (4) a CRLB analysis is carried out for the problem of estimating distances via connectivity; (5) it is shown that the proposed method contributes to the quality of connectivity-based sensor localization.

The remainder of the paper is organized as follows. Section II introduces the network model and the (radio) channel model. Section III proposes the method under a generic channel model. Under the log-normal model, Section IV analyzes the performance of the proposed method; Section V provides a CRLB analysis for the general distance estimation problem using connectivity; Section VI implements the proposed method in a real environment; Section VII reports the contributions of the proposed method to connectivity-based sensor localization by simulations. Finally, we conclude the paper in Section VIII.

II. SYSTEM MODEL DESCRIPTION

This section briefly introduces the system model we shall use, including the network model and the channel model. Throughout this paper, we shall use the following mathematical notations: $\Pr\{\cdot\}$ denotes the probability of an event; $E(\cdot)$ denotes the expectation of a random variable.

A. Network Model

In static wireless sensor networks, nodes are often assumed to be randomly and uniformly distributed on account of the random nature of the network deployment. A homogeneous Poisson process provides an accurate model for a uniform distribution of nodes as the network size approaches infinity. Therefore, we consider a static wireless sensor network which is deployed over an infinite plane according to a homogeneous Poisson process of density $\lambda$.

B. Channel Model

Let $P_T$ be the transmitted signal power by a transmitter and $P_R(d)$ be the received signal power by a receiver located at distance $d$ from the transmitter. According to [11], the log-normal model predicts $P_R(d)$ to be log-normally distributed, and $P_R(d)$ is typically modeled as follows

$$P_R(d)(\text{dBm}) = P_R(d_0)(\text{dBm}) - 10\alpha \log_{10} \frac{d}{d_0} + Z,$$

$$P_R(d) = \tau^2 G_T G_R P_T \frac{1}{(4\pi)^2 d^\alpha},$$

(1)
where $P_R(d_0)$ (dBm) is the mean received signal power (in dBm) at a reference distance $d_0$, $\alpha$ is the path-loss exponent, $G_T$ and $G_R$ are the transmitter and receiver antenna gains, $\tau$ is the wavelength of the propagating signal in meters, and $Z$ is a random variable representing the shadowing effect, normally distributed with mean zero and variance $\sigma_{dB}^2$ (in dBm). Typically, $\alpha$ can vary between 2 in free space to 6 in heavily built urban areas and $\sigma_{dB}$ is as low as 4 and as high as 12 according to [11].

If $P_R(d)$ is no less than some specified value $P_c$, the receiver is able to communicate with the transmitter. Particularly, if $\sigma_{dB} = 0$, the log-normal model is equivalent to the unit disk model with the transmission range

$$r = \left(\frac{\tau^2 G_T G_R P_T}{(4\pi)^2 P_c}\right)^{\frac{1}{\alpha}}. \quad (3)$$

Hence, under the unit disk model, the communication coverage of each node is a perfect disk of radius $r$. For ease of presentation, $r$ is generalized to be a parameter defined by (3) other than the transmission range of the unit disk model.

In effect, the randomness on the received signal power $P_R(d)$ can be described by a function $g(d)$ denoting the probability that a directional communication link exists from transmitter to receiver with distance $d$. Based on $g(d)$, a generic channel model can be defined once $g(d)$ satisfies the following restrictions:

$$\begin{cases} g(d_1) = g(d_2), & \text{if } d_1 = d_2; \\ g(d_1) \leq g(d_2), & \text{if } d_1 \geq d_2; \\ 0 < \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\sqrt{x^2 + y^2}) \, dx \, dy < \infty. \end{cases} \quad (4, 5, 6)$$

The generic channel model has been treated intensively in percolation theory [12], [13]. The first restriction indicates that the propagation path is symmetric; the second one indicates that $g(d)$ must be a non-increasing function of $d$; the third one avoids the trivial cases that any two nodes are directly connected with probability 1 and that any two nodes are isolated with probability 1. It can be easily shown that both the unit disk model and the log-normal model satisfy these restrictions [14].

Throughout the paper, we make the following assumptions (as is commonly the case in the literature):

**Assumption 1**: The attenuations caused by shadowing effects (i.e. $Z$) between any pairs of nodes are independent and identically distributed;
Assumption 2: Communication links are symmetric, namely that node $v$ can directly receive packets from node $u$ as long as node $u$ can directly receive packets from node $v$.

Even though field measurements in real applications seem to indicate that the attenuations between two links with a common node are correlated [15], Assumption 1 is generally considered appropriate for far field transmission and is widely used in the literature [14]–[19]. Although the assumptions may not fully reflect a real network environment, they still enable us to obtain some results as estimates for more realistic situations.

As transmit power of each node is actually tunable in many wireless sensor networks [20], we assume

Assumption 3: During the period of running the proposed distance estimation method, all nodes transmit at a fixed power level, i.e. $P_T$.

III. THE DISTANCE ESTIMATION METHOD

In this section, we present the method of estimating distances via connectivity and detail its implementation under the log-normal model.

A. Estimating Distances under the Unit Disk Model

In a static wireless sensor network, given two nodes $A$ and $B$ with coordinates $(x_A, y_A)$ and $(x_B, y_B)$, their distance is defined to be $d$ ($d \leq r$) and two disks with the same radius $r$ represent their individual communication coverage under the unit disk model, as shown in Fig. 1. Due to
$d \leq r$, the two disks intersect and create three disjoint regions. Regarding $r$ as a constant, we define $S = \pi r^2$ and $f(d)$ to be the area of the middle region in Fig. 1, where

$$f(d) = \frac{2S}{\pi} \arccos\left(\frac{d}{2r}\right) - d\sqrt{r^2 - \frac{d^2}{4}}.$$ (7)

It is obvious that the nodes residing in the middle region are common one-hop neighbors of $A$ and $B$, the nodes residing in the left (right) one are non-common one-hop neighbors of $A$ ($B$). Define three random variables $M, P$ and $Q$ to be the numbers of the three categories of neighbors. Obviously, they are mutually independent and Poisson with means $\lambda f(d), \lambda(S - f(d))$ and $\lambda(S - f(d))$, as pointed out in [13]. The actual values of $M, P$ and $Q$ can be easily obtained after $A$ and $B$ exchange their neighborhood information. Based on the observations of $M, P$ and $Q$, an MLE for estimating $d$ is summarized as follows:

**Theorem 1:** $M, P$ and $Q$ are mutually independent Poisson random variables with means $\lambda f(d), \lambda(S - f(d))$ and $\lambda(S - f(d))$, respectively. If $f(d)$ is invertible and $S$ is a non-zero constant, then the MLE for $d$, termed $\hat{d}$, is

$$\hat{d} = \begin{cases} f^{-1}(S), & \text{if } M=P=Q=0; \\ f^{-1}(\hat{\rho}S), & \text{otherwise}; \end{cases}$$

where $\hat{\rho} = \frac{2M+P+Q}{2M+P+Q}$. (8)

**Proof:** See Appendix A. \hfill \blacksquare

Note that the actual value of $\lambda$ is not needed in obtaining $\hat{d}$. Though nodes are assumed to follow a random and uniform distribution of density $\lambda$, the derivation of the MLE indicates that as long as nodes in a local region which covers the communication coverage of two neighboring nodes admits a uniform density, the proposed method is reasonably applicable. Moreover, in some sensor network applications, nodes may be placed based on some regular structures but with certain noises. For example, in a two-dimensional sensor network, the $x$- and $y$-coordinates of each node are Gaussian with same variance and the mean values at one grid point. Compared to a random and uniform distribution, such a distribution is even closer to be uniform, and the proposed method can thus attain better performance. In addition, if the least squares method instead of the MLE is applied here, the resulting expression of the distance estimator is actually the same as that in Theorem 1.
B. Extension under the Generic Channel Model

Under the generic channel model defined by $g(d)$, $M$, $P$ and $Q$ are still used to denote the numbers of common and non-common one-hop neighbors associated with two nodes. First, we can compute their expectations as follows:

$$E(M + P) = E(M + Q) = \lambda \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\sqrt{(x - x_B)^2 + (y - y_B)^2})
dxdy, \quad (10)$$

$$E(M) = \lambda \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\sqrt{(x - x_A)^2 + (y - y_A)^2})g(\sqrt{(x - x_B)^2 + (y - y_B)^2})
dxdy. \quad (11)$$

Then, from the third restriction on $g(d)$, i.e. (6), it follows that $E(M) < E(M + P) < \infty$.

Unlike the unit disk model where the independence among $M$, $P$ and $Q$ is straightforward due to having three disjoint regions, the generic channel model does not necessarily lead to such three disjoint regions. The following theorem guarantees the mutual independence.

Theorem 2: Suppose a static wireless sensor network is deployed in an infinite plane according to a homogeneous Poisson process of density $\lambda$ and conforms to the generic channel model defined by $g(d)$; given two nodes in this wireless sensor network, let $M$ be the number of their common one-hop neighbors and $P$ and $Q$ be the numbers of their non-common one-hop neighbors. Then $M$, $P$ and $Q$ are mutually independent Poisson random variables.

Proof: See Appendix B.

Under the generic channel model, $S$ and $f(d)$ are just tools used to specify the expectations of $M$, $P$ and $Q$ other than the areas defined under the unit disk model, and are formulated as follows:

$$S = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\sqrt{(x - x_B)^2 + (y - y_B)^2})dxdy, \quad (12)$$

$$f(d) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\sqrt{(x - x_A)^2 + (y - y_A)^2})
g(\sqrt{(x - x_B)^2 + (y - y_B)^2})dxdy}{g(\sqrt{(x - x_B)^2 + (y - y_B)^2})}. \quad (13)$$

Therefore, if $S$ and $f(d)$ satisfy the conditions in Theorem 1, the MLE is applicable here.

In reality, however, sensor networks are deployed in regions of finite areas, and thus the expectations of $M$, $P$ and $Q$ associated with the nodes, especially those near the network boundaries, cannot be derived by simply integrating over an infinite plane to compute $S$ and $f(d)$. This is termed boundary effects. In this study, we concentrate on the theoretical foundations of the proposed method and will tackle boundary effects in our future work.
Prior to implementing the proposed method in a static wireless sensor networks, it is requested to know the wireless channel, i.e. \( g(d) \), such that the quantity \( S \), the function \( f(d) \) and its inverse can be determined offline and then programmed into each node. Due to the simple mechanism of the proposed method, a distributed protocol can be easily developed for collecting and exchanging local connectivity information by each node through broadcasting operations. Once a node obtains neighboring information of all its immediate neighbors, this node is able to estimate the distances from its immediate neighbors by using the inverse of \( f(d) \), \( S \) and the MLE in Theorem 1.

If Assumption 2 holds, each pair of neighboring nodes will have identical information for estimating their distance and thus will obtain the same distance estimate; otherwise, provided that \( A \) can hear \( B \) but \( B \) cannot hear \( A \), \( A \) will estimate their distance, whereas \( B \) will not. Such asymmetry in distance estimation can be alleviated by allowing each node to exchange two-hop neighborhood information with its immediate neighbors.

Clearly, if the size of each sensor’s neighborhood has the magnitude of \( O(1) \), the complexities for communications and computations of running the proposed method in this network are both \( O(n) \), where \( n \) is the number of nodes in a wireless sensor network. Therefore, the proposed method is efficient and scalable.

C. Implementation under the Log-normal Model

Provided that the wireless channel is known to be log-normal with known parameters \( \sigma_{dB} \) and \( \alpha \), we shall demonstrate how to determine \( g(d) \), \( S \) and \( f(d) \) involved in the proposed method.

1) Formulating \( g(d) \): Under the log-normal model, for a transmitter and receiver pair with distance \( d \), if the received signal power described by (1) is no less than \( P_c \), a bi-directional communication link exists between them (as a result of the symmetry in Assumption 2). The probability that the two nodes are able to communicate with each other, i.e. \( g(d) \), is

\[
g(d) = \int_{k \log \frac{d}{r}}^{\infty} e^{-\frac{z^2}{2\sigma_{dB}^2}} dz
\]

where \( k = \frac{10\alpha}{\log 10} \) and \( \log \) denotes natural logarithm. Evidently, \( g(d) \) is determined by \( r \), \( \alpha \) and \( \sigma_{dB} \), where \( r \) depends on \( P_T \) and \( P_c \), and the other two parameters can be derived by measurement obtained prior to the deployment of sensor networks or empirically assigned based on characteristics of the deployment environment [11]. Alternatively, using the technique
developed in [21], an online estimation of $\alpha$ and $\sigma_{dB}$ is applicable, where the estimate of $\alpha$ is comparatively accurate but that of $\sigma_{dB}$ involves a slightly larger error.

We plot $g(d)$ with respect to $d$ and $\sigma_{dB}$ given $\alpha = 4$ and $r = 1$ in Fig. 2(a). It can be seen that: the smaller is $d$, the higher is the probability that a communication link exists; a larger $\sigma_{dB}$ tends to inhibit communications for a smaller $d$ but promotes communications for a larger $d$ in comparison with a smaller $\sigma_{dB}$.

In view of the restrictions on $g(d)$, it is straightforward to obtain $\lim_{d \to \infty} g(d) = 0$; as such, for an extremely small and positive $\varepsilon$, there exists $d_{th}$ such that $g(d) < \varepsilon$ if $d > d_{th}$. That is to say, nodes with distances to a node longer than $d_{th}$ hardly communicate with this node directly; as such, $d_{th}$ is a surrogate of the transmission range. This phenomenon can be observed in Fig. 2(a).

2) Formulating $S$ and $f(d)$: In [16], [18], the expectations $E(M + P)$ (or $E(M + Q)$) has been well studied. Thus, we can have

$$S = \pi r^2 e^{\frac{2\sigma_{dB}^2}{\alpha r^2}}. \quad (15)$$

By (11), (13) and (14), we can derive the formula for $f(d)$ under the log-normal model. By letting $\alpha = 4$ and $r = 1$, we plot $f(d)$ with respect to different values of $d$ and $\sigma_{dB}$ in Fig. 2(b). As can be seen, $f(d)$ is monotonically decreasing and invertible; hence, Theorem 1 is applicable under the log-normal model. But the closed-form formulas for $f(d)$ and its inverse are not available. Alternatively, we can establish a piecewise linear function to approximate its inverse and for each affine segment, a linear regression model is applied to predict $d$. 

Fig. 2. The functions $g(d)$ and $f(d)$ under the log-normal model with $\alpha = 4, r = 1$. 

![Graphs of g(d) and f(d) under log-normal model](graph.png)
Considering the fact that two nodes with distance longer than $d_{th}$ hardly communicate with each other directly, we restrict the distance estimates to be between 0 and $d_{th}$. But in a real estimation process, $\hat{\rho}S$ and $S$ may exceed $[f(d_{th}), f(0)]$ and as a result, $\hat{d}$ may exceed $[0, d_{th}]$. Therefore, we adapt the distance estimator as follows

$$\hat{d} = \begin{cases} 
0, & \text{if } M = P = Q = 0 \text{ or } \hat{\rho}S > f(0); \\
\rho^{-1}(\hat{\rho}S), & \text{if } f(d_{th}) \leq \hat{\rho}S \leq f(0); \\
d_{th}, & \text{if } \hat{\rho}S < f(d_{th}).
\end{cases}$$

IV. PERFORMANCE ANALYSIS

In this section, we evaluate the performance of the proposed method under the log-normal model from different respects.

A. Impact of Imprecise $\alpha$ and $\sigma_{dB}$

In the proposed method, the parameters $\alpha$ and $\sigma_{dB}$ are supposed to be known precisely. But what if their values are imprecise? To answer this question, we define a function as follows

$$\rho_{\alpha, \sigma_{dB}}(d) = \frac{f(d)}{S}, \quad (16)$$

where $f(d)$ and $S$ are computed using (13), (15) and (14) given $\alpha$ and $\sigma_{dB}$. Thus, the distance estimator is equivalently

$$\hat{d} = \begin{cases} 
0, & \text{if } M = P = Q = 0 \text{ or } \hat{\rho} > \rho_{\alpha, \sigma_{dB}}(0); \\
\rho^{-1}_{\alpha, \sigma_{dB}}(\hat{\rho}), & \text{if } \rho_{\alpha, \sigma_{dB}}(d_{th}) \leq \hat{\rho} \leq \rho_{\alpha, \sigma_{dB}}(0); \\
d_{th}, & \text{if } \hat{\rho} < \rho_{\alpha, \sigma_{dB}}(d_{th}).
\end{cases} \quad (17)$$

Therefore, using imprecise $\alpha$ and/or $\sigma_{dB}$ results in an incorrect function $\rho^{-1}_{\alpha, \sigma_{dB}}(\hat{\rho})$, with the result that the distance estimate $\hat{d}$ is degraded. We plot the function $\rho^{-1}_{\alpha, \sigma_{dB}}(\hat{\rho})$ with respect to different values of $\alpha$, and $\sigma_{dB}$ in Fig. 3.

Supposing $\sigma_{dB}$ is known to be exactly 4, we investigate the impact of imprecise $\alpha$. As shown in Fig. 3(a), $\rho^{-1}_{\alpha, \sigma_{dB}}(\hat{\rho})$ is much more sensitive to a small $\alpha$ than to a large $\alpha$; in other words, for a small $\alpha$, using an imprecise version of $\alpha$ tends to degrade the distance estimate $\hat{d}$ more seriously than for a large $\alpha$. Moreover, if $\alpha$ is overestimated, then an underestimated $\hat{d}$ will be produced for a small $d$ but an overestimated $\hat{d}$ for a large $d$, and vice versa.
However, $\rho^{-1}_{\alpha,\sigma_{dB}}(\hat{d})$ does not demonstrate the same sensitivity to $\sigma_{dB}$ as is observed for $\alpha$, as illustrated in Fig. 3(b). We can conclude that, if $\sigma_{dB}$ is overestimated, then an overestimated $\hat{d}$ will be produced for a small $d$ but an underestimated $\hat{d}$ for a large $d$, and vice versa.

B. Bias and Standard Deviation

According to Theorem 1, all possible values of $\hat{\rho}$ are rational numbers within $[0, 1]$, so that $\hat{d}$ is a discrete random variable and its $j$-th moment is as follows

$$E(\hat{d}^j) = \sum_a \left[ \hat{d}^k Pr(\hat{d} = a) \right]. \tag{18}$$

We divide the range of $\hat{d}$, i.e. $[0, d_{th}]$, into $w$ equal intervals: $\mathcal{I}_1 = [z_0, z_1), \cdots, \mathcal{I}_w = [z_{w-1}, z_w]$ with $z_i = \frac{i \times d_{th}}{w}$. Given a sufficient large $w$, $\hat{d}$ is approximately constant over each interval, denoted $\tilde{d}_i$. Then we can approximately reformulate (18) as

$$E(\hat{d}^j) \approx \sum_{i=1}^w \left[ (\tilde{d}_i)^k Pr(\hat{d} \in \mathcal{I}_i) \right]. \tag{19}$$

Towards the probability associated with the $i$-th interval $\mathcal{I}_i$, we have

$$Pr(\hat{d} \in \mathcal{I}_i) = \begin{cases} Pr(f(z_1) < \rho S \leq S), & \text{if } i = 1; \\ Pr(f(z_i) < \rho S \leq f(z_{i-1})), & \text{if } 1 < i < w; \\ Pr(0 \leq \rho S \leq f(z_{w-1})), & \text{if } i = w. \end{cases}$$
By letting \( Y = P + Q \), we have
\[
Pr(b < \hat{\rho}S < c) = \sum_{y=0}^{\infty} [Pr(b < \hat{\rho}S < c|Y = y)Pr(Y = y)]
\] (20)
which makes it possible for us to numerically evaluate the the moments of \( \hat{d} \) and thus the bias and standard deviation. Let \( \mu \) be the expected number of one-hop neighbors of a node, namely \( \mu = E(M + P) = E(M + Q) \), and the values of \( \lambda \) with respect to different \( \sigma_{dB} \) and \( \mu \) are listed in Tab. I. For clear of presentation, the connectivity index \( \mu \) will be used in the following discussions instead of the node density \( \lambda \).

Fig. 4 depicts the numerical results with \( w = 1000 \) and simulation bias and standard deviation associated with the proposed method given \( \alpha = 4, r = 1 \) with \( \mu \) varying from 5 to 40. The two groups of results are highly consistent, and the comparatively non-smooth aspect of some of the curves, e.g. Fig 4(a), is probably attributable to the fact that all observations of \( M, P \) and \( Q \) in related results are necessarily integer, and such observations are used in determining the curves.

It can be seen from Fig. 4 that, distance estimates derived by our method are obviously biased, but the absolute bias is much less than the standard deviation in most cases; except for \( \sigma_{dB} = 0 \), the absolute bias and standard deviation are comparable to true distances, especially for short distances and sparse sensor networks, and particularly, when \( \mu = 5 \), their values are extraordinarily large and nearly twice the corresponding values when \( \mu = 10 \). Moreover, with \( \mu \) increasing, the standard deviation always reduces, while the absolute bias reduces in most cases. An intuitive explanation is that with \( \mu \) increasing, the variances of the ratios \( 2M/E(2M) \) and \( (2M + P + Q)/E(2M + P + Q) \) both decrease, the variance of \( \hat{\rho} \) is reduced, and so is the variance of \( \hat{d} \). As mentioned in the previous section, a large \( \sigma_{dB} \) promotes communications between distant nodes but inhibits communications between close nodes; as a result, connectivity is related to a wide range of distances, so that the geometric information implied by connectivity becomes less accurate. Hence, the larger is \( \sigma_{dB} \), the worse are both the bias and standard deviation.

C. RMSE

As a performance measure, the RMSE is defined to be the square root of the sum of the square bias and variance of estimation errors. We plot the RMSE of \( \hat{d} \) produced by the proposed method in Fig. 5. As can be seen, the RMSE decreases with \( \mu \) increasing and \( \sigma_{dB} \) decreasing, which is consistent with how the bias and standard deviation of \( \hat{d} \) depend on \( \mu \) and \( \sigma_{dB} \). When \( d \) is near
Fig. 4. The bias and standard deviation of distance estimation from numerical evaluations (solid lines) and simulations (dashed lines) with $\mu = 5, 10, 20, 30, 40$ and $\alpha = 4, r = 1$. For the standard deviation, a larger $\mu$ corresponds to a line to the bottom; for the bias, a larger $\mu$ corresponds to a line with the bias closer to 0.
0, the RMSE is extraordinarily large compared to the true value of $d$, implying that the proposed method fails to provide reasonable estimates for short distances. This underperformance of the proposed method with short distances limits its usage in practice, and is due to a mixed impact of the following facts:

- It is evident that the variance of $2M + P + Q$ is constant no matter what $d$ is, but the variance of $2M$ increases with $d$ decreasing; as a result, $\hat{\rho}$, i.e. $\frac{2M}{2M + P + Q}$, is more likely to suffer bigger variances when $d$ is small than when $d$ is large.

- As depicted in Fig. 3, $\rho_{\alpha,\sigma_{dB}}^{-1}(\hat{\rho})$ is quite sensitive to $\hat{\rho}$ when $d$ is small, namely that a small perturbation in $\hat{\rho}$ leads to a big change in $\hat{d}$ and thus a big distance estimation error.

- In light of (17), $\hat{d}$ is roughly set 0 when $\hat{\rho}$ is greater than $\rho_{\alpha,\sigma_{dB}}(0)$, but a small $d$ often causes $\hat{\rho}$ to be within $[\rho_{\alpha,\sigma_{dB}}(0),1]$ and hence the poor performance is attained.

To conclude, for short distances, the non-smooth aspect and the sensitivity to $\hat{\rho}$ of the function defined in (17) are responsible for the poor performance.

Under the log-normal model with $\sigma_{dB} > 0$, distance estimation can be realized by using the RSS measurements, i.e. received signal powers. The bias and variance of the resulting distance estimate (denoted $\hat{d}_{\text{RSS}}$) are provided in [22], so that we can compute the RMSE of $\hat{d}_{\text{RSS}}$ and compare it with that of $\hat{d}$ in Fig. 5. It can be seen that: (1) the RMSE of $\hat{d}_{\text{RSS}}$ increases in direct proportion to $d$, but that of $\hat{d}$ appears to have comparatively small variations with $d$ increasing; (2) the proposed method outperforms the RSS method for long distances by a large margin.

V. Analysis Based on the CRLB

In this section, we formulate the CRLB regarding the distance estimation problem via connectivity, i.e. estimating $d$ from $M$, $P$ and $Q$, under the log-normal model. For this estimation problem, the unknown parameters are $d$ and $\lambda$. The Fisher Information Matrix (FIM) for this estimation problem, denoted $\text{FIM}(d, \lambda)$, is

$$\text{FIM}(d, \lambda) = \begin{vmatrix} \lambda (f'(d))^2 \left( \frac{1}{f(d)} + \frac{2}{S-f(d)} \right) - f'(d) \\ - f'(d) \\ \lambda S - f(d) \end{vmatrix}$$

where $f(d)$ is differentiable to the first order in (13) (see Appendix C). Then, the CRLB for $d$ by using any unbiased estimator, denoted $\text{CRLB}(d)$, is

$$\text{CRLB}(d) = \frac{(S-f(d))(2S-f(d))f(d)}{2\lambda S^2(f'(d))^2}.$$
Though the CRLB is only valid for unbiased distance estimates and the proposed method is known to be biased, it is still helpful to understand the essential features of the distance estimation problem. In what follows, we shall investigate the influences of various parameters.

A. Influence of $\lambda$

It is clear that the CRLB is inversely proportional to $\lambda$. In other words, better estimation accuracy can be attained in dense wireless sensor networks, which is intuitive and is also illustrated in Fig. 5. Hence, it is attractive to apply the proposed method in dense wireless sensor networks. Dense wireless sensor networks, however, are really required in some circumstances. For example, due to the limited energy resource in each node, nodes are usually deployed in high density and may take turns to be active in order to prolong the network lifetime [23]; accordingly,
many scheduling strategies have been developed to determine when and which sensors should be
powered up and which sensors should be put into energy saving mode while satisfying certain
coverage and connectivity requirements [24]–[29].

B. Influence of d

According to (22), it is difficult to directly observe the influence of d on the CRLB, for we do
not have the closed-form formulas for f(d) and f'(d) except for the case of σdB = 0. But, since
it is easily justified that the numerator of (22) is bounded in a narrow range, if the denominator
can be very small, the CRLB will be seriously affected by the denominator. Based on Fig. 2(b),
we can obtain some preliminary knowledge about the key component in the denominator, i.e.
f'(d). As can be seen from Fig. 2(b), with d increasing from 0, |f'(d)| firstly experiences a rise
and then decreases after d is greater than some value which differs from σdB; when d increases
further, |f'(d)| continuously decreases and approaches 0. Hence, it is postulated that the CRLB
will experience a rise with d increasing.

As shown in Fig. 4, in the cases of σdB = 8, 12, the standard deviation displays an evident rise
when d is larger than some value, and then drops when d approaches dth. The reason causing
such a drop is that we restrict the maximal distance estimate to be dth, so that estimates for d
near dth are improved. In the case of σdB = 4, |f'(d)| is not so close to 0 when d is near dth, but
is comparatively small when d is near 0; as a result, the expecting rise does not happen.

C. Influences of PT and Pc

Provided that GT, GR and α are known in (3), r is proportional to (PT/Pc)α, where PT is the
transmission power PT and Pc is the threshold of power for communications. If both PT and
Pc are tunable in wireless sensor networks (which does not break Assumption 3), it will be
meaningful to analyze their influences on the CRLB. For simplicity, we shall use r instead of
PT and Pc to carry out the analysis. To do so, we derive the following theorem.

Theorem 3: Consider the CRLB which is defined based on (13), (15) and (22) and suppose
only d and r are variables. Then, the CRLB for a given distance d with r = r0 is equal to the
CRLB for a distance d/r0 with r = 1.

Proof: See Appendix D.
This theorem reveals that: (1) the CRLB is virtually determined by the ratio $\frac{d}{r}$; (2) the value field of the CRLB is invariant no matter how large $r$ is. That is, if the ratio $\frac{P_T}{P_c}$ is raised, distant nodes will tend to become connected, so that estimates for long distances will be available, but the CRLB will not exceed the value field of the CRLB associated with the original small value of $\frac{P_T}{P_c}$. Consequently, estimates for long distances will generally suffer less relative errors (i.e. the ratio of the estimation error to $d$) than those for short distances.

Notice that, because the CRLB is not monotonic with $d$, tuning $\frac{P_T}{P_c}$ does not definitely increase or decrease the corresponding CRLB associated with one given value of $d$. Since raising $\frac{P_T}{P_c}$ results in more one-hop neighbors for each node and consequently more distance estimates, though any distance estimate is not necessarily improved, more available distance estimates will benefit other applications, e.g. sensor network localization.

Furthermore, this feature can be exploited in the implementation of the proposed method. Considering the fact that in static wireless sensor networks the procedure of estimating distances is usually executed only once and probably in the beginning of the network lifetime, the ratio $\frac{P_T}{P_c}$ can be initially set a high value to achieve a high “sensor density” by increasing $P_T$ and/or decreasing $P_c$, and then is tuned to be a normal value after the phase of estimating distances. As a result, more estimates of long distances with comparatively good accuracies will be available.

VI. IMPLEMENTING THE METHOD IN PRACTICE

In this section, we improve the proposed method when dealing with short distances and then test the proposed method in a practical environment.

A. Dealing with Short Distances

Given $\alpha$ and $\sigma_{dB}$, define $\epsilon_{\alpha,\sigma_{dB}}$ to be the RMSE when $d = 0$. As illustrated in Fig. 5, the RMSE of distance estimates produced by the proposed method experiences small variations as $d$ increases from 0 up to $d_{th}$, so that if $d \geq \epsilon_{\alpha,\sigma_{dB}}$ the RMSE tends to be under $d$, implying that relatively good performance is attained. Moreover, on the grounds of the analysis in Subsection IV-C, we focus on the function defined by (17) with $\hat{\rho} \in [\epsilon_{\alpha,\sigma_{dB}}, 1]$, and reformulate it by a linear function

$$\frac{(1-\hat{\rho})\epsilon_{\alpha,\sigma_{dB}}}{1-\rho_{\alpha,\sigma_{dB}}(\epsilon_{\alpha,\sigma_{dB}})}$$

which smoothly transforms any $\hat{\rho}$ between $\rho_{\alpha,\sigma_{dB}}(\epsilon_{\alpha,\sigma_{dB}})$ and 1.
to a distance estimate between 0 and $\epsilon_{\alpha,\sigma dB}$. Consequently, (17) is updated to be

$$\hat{d} = \begin{cases} 
0, & \text{if } M = P = Q = 0; \\
\frac{(1-\hat{\rho})\epsilon_{\alpha,\sigma dB}}{1-\rho_{\alpha,\sigma dB}(\epsilon_{\alpha,\sigma dB})}, & \text{if } \hat{\rho} > \rho_{\alpha,\sigma dB}(\epsilon_{\alpha,\sigma dB}); \\
\rho_{\alpha,\sigma dB}^{-1}(\hat{\rho}), & \text{if } \rho_{\alpha,\sigma dB}(d_{th}) \leq \hat{\rho} \leq \rho_{\alpha,\sigma dB}(\epsilon_{\alpha,\sigma dB}); \\
d_{th}, & \text{if } \hat{\rho} < \rho_{\alpha,\sigma dB}(d_{th}).
\end{cases} \quad (23)$$

### B. Test the Method in a Real Environment

In [30], a sensor network consisting of 44 nodes was deployed in a real environment and RSS measurements between any two nodes were reported. Based on their measurement data, we can simulate a realistic environment to implement our method. According to [30], $\alpha = 2.3$, $\sigma dB = 3.92$ and $P_R(R_0)(dBm) = -37.47dBm$. But, to proceed with the experiment, we also need to specify the threshold power $P_c$, which essentially “defines” whether two nodes are connected, and $\epsilon_{\alpha,\sigma dB}$ in (23). After that, we can compute the function $g(d)$ associated with this channel, and then obtain the distance estimators based on (17) and (23), respectively. To avoid boundary effects as much as possible, we consider the four nodes near the center of the deployment region, i.e. nodes 15, 23, 24, 25, and only estimate the inter-node distances associated with the four nodes by using the originally proposed method and the method with the adjustment.

In this experiment, by letting $\epsilon_{\alpha,\sigma dB}$ be 0.5r and raising $P_c(dBm)$ from $-61dBm$ to $-52dBm$, the average distance estimation errors incurred by the original and adjusted methods are listed in Table II. According to the distance estimates produced by the RSS method (which were also provided by [30]), we compute the corresponding average distance estimation error, i.e. 1.07 m. As listed in the table, (i) the adjusted method always outperforms the originally proposed method; (ii) the original method outperforms the RSS method only in the case of $P_c(dBm) = -52dBm$, while the adjusted method outperforms the RSS method if $P_c(dBm)$ is between $-56dBm$ and $-52dBm$; (iii) though the average node degree increases with $P_c$ decreasing, the average error obtained with the adjusted method increases in general, a phenomenon which is attributable to boundary effects.

### VII. Applying the Method in Localization

In this section, we report the improvement in connectivity-based sensor localization by using the proposed method.
A. Connectivity-based Localization Algorithms

Connectivity-based localization algorithms, e.g. [6], [8], generally involve as a crucial component a mechanism of converting connectivity information into rough distance estimates, which are then used for localization as in distance-based localization algorithms.

The ad-hoc positioning system (APS) [6] approach includes three variants according to the propagation model of distances, termed by the authors DV-hop, DV-distance and Euclidean schemes. Here, we briefly introduce the first two schemes. The DV-hop scheme employs distance vector exchange. Both sensor and anchor exchange distance tables that contain the locations of and the hop counts to anchors with their corresponding neighboring nodes. Once an anchor obtains the distance tables from other anchors, it estimates an individual average distance per hop and broadcasts this average distance into the network. A sensor approximates its geographic distance to an anchor by multiplying the hop count to this anchor and the associated average distance per hop, and then estimates its location by performing trilateration if a sufficient number of distance estimates are obtained. The DV-distance scheme is almost the same as the DV-hop scheme except that it employs the geographic distances measured with the use of radio signals other than hop counts.

MDS-MAP [8] approximates the distance between any pair of nodes by the length of the shortest path between them, and then uses multidimensional scaling (MDS) to generate a relative map that represents the relative positions of nodes. Once a sufficient number of anchors are known, MDS-MAP estimates the absolute coordinates of all the sensors in the network. Like DV-distance, MDS-MAP is also able to employ geographic distance measurements; we term it MDS-MAP distance.

In both DV-hop and MDS-MAP, the distance between two nodes is roughly estimated according to the length of the shortest path between them; in other words, the one-hop distances along any shortest path are assumed to be equal. As opposed to this assumption, our method provides comparatively accurate estimates of one-hop distances, and thus helps to improve the quality of connectivity-based sensor localization, which will be demonstrated by simulations in the following subsection.

Many methods have been developed in the literature to improve DV-hop. For instance, in [31], estimating the distance from a sensor to an anchor not only uses the length of the shortest
path between them as in DV-hop, but also exploits the lengths of the shortest paths from this sensor’s one-hop neighbors to the anchor. Moreover, DV-hop suffers large errors in anisotropic networks, because the estimates of distances from sensors to anchors can be extraordinarily inaccurate. Accordingly, [32]–[34] were developed to alleviate the impacts of the anisotropic network topology on the estimates of distances. Since comparatively accurate estimates of one-hop distances provided by our method are the basis for estimating distances from sensors to anchors, it is attractive to combine our method with these DV-hop related methods to improve the estimates of distances from sensors to anchors and thus to improve localization accuracy.

B. Simulations

We simulate connectivity-based sensor localization under the log-normal model using: DV-hop and MDS-MAP, and their distance-based counterparts DV-distance and MDS-MAP distance (with distance measurements from the adjusted method).

To avoid boundary effect, we actually generate wireless sensor networks over a large square with side of 18, but only localize the nodes inside of a small one with side of 6 and concentric to the large one. However, the nodes outside of the small one are sometimes used in estimating distances between nodes inside. Four nodes inside the small square and closest to its four corners are chosen as anchors. Regarding the constants parameterizing the log-normal model, $\alpha$ is known to be 4, $\sigma_{dB}$ takes values from 0, 4, 8, 12, and $P_T$ and $P_c$ are properly assigned such that $r = 1$. Furthermore, $\lambda$ takes proper values such that $\mu$ varies from 10 to 40 with step size 5 (see Tab. I).

For each pair of $\sigma_{dB}$ and $\mu$, 100 independent runs are carried out. In each run, first, a static wireless sensor network is generated according to a homogeneous Poisson process of density $\lambda$; second, distance between any pair of neighboring nodes is estimated based on Theorem 1 and accordingly, the average absolute distance estimation error is computed; third, sensors are localized by using four localization algorithms: DV-hop, MDS-MAP and their distance-based counterparts DV-distance and MDS-MAP distance (with distances coming from the second step); finally, the average absolute distance estimation error and the average position estimation error are computed for each localization algorithm. Then, the average absolute distance estimation errors and the average position estimation errors are averaged over the 100 independent runs and plotted in Fig. 6 and 7.

As mentioned before, DV-hop and DV-distance are almost same, except that DV-hop assumes
identical one-hop distances along any shortest path, while DV-distance uses distance estimates with comparatively good accuracies; this is also true for MDS-MAP and MDS-MAP distance. Because only connectivity information with the assistance of four anchors is exploited to realize sensor localization in the simulations, the fact that DV-distance and MDS-MAP distance use distance estimates produced by our method and their superior performance imply the advantages of our method.

VIII. CONCLUSIONS

In this paper, we proposed the method of estimating distances via connectivity in static wireless sensor networks by dealing with a generic channel model, including the realistic log-normal model. The advantages of the proposed method include: not relying on extra hardware; totally distributed; energy efficient due to its simple mechanism and computations. Under the log-normal model, the bias and standard deviation of distance estimates from the proposed method were numerically evaluated and verified by simulations; in addition, it was shown that the proposed method outperforms the RSS method for long distances; we carried out a CRLB analysis for the problem of estimating distances using connectivity and derived useful guidelines for implementing the proposed method; on applying the proposed method based on realistic measurement data and also in connectivity-based sensor localization, the superiority of the proposed method is confirmed.

In future work, we may tackle estimating distances involving nodes near the network bound-
Fig. 7. Average position estimation errors with $\alpha = 4, r = 1$.

Appendix A

Proof of Theorem 1

Establish a statistical model: observations of $M, P$ and $Q$ provide measured data $\phi = [m \ p \ q]$ where $m, p, q$ are non-negative integers; the unknown parameters are $\theta = [d \ \lambda]$. By formulating the likelihood function, we can derive that the MLE for $d$ is the solution for $d$ in the following equation set

$$
\begin{align*}
    m f(d) - \frac{p + q}{S - f(d)} + \lambda &= 0, \\
    m + p + q \frac{1}{\lambda} - (2S - f(d)) &= 0.
\end{align*}
$$

(24) (25)
By eliminating $\lambda$, we can obtain

$$2mS = (2m + p + q)f(d).$$

(26)

If $2m + p + q > 0$, $\hat{d} = f^{-1}(\frac{2m}{2m+p+q}S)$; otherwise, the solution for $d$ is not well-defined, but because $f(d) = S$ maximizes the likelihood, we have $\hat{d} = f^{-1}(S)$. Thus, we prove the theorem.

**APPENDIX B**

**PROOF OF THEOREM 2**

Let the two nodes be $A$ and $B$ and consider a finite region of area $D$ covering $A$ and $B$. As pointed out in [13], apart from $A$ and $B$, the rest of the Poisson process is not affected, namely that the number of remaining nodes in this region, denoted $N$, is still Poisson with mean $\lambda D$. Choosing an arbitrary node $C$ from the $N$ nodes, one of the following cases hold: (1) $C$ can directly communicate with $A$ and $B$; (2) $C$ can directly communicate with $A$ but not $B$; (3) $C$ can directly communicate with $B$ but not $A$; (4) $C$ cannot directly communicate with $A$ or $B$.

Conduct a trial for each of the $N$ nodes to decide how it communicates with $A$ and $B$, and each trial results in the above four cases with probabilities $p_1, p_2, p_3, p_4$, respectively. Due to the independence of connectivity assumed in Assumption 1, the $N$ trials are then independent from each other. Evidently, $M, P, Q$ represent the numbers of nodes belonging to the first three cases. In addition, let the random variable $L$ denote the number of nodes belonging to the last case. Due to $p_1 + p_2 + p_3 + p_4 = 1$, $M, P, Q, L$ follow a multinomial distribution with parameters $N$ and $p_1, p_2, p_3, p_4$. Considering $N$ is Poisson with mean $\lambda D$, from the theorem on Page 8 in [13], it follows that $M, P, Q, L$ are mutually independent Poisson random variables with means $\lambda Dp_1, \lambda Dp_2, \lambda Dp_3, \lambda Dp_4$, respectively. Now, we let the region approach the infinite plane, and can conclude that $M, P, Q$ are mutually independent Poisson with finite means.

**APPENDIX C**

**THE EXISTENCE OF $f'(d)$**

At first, consider the following expression

$$\lim_{\varepsilon \to 0} \left( -\frac{1}{\varepsilon} \left[ g(\sqrt{x^2 + d^2} - 2xd\cos\theta) - g(\sqrt{x^2 + (d + \varepsilon)^2} - 2x(d + \varepsilon)\cos\theta) \right] \right)$$

$$= \frac{k(x \cos \theta - d)e^{-\frac{(k\log \sqrt{x^2 + d^2} - 2xd\cos\theta - \log r)^2}{2\sigma^2_{dB}}}}{\sqrt{2\pi \sigma_{dB}(x^2 + d^2 - 2xd \cos \theta)}}$$
which is bounded for \( x \in [0, +\infty) \). Moreover, the derivative of \( f(d) \) can be formulated as

\[
\frac{df}{d} = \int_0^\infty \int_0^{2\pi} g(x) x \lim_{\varepsilon \to 0} \left( -\frac{1}{\varepsilon} \left[ g(\sqrt{x^2 + d^2} - 2xd \cos \theta) - g(\sqrt{x^2 + (d + \varepsilon)^2} - 2x(d + \varepsilon) \cos \theta) \right] \right) d\theta dx.
\]

(27)

Because \( \int_0^\infty \int_0^{2\pi} g(x) xd\theta dx \) equals \( E(M + P) \) and is convergent, \( f'(d) \) is also convergent.

**APPENDIX D**

**PROOF OF THEOREM 3**

Regarding \( r \) as a variable, we substitute the notations as follows: \( S \to S(r) \), \( f(d) \to f(r, d) \), \( f'(d) \to \frac{\partial f(r, d)}{\partial d} \), \( CRLB(d) \to CRLB(r, d) \), \( g(d) \to g(r, d) \). According to (14), we have

\[
g(r, dr) = \int_0^\infty \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma dB} d\tau = g(1, d).
\]

By (13), we have

\[
f(r, dr) = \int_0^\infty \int_0^{2\pi} g(r, x) g(r, \sqrt{x^2 + (dr)^2} - 2xdr \cos \theta) x d\theta dx
\]

\[
= \int_0^\infty \int_0^{2\pi} g(r, x) g(r, r\sqrt{x^2 + d^2} - 2xd\cos \theta) r^2 x d\theta dx
\]

\[
= r^2 \int_0^\infty \int_0^{2\pi} g(1, x) g(1, \sqrt{x^2 + d^2} - 2xd \cos \theta) x d\theta dx
\]

\[
= r^2 f(1, d).
\]

Moreover, we can obtain

\[
\frac{\partial f(x, y)}{\partial y} \bigg|_{x=1, y=d} = \frac{1}{r} \times \frac{\partial f(x, y)}{\partial y} \bigg|_{x=r, y=dr}.
\]

(28)

By \( S(r) = r^2 S(1) \) (based on (15)), (22) and the above formulas, we can obtain

\[
CRLB(r, dr) = CRLB(1, d) \text{ equivalently, } CRLB(r, d) = CRLB(1, \frac{d}{r}).
\]

(29)

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REFERENCES


TABLE I
THE VALUES OF $\lambda$ WITH RESPECT TO DIFFERENT $\sigma_{dB}$ AND $\mu$ WHEN $\alpha = 4$.

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TABLE II
THE EXPERIMENTAL RESULTS WITH RESPECT TO DIFFERENT $P_c$.

<table>
<thead>
<tr>
<th>$P_c$(dBm)</th>
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<th>DEE Adjusted (m)</th>
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</table>

Note: average node degree (ND), average distance estimation error (DEE)
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