Spectrally Efficient Wireless Systems with Cooperative Precoding and Beamforming

Wibowo Hardjawana, Student Member, IEEE, Branka Vucetic, Fellow, IEEE, Yonghui Li, Senior Member, IEEE, and Zhendong Zhou, Member, IEEE

Abstract—Interference among multiple base stations that co-exist in the same location limits the capacity of wireless networks. In this paper, we propose a method to design a spectrally efficient cooperative downlink transmission scheme employing precoding and beamforming. The algorithm eliminates interference and achieves symbol error rate (SER) fairness among different users. To eliminate the interference, Tomlinson Harashima precoding (THP) is used to cancel part of the interference while the transmit-receive antenna weights are chosen to cancel the remaining interference. A novel iterative method is applied to generate the transmit-receive antenna weights. The convergence behaviour of the iterative process is investigated. To achieve SER fairness among different users and improve the performance of the system, we develop algorithms that provide equal signal-to-interference-plus-noise-ratios (SINR) across all users under both per Base Station (BS) and total BSs power constraints. Per BS and total BSs power constraints are constraints where the power for each BS and all BSs is limited to a particular value. The simulation results show that the proposed scheme outperforms existing cooperative transmission schemes in terms of the SER performance and complexity and closely approaches an interference free performance under the same configuration.

Index Terms—Cooperative communications, multi-user MIMO, beamforming, precoding, base station cooperation, zero forcing.

I. INTRODUCTION

The spectral efficiency in existing cellular mobile networks [1] and WLANs (Wireless Local Area Network) [2] is limited by interference. In cellular mobile networks, the dominant interference comes from adjacent cells [1], while in co-working WLANs [2], the interference from other networks, operating in the same area, is a major limiting factor [2]. In the cooperative transmission scheme proposed here, multiple base stations (BSs) share information about the transmitted messages to their respective users and wireless channels via a backbone network. Individual BSs are equipped with multiple transmit antennas. Each BS transmitter uses the information of the transmitted signals from other BSs and wireless channel condition to precode its own signal. The precoded signal for each BS is broadcast through all BS transmit antennas in the same frequency band at a given time slot. The precoding operation and transmit-receive antenna coefficients are chosen in such a way as to minimize the interference coming from other BS transmissions. The calculated receive antenna coefficients are then sent from the transmitter to the receiver through the wireless channel prior to the data transmission.

Most of the published papers in this area consider only a downlink multi-user multiple-input-single-output (MISO) broadcast channel. A variety of methods to suppress the interference from other users has been developed. In [3], a method to maximize individual signal-to-interference-plus-noise-ratio (SINR) by jointly adjusting the transmit weights and transmission powers was developed. A different approach was proposed in [4] where a combination of a zero forcing (ZF) method, that determines transmit weights by forcing part of the interference to zero, and dirty paper coding (DPC) [5] was used to suppress interference from other users. A more practical approach than [4] was considered in [6] where DPC is replaced with Tomlinson-Harashima precoding (THP) [7], [8]. These algorithms, however, only consider single receive antenna scenarios, which are not directly applicable to multiple-input-multiple-output (MIMO) systems. Recently, these works on downlink multi-user MISO systems have been extended to incorporate multiple receive antennas. In [9], the authors showed how a ZF method can be used to exploit the availability of multiple receive antennas. Here transmit-receive antenna weights are first jointly optimized by a ZF diagonalization technique and then a water-filling power allocation method is applied to allocate power to each user. The scheme in [9] is further improved in [10] by using an iterative method. Nonlinear methods, utilizing a combination of a ZF method with DPC and a combination of a ZF method with THP [7], [8] for a multi-user MIMO system were considered in [11], [12], respectively. The authors use the ZF method to eliminate part of the inter-link interference. DPC or THP is then used to cancel the remaining interference. These schemes, however, are not practical for cooperative MIMO systems, since their symbol-error-rate (SER) performance varies from user to user. In particular, this SER variation is not desirable since the MIMO systems can be deployed by different operators and they expect the systems to have a similar performance.

In this paper, we propose a cooperative transmission scheme employing precoding and beamforming for the downlink of a MIMO system. In this algorithm, the THP cancels part of the interference while the transmit-receive antenna weights cancel the remaining interference. A new novel iterative method is
used to generate the transmit-receive antenna weights. This transmit-receive antenna weights are optimized based on the iterative optimization method in [13]. The receive and transmit weights are optimized iteratively until the SINR for each user converges to a fixed value. The convergence behaviour of the proposed method is investigated both analytically and numerically. In addition to the iterative joint transmit-receive antenna weights optimization and THP above, we also employ SINR equalization, and an adaptive precoding ordering (APO) in the algorithm. The SINR equalization process [3] is used to allocate power to users in such a way that all users have the same SINRs, which ensures SER fairness among all users. We investigate two types of power constraints. The first one is a total BSs power constraint, where the total power for all BSs is constrained to a particular value. The second one is a total BSs power constraint, where the total power for all BSs is constrained to a particular value. The convergence behaviour of the proposed method is investigated both analytically and numerically. In addition to the iterative joint transmit-receive antenna weights optimization and THP above, we also employ SINR equalization, and an adaptive precoding ordering (APO) in the algorithm. The SINR equalization process [3] is used to allocate power to users in such a way that all users have the same SINRs, which ensures SER fairness among all users.

We investigate two types of power constraints. The first one is a total BSs power constraint, where the total power for all BSs is constrained to a particular value. The second one is a total BSs power constraint, where the total power for each BS is constrained to a particular value. We derive expressions for optimum power allocation under these two power constraints. The APO is then used to further improve the performance of MIMO systems by maximizing the minimum SINR for each user [14]. Simulation results show that the proposed scheme is significantly superior to the existing methods and is only 0.25 dB away from an interference free channel, under the same configuration. The proposed method offers a significant improvement over a nonlinear cooperative precoding algorithm presented in [9]–[12]. The first improvement is the enhancement of the SER performance compared to [9]–[12]. The second improvement is the relaxation of the zero forcing constraints. Unlike [11], [12], here we allow transmit signals intended for different users to interfere with each other. This interference is canceled at the receiver where the signal is multiplied by the receive antenna weights. The third improvement comes from the complexity reduction. The proposed scheme has a much lower computational complexity than the methods in [9]–[12]. The fourth improvement comes from the elimination of the dependency of the number of receive antennas to the number of transmit antennas. In the proposed method, it is not necessary for the number of receive antennas to be at least equal to the number of transmit antennas as required in [11] and [12]. This latter feature allows the proposed algorithm to be applied to a wider range of scenarios than the schemes in [11], [12], while providing a capacity-approaching performance. The proposed method can be used to improve the performance and the capacity of co-working WLANs and cellular mobile systems.

The remainder of this paper is organized as follows. Section II presents the system model. The design of the nonlinear cooperative precoding algorithm is introduced in Section III. Section IV describes the advantage of the proposed method over other existing schemes. Section V presents the numerical results. Finally, the conclusion is drawn in Section VI. The notations used in the paper are as follows. We use boldface lower case letters to denote vectors and boldface uppercase letters to denote matrices. The superscripts $^H, ^T, I$ and $\text{Diag}()$ denote the conjugate transpose, transpose, an identity matrix, and a diagonal matrix, respectively. $C^{a \times b}$ indicates a complex matrix with $a$ rows and $b$ columns. $\| \cdot \|$ and $\lfloor \cdot \rfloor$ are the greatest integer smaller than $x$, the Euclidean distance and the absolute value, respectively. $\text{LoT}(A)$ is defined as the operation to extract the lower triangular components of $A$ and $\text{UpT}(A)$ as defined as the operation to extract the upper triangular components of $A$ and to set the other components to zero. $\text{UpT}(A)$ is defined as the operation to extract the diagonal components of $A$ and to set the other components to zero. Lastly, we define $\mathbf{1}$ as a column vector with all entries equal to $1$.

II. SYSTEM MODEL

In this paper, we consider MIMO systems, where $K$ BSs transmit to $K$ MSs and BSs and MSs are equipped with $N_{BS}$ and $N_{MS}$ antennas, respectively. All BSs cooperate with each other to transmit data to their respective MSs via $KN_{BS}$ antennas. Each of these transmissions is defined as a link. In addition, we also assume that $K$ base stations communicate with $K$ mobile stations. In a practical cellular network or a WLAN, if FDMA or OFDMA multiple access is used, a frequency slot can only be allocated to one user at a given time slot. Therefore, only one user is actually served by a base station at any one time slot or frequency slot. Thus, to simplify the analysis, we consider a scenario whereby each station transmits at a given time slot to a single mobile station. The proposed method aims to enable $K$ such base stations deployed by different network operators in the same location to communicate simultaneously with $K$ respective mobile stations using the same frequency band at a given time slot.

A. Transmitter Structure

The transmitter for the proposed cooperative transmission with precoding and beamforming is shown in Fig. 1(a). Let $\mathbf{x} = [x_1 \cdots x_K]^T$ represent the modulated symbol vector, consisting of $M$-QAM ($M$-ary Quadrature Amplitude Modulation) modulated symbols where $x_k$ is the modulated symbol intended for transmission from BS $k$ to MS $k$. The transmitted symbols for each user are first permuted by a permutation matrix $\mathbf{M}_{\text{perm}} = [m_{i,j}]_{K \times K}$ where $m_{i,j} \in \{0,1\}$. This permutation operation is referred to as the adaptive precoding order (APO). The APO adaptively selects the precoding order of $x_k$ that maximizes the minimum SINR of $K$ users. It selects a suitable permutation matrix $\mathbf{M}_{\text{perm}}$ to permute the transmitted symbol vector. After the APO, $x_k$ for MS $k$ is permuted into $u_j$, which will be transmitted in link $j$. After the reordering, signal vector $\mathbf{u}$ is passed to the THP [7], [8], which performs a precoding operation to create THP precoded symbols arranged in a vector $\mathbf{v} = [v_1 \cdots v_K]^T \in C^{K \times 1}$. The THP precoding order of link $j$ is assumed to be $j$. That is, link 1 is precoded first and link $K$ is precoded last. In other words, we first precode $u_1$ to obtain $v_1$. We then precode $u_2$ by treating $v_1$ as known interference. We repeat the process until we precode $u_K$ by treating $v_1, \ldots, v_{K-1}$ as known interference. THP needs to perform the precoding $K$ times in precoding $u_j$. THP treats the interference from links $1, \ldots, j-1$ to link $j$ as known. To enable the cancellation of the known interference, THP uses a feedback matrix $\mathbf{M}_{TFHP}$ in the precoding operation. As discussed in [15], $\mathbf{M}_{TFHP}$ is strictly lower triangular to allow data precoding in a recursive fashion. The derivation of the feedback matrix will be explained in the
next section. The output of the THP precoder can be obtained as
\[ v_j = \text{mod}_M(u_j - \sum_{l=1}^{j-1} [M_{THP}]_{j,l} v_l), \quad j = 1, \ldots, K \] (1)
where \([M_{THP}]_{j,l}\) denotes the \((j,l)\)th component of \(M_{THP}\) and \(\text{mod}_M(u) = \text{mod}_M(u_1) \ldots \text{mod}_M(u_K)^T\) is an element-wise modulo operator [6]
\[ \text{mod}_M(u_j) = u_j - \sqrt{M} \left( (u_j + \sqrt{M}) / \sqrt{M} \right), \quad j = 1, \ldots, K. \] (2)
The SINR equalization module then allocates powers to each of \(K\) links in such a way that the received SINRs for all links are equal. This is done by multiplying \(v\) with the matrix \(P = \text{Diag}(\sqrt{p_1}, \ldots, \sqrt{p_K})\) where \(p_j\) is the power allocated to the precoded symbol \(v_j\) in link \(j\). The THP, however, cannot cancel the interference from links \(j + 1, \ldots, K\) to link \(j\), since this interference is unknown to link \(j\). This remaining interference needs to be suppressed by multiplying the transmitted signal from each link by the transmit antenna weights vectors of all BSs, denoted by \(T\), where \(T \in \mathbb{C}^{K \times N_{BS} \times K}\) and by the receive antenna weights vector, denoted by \(r_j\), where \(r_j \in \mathbb{C}^{N_{BS} \times 1}\), at the receiver of link \(j\). The transmitted signal, thus is given as \(x_T = TPv\).

B. Receiver Structure

The receiver for each link is shown in Fig. 1(b). Note that there is no cooperation among the receivers. We first denote the received signal vector for each link as \(y_j\) where \(y_j \in \mathbb{C}^{N_{BS} \times 1}\). The received power vector for \(K\) links, denoted by \(y\), \(y = [y_1 \ldots y_K]^T\), can be written as \(y = \text{HTHPv + RN}\) where \(H = [H_1 \ldots H_1 H_2 \ldots H_2]^T\), \(N = [n_1 \ldots n_K]^T\) and \(T = [t_1 \ldots t_K]\). \(n_j \in \mathbb{C}^{N_{BS} \times 1}\) is the noise vector for link \(j\), \(H_j\) and \(t_j \in \mathbb{C}^{K \times N_{BS} \times 1}\) are the channels matrix and transmit weights for link \(j\), respectively. After multiplying \(y\) by the receive weights \(R\), the received signal vector becomes \(\tilde{y} = \text{RHTPv + RN}\), where \(R = \text{Diag}(r_1 \ldots r_1 r_2 \ldots r_2)^T\) and \(\tilde{y} = [\tilde{y}_1 \ldots \tilde{y}_K]^T\) and \(\tilde{y}_j\) is the received signal at the input of the decoder used by link \(j\). \(r_j\) is the receive antenna weight vector for link \(j\), \(j = 1, 2, \ldots, K\). Here, the receive antenna weights are calculated at the transmitter and sent from the transmitter to the receiver through a wireless channel prior to each data block. The estimates of the transmitted symbols for link \(j\), denoted by \(\hat{u}_j\), can be recovered from \(\tilde{y}_j\) by applying an element-wise modulo operator in (2) to each \(\tilde{y}_j\), as \(\hat{u}_j = \text{mod}_M(\tilde{y}_j), \quad j = 1, 2, \ldots, K\). This decoding process is shown in Fig. 1(b). The received signal \(y\) can be further written as
\[ y = \text{RHTPv + RN} = (D + F + B)Pv + RN \] (3)
where \(D = \text{DiT(RHT)}, \quad B = U p_t^T(\text{RHT})\) and \(F = \text{LoT(RHT)}\). \(DP\) is a vector of scaled replicas of the transmitted symbols for \(K\) links. \(FP\) is defined as the front-channel interference matrix, since the rows \(j = 1, \ldots, K\) of \(FP\) represent the interference caused by front links \(1, \ldots, j - 1\) to link \(j\). \(BP\) is defined as the rear-channel interference matrix, since the rows \(j = 1, \ldots, K\) of \(BP\) represent the interference caused by rear links \(j - 1, \ldots, K\) to link \(j\). In the proposed scheme, THP cancels the interference caused by the front-channel interference, while the interference caused by the rear-channel interference is eliminated by the transmit-receive antenna weights optimization process.

III. NONLINEAR COOPERATIVE PRECODING ALGORITHM

A. THP Design at the Transmitter

We assume that the channel state information (CSI) for all users is available at the transmitter. The THP operation in Fig. 1(a) aims to cancel the front-channel interference by performing \(K\) successive precoding operations. This operation uses the feedback matrix \(M_{THP}\) which is strictly lower triangular and the modulo operator \(\text{mod}_M(\cdot)\). The THP operation to generate THP precoded symbols \(v = [v_1 \ldots v_K]^T\) can then be represented as [15]
\[ v_j = u_j + d_j - \sum_{l=1}^{j-1} [M_{THP}]_{j,l} v_l, \quad j = 1, \ldots, K \] (4)
where \([M_{THP}]_{j,l}\) denotes the \((j,l)\)th component of \(M_{THP}\).
\[ d_j = 2\sqrt{M}\Delta \quad \text{and} \quad \Delta \text{is a complex number whose real and imaginary parts are suitable integers selected to ensure the real and imaginary parts of } u_j \text{ are constrained into } (-\sqrt{M}, \sqrt{M}). \] Here, the integers for \(\Delta\) can be found by an exhaustive search across all integers [15]. Note that if \(d_j\) is selected as above, adding \(d_j\) to \(u_j\) is equivalent to performing a modulo operation to \(d_j + u_j\) [1], [7], [12], [15].

\[ u_j = \text{mod}_M(\tilde{v}_j) = \text{mod}_M(d_j + u_j). \] (5)
Here, \(\tilde{v} = [\tilde{v}_1 \ldots \tilde{v}_K]^T\) where \(\tilde{v}\) represents the modified symbol constructed by adding \(d_j\) to \(\hat{u}_j\). By using this notation, (4) can be written in a matrix format as
\[ \tilde{v} = (I + M_{THP})\nu. \] (6)
At each receiver, the signal at the input of the receive antennas, is the sum of the scaled replicas of the transmitted signals, and the rear-channel interference \(BP\nu\) and the front-channel interference \(FP\nu\). The rear-channel interference is cancelled by the jointly optimized transmit-receive antenna weights described in Section III-C. The received signals at the input of the THP decoder can be represented as \((D + F)\nu(I + M_{THP})^{-1}\tilde{v}\). The desired signals at the input of the THP decoder is given by \(DP\tilde{v}\). It consists of the scaled replicas of the transmitted symbols. By equating the actual received signal at the input of the THP decoder with the desired received signal and ignoring AWGN, we have
\[ (D + F)\nu(I + M_{THP})^{-1}\tilde{v} = DP\tilde{v}. \] (7)
The matrix \(I + M_{THP}\) on the left hand side of (7) is used to cancel the front-channel interference. In order to derive the feedback matrix \(M_{THP}\), we use (7) and (6) to obtain \(M_{THP} = (DP)^{-1}FP\). By using (7), the composite received signal for all \(K\) receivers, at the input of the THP precoder, from (6) and (3), can be represented as
\[ y = DP\tilde{v} + BF(I + M_{THP})^{-1}\tilde{v} + RN. \] (8)
It can be seen that the front-channel interference \( FPv = FP(I + M_{THP})^{-1}v \) term no longer exists in (8). This is so, because we are using \( M_{THP} \) as in (6), to force the summation of the front-channel interference and the scaled replica of transmitted signals to be equal with the desired received signal at the receiver end. Thus, as long as \( \hat{u} \) transmitted signals to be equal with the desired received signal of the front-channel interference and the scaled replica of transmitted signal estimate of link \( j \) at each receiver, \( \hat{y}_j \), can be obtained from (8) and expressed as

\[
\hat{y}_j = \sqrt{p_j}r_j^Hj(H_jt_j)v_j + \sum_{i=j+1}^{K} p_i r_i^Hj(H_jt_j)v_i + r_j^Hn_j. \tag{10}
\]

The SINR for link \( j \), can then be written as

\[
SINR_j = \frac{p_j r_j^Hj(H_jt_j)^H r_j E[v_j^Hv_j]}{r_j^H(\sum_{i=j+1}^{K} p_i H_i t_i (H_i t_i)^H E[v_i^Hv_i]) + \sigma^2 I_r j}. \tag{11}
\]

Maximizing the minimum SINR for each link, while maintaining it equal for all links, can be formulated as follows

\[
\max_{R,T,P} \min_{1 \leq i \leq K} SINR_i \quad \text{subject to} \quad (1) \quad T^HT = I, \quad (2) \quad r_j^Ht_j = 1 \quad \forall j, \quad (3) \quad 1^TP = P_{\text{max}}, \quad (4) \quad r_j^Hjt_j = 0 \quad (12)
\]

for \( j = 1, ..., K, i = j + 1, ..., K \) where \( P_{\text{max}} \) and \( p = [p_1 ... p_K]^T = P^{21} \) are the power constraint at the cooperative transmitter and the set of powers assigned to each link, respectively. Here the objective of (12) is to maximize the minimum SINR for each link. The first, second and third constraints in (12) are to ensure that the transmit-receive weight vectors are unitary vectors and the sum of the power allocated to each link does not exceed the maximum power available at the transmitter (total BSs power constraint). These constraints will bound the possible solution for \( R, T, \) and \( P \) and ensure the convergence of (12) to a solution. Finally, the fourth one is the ZF constraint which ensures the interference from links \( j + 1, ..., K \) to link \( j \) are fully cancelled. Here, to maximize the minimum SINR in (12), we reduce the SINR of the best link until the SINR of all links is equal. Thus, the optimal solution is reached when all links attain an equal SINR [3], [16]. This optimization problem, however, is difficult to solve as it is not jointly convex in variables \( R, T, \) and \( P \). To solve (12), we propose a sub-optimal solution that splits the problem into a 2-step optimization. The first step is to solve \( R, T \) and \( P \) iteratively, when \( p \) is fixed. Hence in this step we simply ignore the equalization of SINRs among all links. The second step is to solve \( p \) in a way that equalizes SINRs for all links under fixed \( R, T, \) and \( p \). Once \( R, T, \) and \( p \) are obtained, \( M_{THP} = DP^{-1}FP \) is computed. The process is described in Fig. 2, where \( i, f_i(\cdot) \) and \( g_j(\cdot) \) are the iteration number, a
function to generate transmit antenna weights for $K$ links and a function to generate the receive antenna weights vector for link $j$, respectively.

C. First Step

In the first step, we assume an equal power allocation for each link by setting $P = I$. (12) can then be simplified as

$$\max_{R, T} \quad SINR_i$$
subject to

$$T^H T = I, \quad (2) \quad r_j^H r_j = 1$$

$$1^T p = P_{max} \quad (13)$$

for $j = 1, ..., K$. To solve (13), we propose to alternately optimize $R$ and $T$ until they converge, under the ZF constraint in (12). We first assign the initial value of the receive antenna weights for $K$ links. The initial receive weights of $K$ links are given as $r_j^{(0)} = v_{svo}(H_j^H)$, $j = 1, ..., K$ where $v_{svo}(\cdot)$ is the Singular Value Decomposition operation (SVD) [17], to select the left singular vector of $H_j^H$, corresponding to the largest singular value. We then transform the system into a downlink multi-link MISO system by fixing $R = Diag(r_1^{(0)}, ..., r_K^{(0)})$. (3) can then be written as

$$\mathbf{y} = RH^T \mathbf{v} + \mathbf{R} \mathbf{n} = H^T \mathbf{v} + \mathbf{N}. \quad (14)$$

Here, we know from (9) that the interference from links $1, ..., j - 1$ to links $j = 1, ..., K$ does not exist at the receiver, after performing decoding, since this front-channel interference is totally canceled by the THP described in Section III-A. The remaining interference is the rear-channel interference, coming from links $j + 1, ..., K$ to links $j = 1, ..., K$ which needs to be canceled. At each iteration, we apply a QR decomposition [17] to $H^H_j$ to find $T$ that forces this interference to zero,

$$T = f_1(R), \quad f_1(R) = [Q | QR(H_j^H)]. \quad (15)$$

We choose the unitary matrix $Q$ obtained from the QR decomposition of $H^H_j$ in (15) as $T$. We need to compute $R$ that gives a maximum SINR for each link for the derived $T$. This can be calculated as

$$r_j = g_j(T) \quad (16)$$

where $j = 1, ..., K$. $g_j(T)$ is a function that generates the receive weights vector $r_j$ for the derived $T$ such that SINR for each link is maximized. We now describe how $g_j(T)$ operates. By using (11), the SINR maximization for each link can be written as

$$\max_{r_j} \frac{p_j r_j^H \tilde{h}_j r_j}{r_j^H R_j r_j} \quad (17)$$

where $\tilde{h}_j = H_j t_j$ and

$$R_j = \sum_{i=j+1}^{K} p_i H_i t_i (H_i t_i)^H + \sigma^2 I \quad (18)$$

is the interference in link $j$. To obtain $r_j$ that maximizes SINR in (17) we use the Spectral/Eigenvalue Decomposition [17]. Thus the functions to generate $r_1, ..., r_K$, $g_j = 1, ..., K$ can now be written as

$$g_j(T) = v_{EVD}(R_j^{-1} \tilde{h}_j \tilde{h}_j^H), \quad j = 1, ..., K \quad (19)$$

where $v_{EVD}(\cdot)$ is the Spectral/Eigenvalue Decomposition operation [17], to select the left singular vector of $p_j R_j^{-1} \tilde{h}_j \tilde{h}_j^H$, corresponding to the largest singular value. We can obtain a simpler expression for $r_j$ that gives the same maximum SINR, as in (17), by using the following fact,

$$\max_{r_j} \frac{r_j^H \tilde{h}_j \tilde{h}_j^H r_j}{r_j^H R_j r_j} = \max_{r_j} \frac{r_j^H \tilde{h}_j}{r_j^H R_j r_j} \quad (20)$$

Here we state that the optimum $SINR_j$ obtained by using the term on the left hand side of (20) is equal to the optimum $SINR_j$ obtained by using the term on the right hand side of (20). The proof of their equivalence is shown in Appendix A. By solving the term on the right hand side of (20), the normalized receive antenna weight vector for link $j$ can be obtained as [18]

$$r_j = g_j(T) = \frac{R_j^{-1} H_j t_j}{\|R_j^{-1} H_j t_j\|.} \quad (21)$$

It is straightforward to show that the $SINR_j$ generated by using the receive antenna weights vector from (21) yields the optimum $SINR_j$ given in (20). We can conclude from this fact and (20) that the normalization process of the receive weights vector in (21) will not affect the SINR. Note that this receiver design is also known in the literature as the Minimum Variance Distortionless Response (MVDR) design [19]. The iterative calculations of $R$ and $T$ continue by fixing one and optimizing the other one, until they converge to a fixed solution. It is proved in Appendix B that the proposed iterative method always converges. This can be summarized in Lemma 1 as follows,

Convergence Lemma 1: The proposed iterative method to solve (13) converges to a local maximum and satisfies (15) and (16) as the number of iterations increases.

D. Second Step

In the second step, we use $R$ and $T$ obtained in the first step to find $p$. Using the fact that at the optimal solution all links will attain equal SINR and letting $a_{i,j} = r_i^H H_j t_j$, (11) can be written as

$$\sum_{i=j+1}^{K} |a_{i,j}|^2 p_i + \sigma^2 = \frac{p_j |a_{j,j}|^2}{SINR}. \quad (22)$$
(22) can be further represented in a matrix format as
\[
A^{-1}Bp + \sigma A^{-1}1 = \frac{p}{SINR}
\]  
(23)
where \( A = DiT(M) \), \( B = UpT(M) \) and \( M \) is a \( K \) by \( K \) matrix with entries \( |a_{i,j}|^2 \) in row \( i \) and column \( j \). By multiplying both sides of (23) with \( 1^T \), we obtain [3]
\[
\frac{1}{P_{max}} (1^T A^{-1}Bp + \sigma 1^T A^{-1}1) = \frac{1}{SINR}.
\]  
(24)
By defining the extended power vector \( p_e = [p^T \; 1]^T \), we can then combine (23) and (24) to obtain an equations matrix given as [3]
\[
\begin{bmatrix}
A^{-1}B \\
1^T A^{-1}B P_{max} \\
\sigma A^{-1}1 \\
\sigma 1^T A^{-1}1 P_{max}
\end{bmatrix}
\begin{bmatrix}
p \\
1^T P_{max} \\
p_e \\
1^T P_{max}
\end{bmatrix}
\]  
(25)
Hence the optimum \( p \) can be obtained by selecting \( p_e \) that corresponds to the maximum eigenvalue of \( \Psi \). This is the only possible solution for (25) satisfying \( p_j \geq 0 \) for \( j = 1, ..., K \) and \( SINR \geq 0 \). The proof is described in detail in Theorem 1 and 2 of [20].

Convergence Lemma 1: \( \prod_j \beta_j^{(i)} \leq det(R^*H^*T^*) \), where \( \beta_j^{(i)} = (H_j^* r_j^{(i)})^H t_j^{(i)} \) and \( (R^*)^* \) and \( (T^*)^* \) are the optimal transmit-receive antenna weights vectors for \( K \) links satisfying Convergence Lemma 1.

The proof of the lemma is presented in Appendix C. From Convergence Lemmas 1 and 2, we know that at the local maximum, \( 1 \prod_j \beta_j^{(i)} \) achieves the maximum value equal to \( det(R^*H^*T^*) \), 2) The front-channel interference matrix \( B \) converges to 0 since (15) forces \( R \) and \( T \) to have a lower triangular structure. Hence, we could simply maximize \( \beta_j^{(i)} \) to achieve the local maxima. By using the Matrix Inversion Lemma [21] (18) and (21) in \( \beta_j^{(i)} \), we can rewrite \( \beta_j^{(i)} \) for link \( j \) as
\[
\beta_j^{(i)} = (H_j^* r_j^{(i)})^H (\sigma^{-1} I - (Z^{-1} + \sigma I)^{-1}) H_j t_j^{(i)}
\]  
(29)
where \( Z = \sum_{a=j+1}^{K} \sum_{j=1}^{K} (H_j a^{(i)} (H_j a^{(i)})^H) \), representing a scaling/normalization factor. It is obvious that \( (H_j t_j^{(i)})^H (Z^{-1} + \sigma I)^{-1} H_j t_j^{(i)} \) in (29) reduces the value of \( \beta_j^{(i)} \). Therefore, if we omit this term in calculating the receive antenna weights, we can reach the maximum \( \beta_j^{(i)} \) faster. Thus, we can simply ignore this term to speed up the convergence of the iterative process. Therefore by omitting the term \( (H_j t_j^{(i)})^H (Z^{-1} + \sigma I)^{-1} H_j t_j^{(i)} \), we have \( \beta_j^{(i)} \sim (H_j t_j^{(i)})^H (\sigma^{-1} I t_j^{(i)}) = \sigma^{-1} (r_j^{(i)})^H H_j t_j^{(i)} \). The maximum \( \beta_j^{(i)} \) can be obtained by aligning \( r_j^{(i)} \) in the direction of \( H_j t_j^{(i)} \). The total power of the receive weights vector, \( r_j^{(i)} \) is normalized to 1, to ensure it satisfies the second constraint in (12),
\[
r_j^{(i)} = \frac{H_j t_j^{(i)}}{\| H_j t_j^{(i)} \|}.
\]  
(30)
We refer to this receiver structure as a Matched Filter (MF) design.

F. Adaptive Precoding Order

In the THP and the 2-step optimization process described in the previous sections, we fix the order of \( u_j \), resulting in a fixed permutation matrix \( M_{\text{perm}} \). The performance of the system, however, differs when a different \( M_{\text{perm}} \) is used. In addition, the performance of the system also depends on the weakest link. In this section we propose an APO scheme. APO arranges the order of \( x \) by selecting \( M_{\text{perm}} \) that maximizes the minimum SINR for each user. We formulate the optimization process to find a permutation matrix \( M_{\text{perm}} \in M_{\text{perm}} \) that gives the maximum SINR as 

\[
M_{\text{perm}} = \arg\max_{M_{\text{perm}}} \min_{j} \left( \frac{\text{SINR}_{1}(M_{\text{perm}}), ..., \text{SINR}_{K}(M_{\text{perm}})}{1} \right)
\]

where \( \text{SINR}_{j}(M_{\text{perm}}) \) is the SINR of link \( j \), given that the permutation matrix \( M_{\text{perm}} \) is used. To find the \( M_{\text{perm}} \) without searching from \( K! \) possible orderings, we use the idea of the Myopic Optimization method proposed in [14], which is proven to be optimal. Using this idea we now only need to search \( \sum_{i=0, i \neq 1}^{K-1} K-i \) possible orderings.

IV. THE COMPLEXITY COMPARISON OF THE PROPOSED AND OTHER KNOWN SCHEMES

In this section, we discuss the advantages of the proposed scheme over other existing schemes. We first compare the proposed method with the scheme in [9] and the iterative scheme in [10] where the transmit-receive weights are found by diagonalizing the receive signal matrix of \( K \) users without the receiver noise in (3). To have a fair comparison with the proposed method, we replace the water-filling power allocation in [9] with (26) and (28), that equalizes SINR for all links under the total BSs and per BS power constraints, respectively. This is required as the water-filling power allocation used in [9] tends to assign more power to stronger links and less power to weaker links. Hence, the performance of a weaker link will decrease the overall SINR for all links. The main differences between the methods in [9], [10] and the proposed method are 1) [9], [10] suppress both the front-channel and rear-channel interference using transmit-receive weights, while the proposed method suppresses the rear-channel and front-channel interference using THP and iterative transmit-receive weights, respectively. 2) Unlike [9], [10], the proposed scheme does not calculate null spaces. To compute these null spaces, the iterative scheme in [10] and the non-iterative scheme in [9] perform \( K \) SVD operations per iteration and \( K \) SVD operations, respectively. 3) Within a single iteration, a QR decomposition [17] and \( K \) MF receiver calculations are required to find all transmit-receive antenna weights while in [10], \( K \) SVD operations per iteration are required to find the transmit-receive weights of all links. Note that [9] requires \( K \) SVD operations to find the transmit-receive weights of all links. The complexity requirements in terms of the number of floating point operations (flops), for the proposed method and the methods in [9], [10] are listed in Table II, where \( i \) denotes the total number of iterations. Hence for \( K = 3 \), \( N_{MS} = 2 \), and \( N_{BS} = 2 \), the proposed method under total power and per BS power constraints has 339 and 387 flops per iteration, while the methods in [9] and [10], under the total BSs and per BS power constraints, have 13080 and 13080 flops per iteration, respectively.

The second comparison is done with the non-linear precoding methods in [11], [12]. Again to have a fair comparison with the proposed method, after the algorithm in [11], [12], we apply (25) to equalize the SINR, instead of using the original power allocation. Unlike [11], [12], we do not require the constraint of \((K-1)N_{MS} < KN_{BS}\) because we do not create null spaces. Thus, there is no relationship between the required number of transmit antennas and receive antennas. This is a definite advantage, since to support say 5 users with \( N_{MS} = 4 \), the proposed method only needs 5 transmit antennas while [9] needs 12 transmit antennas. Another important difference is in the zero forcing condition definition. In our scheme we have \( r_{j}^H H_{ij} t_i = 0 \), \( j = 1, ..., K, i = j + 1, ..., K \), whereas in [11] and [12] \( H_{ij} t_i = 0 \), \( j = 1, ..., K, i = j + 1, ..., K \). Using the zero forcing condition in our scheme, the proposed algorithm allows some inter-link interference to be transmitted (e.g. \( H_{ti} \neq 0 \)), and cancels the interference by steering \( H_{j} t_j \) to be perpendicular with the receive antenna weights vector \( r_j \). Hence the receive and the transmit antenna weights jointly cancel the interference. The zero forcing constraint in [11], [12], on the other hand, does not allow any inter-link interference to be transmitted. Here, the receive antenna weights are not used at all to cancel the interference. Lastly, the computational complexity required to find the null spaces and the transmit-receive antenna weights for the method in [11] is shown in Table II. The complexity of the method in [11], under the total BSs and per BS power constraints, for a system with \( K = 3 \), \( N_{MS} = 2 \), and \( N_{BS} = 2 \), is 8868 and 8916 flops, respectively.

V. NUMERICAL RESULTS AND DISCUSSION

For convenience, in our simulations, we will use the notations \((N_{BS}, N_{MS}, K)\) in all figures to denote a system with \( N_{BS} \) transmit antennas per BS, \( N_{MS} \) receive antennas per MS and \( K \) BSs. Monte Carlo simulations have been carried out to assess the performance of the proposed method. We investigate its performance and compare it with [9]–[12] and with an interference free performance. Here, an interference free performance is defined as the performance of any random single link \( i \) transmitted assuming there is no interference from other links at all. The transmitted power of this single user is set to 1. In this case, the received signal of the cooperative transmission system is given as \( y_i = r_i^H (H_i t_i x_i + n_i) \) where \( r_i \) and \( t_i \) are the left and right eigenvectors associated with the maximum eigenvalue of \( H_i^H H_i \) using SVD. The comparison of the schemes is performed at SER=10^{-4}. We use a fixed permutation matrix that orders MSs \( 1, ..., K \) as links \( K, ..., 1 \), when we are not using APO, for all the simulation results except when stated otherwise. Perfect CSI is assumed to be available at both ends. Rectangular 64-QAM \((M=64)\) modulation is used for all transmissions. The wireless channel model we used is a Rayleigh fading channel. This channel model is commonly used for cellular networks or WLANs, since in most cases there is no line-of-sight path between the transmitter and receiver in these networks. In order to simulate the wireless channel, we set each entry of the \( H_j \)
channel matrix as an i.i.d. complex Gaussian variable with a zero mean and unit variance. In all simulations, we fix the Signal-to-Noise-Ratio of each THP precoded symbol to be $SNR = \frac{E[v_1^2]}{\sigma^2}$, where $E[v_1^2]$ is normalized to 1, $P_{max} = K$ and $P_{max,i}=\ldots=K=P_{max}$. In all simulation figures, the proposed method refers to the algorithm with THP, joint iterative transmit-receive weights optimization, SINR equalization (SINRE) under the total BSs power constraint unless stated otherwise and Adaptive Precoding Order (APO).

### A. Convergence Study

Figs. 3 and 4 show the convergence characteristics of the proposed method with the total BSs power constraint for (2,2,3) and (1,2,4) systems. Note that here it does not matter whether total power or per BS constraints are used. This is because, as shown in Fig. 2, the SINR equalization is not an iterative process. We plot the number of iterations versus the average error and scaled output SINR (after SINR equalization), while fixing the SNR at 21 dB. The average error is defined as the average of the maximum entries of the front-channel interference BP, $e^{(i)} = \max_j |e^{(i)}_j|$, $j = 1, \ldots, K$, $i = 1, \ldots, K$ over all channel realizations. The output SINRs for (1,2,4) and (2,2,3) systems are scaled up by 4 dB and 0 dB to fit in one figure. The scaling does not matter here since we only want to observe the convergence rate. The figures also show the convergence characteristics when the MF receiver design, represented by (30), and the MVDR receiver design, represented by (21), are used.

An interesting observation is that, during the first few iterations, MVDR outperforms MF design. This improvement is due to smaller errors obtained using MVDR and not due to a higher signal gain $\beta_j^{(i)}$. During the first few iterations, the second term of (29) for the MF receiver design is larger than for the MVDR receiver design, thus leading to a higher average error for the MF receiver. This happens because MF ignores the interference when calculating the receive antenna weights. However, its average error decreases rapidly. The average error for (1,2,4) and (2,2,3) systems, denoted by (1,2,4)-MF and (2,2,3)-MF in Fig. 4, approaches the average error of the MVDR method. The SINR using the MF method from that point onwards is always greater than the SINR using the MVDR method. This is shown in the analysis in Section III-E. This analysis is consistent with the results shown in both Figs. 3 and 4. MF converges much faster to the optimal SINR solution than MVDR. This is shown in Fig. 3. This confirms Convergence Lemma 2 and the previous analysis. MF's SINR reaches a plateau after 8 iterations, since it almost converges to the optimal solution while MVDR’s SINR is still rising. Not much performance improvement can be obtained by increasing the number of iterations further. In all simulations for SER comparison, we set the maximum iteration number for the proposed scheme to 10.

### B. Performance of the individual links

Fig. 5 shows the SER of the worst user and the best user versus SNR in a (2,2,3) system. Note that it does not matter whether the total power or per BS constraints are used to compare the performance of the individual links. This is because, as shown in Section III-D, the SINR for all links will be the same under both constraints. As shown in Fig. 5, when the proposed method does not perform SINR equalization and APO (denoted by w/o in the figures), MS 3 has the best performance while MS 1 has the worst performance. The SER performance difference between links 1 and 3 exceeds 3 dB. Once SINR equalization is used, the SER difference between links disappears. This is shown in Fig. 5. Here, the performance of link 1 is improved at the expense of links 2 and 3. This results in a similar SER across all the links. An interesting point here is that APO tends to
equalize the performance of $K$ users even without the use of SINR equalization. This is shown in Fig. 5. Hence, it seems sufficient to use APO without SINR equalization to maximize the minimum SINR in the system. Its performance, however, is still worse than when the proposed method does not perform APO. This is denoted as "Proposed w/o APO" in Fig. 5. This suggests that SINR equalization plays a more important role than APO in performance improvement. In other words, using a good power allocation scheme might be more beneficial than searching for the best order of the users to achieve a higher diversity gain.

C. Performance of the overall SER

Here, we studied the performance of the overall SER. Overall SER is defined as the average SER of $K$ links. The overall SER performance for a $(2,2,3)$ system for both the proposed method with or without APO and for [9]–[12], is shown in Fig. 6. The proposed method without APO outperforms the methods in [11], [12] and [9] by 5 dB and 3 dB, respectively, and is only 1 dB away from an interference free performance when SER=$10^{-4}$. The large performance improvement in the proposed scheme with respect to [11], [12] comes from an increase in the degree of freedom and the iteration process used in determining the transmit-receive antenna weights. In addition, the proposed method without APO is able to achieve much better performance with much less complexity (we only use 10 iterations). The computational complexity of the proposed method for a $(2,2,3)$ system is on average about 75\% less than the complexity of methods in [10]–[12]. As for a $(1,2,4)$ system, we have on average about 50\% complexity reduction. In essence, the proposed method fully utilizes THP, transmit antennas and receive antennas in a more optimal way with much less complexity to create non interfering spatial channels. Fig. 6 also shows the performance of the iterative scheme in [10]–[12]. As for a $(1,2,4)$ system, we have on average about 50\% complexity reduction. In essence, the proposed method fully utilizes THP, transmit antennas and receive antennas in a more optimal way with much less complexity to create non interfering spatial channels. Fig. 6 also shows the performance of the iterative scheme in [10]–[12]. As for a $(1,2,4)$ system, we have on average about 50\% complexity reduction.
point. Here, we must stress that the performance of [10] can be further improved by increasing the number of iterations. This is shown in Fig. 6 when we increase the number of iterations to 11. However, the performance of the scheme in [10] is worse than the proposed method and is 4 times more complex than the proposed method. Lastly, we also plot the performance of the proposed method under the per BS power constraint. There is only a 1 dB performance degradation by switching from the total BSs to the per BS power constraints. The performance of the proposed method under the per BS power constraint is much better than other schemes under both total BSs and per BS power constraints, making the proposed method the most practical to implement.

In Fig. 7, we show how the proposed method performs under a different configuration. We show the performance when the number of users, $K$, the number of transmit antennas per BS, $N_{BS}$, and the number of receive antennas per MS, $N_{MS}$, are 4, 1 and 2 respectively. Here, the total number of transmit antennas $KN_{BS}$ is equal to the number of MSs. Even when the proposed method does not perform APO, it still significantly outperforms the one in [9]. This improvement is even greater than the one in Fig. 6 ($> 4$ dB). Here, however, the performance gap between the proposed method and an interference free channel is 4 dB when $\text{SER}=10^{-4}$. The reason for this wider gap is the lack of spatial diversity of the transmitter since we have $KN_{BS} = 4$ transmit antennas broadcasting to $K = 4$ users. In addition the fact there are only two antennas at each receiver, also limits the overall spatial diversity of the system. In Fig. 7, we can also see clearly that [10] with 5 iterations is much worse than the proposed method. Note that, here the saturation point occurs at 24 dB since no further performance improvement can be obtained in [10] by raising SNR above 24 dB. Fig. 7 also shows the performance of [10] when the number of iterations is increased to 11. Here, we essentially shift the saturation point further to the right. However, by doing this, the scheme [10] is now 3 times more complex that the proposed method, making it much less desirable for a practical implementation. Note that the computational complexity of the proposed method with 10 iterations is 25132 and 28772 flops under the total BS and per BS power constraints, respectively. The computational complexities of the scheme in [10] with 5 iterations under the total BSs and per BS constraints and 11 iterations under the total BSs and per BS power constraints are 36244, 79636, 36504, and 80208 flops, respectively. In addition, the complexity of the proposed method without APO is 3442 and 3962 flops under total power and per BS power constraints, respectively. We plot the performance of the proposed method under a per BS power constraint for a $(1, 2, 4)$ system. Its performance is still better than other schemes under both the total BSs and per BS power constraints. Fig. 8 shows the performance for a $(2, 1, 3)$ system, where each MS is equipped with only one antenna. It can be seen that the performance of the proposed scheme outperforms [9], [10] by more than 2 dB and is within 0.5 dB of an interference free performance when $\text{SER}=10^{-4}$. Even though the performance of the proposed scheme is the same as [11], [12], its complexity is 70% less than the complexity of the reference scheme [11], [12], making the proposed scheme a better choice for implementation.

As the system has an error performance close to an interference free system, its capacity is approaching the capacity of individual interference free links. In typical cellular networks or WLANs, there can be only one user transmitting in the same frequency band at a given time slot. The proposed method enables K base stations in the same location to simultaneously transmit to K users using the same frequency band and time slots. By using the proposed method, instead of transmitting to one user at a time with a power of 1, we can simultaneously transmit to K users with a power of K with the performance of each user approaching an interference free performance. As a result, the capacity of both WLANs and cellular mobile networks for a $(1, 2, 4)$ system can be increased by up to K times as shown in Fig. 9. Note that the capacity of the proposed method is higher than its best competitor [10]. Due to space limitations, we do not show the capacity result for a $(2, 2, 3)$ system. Our simulations show that the capacity of the proposed method for a $(2, 2, 3)$ system is also higher than for the schemes in [9]–[12] under both the total BSs and per
BS power constraints.

VI. CONCLUSION

In this paper, we propose a method to design a spectrally efficient cooperative downlink transmission scheme employing precoding and beamforming. THP and iterative transmit-receive weights optimization are used to cancel multi-user interference. A new method to generate transmit-receive antenna weights is proposed. SINR equalization and APO are used to achieve symbol error rate (SER) fairness among different users and further improve the system performance, respectively. The error performance for two sets of system parameters \((N_{BS}, N_{MS}, K)\) is shown. For a \((2, 2, 3)\) cooperative system, the proposed method outperforms the existing schemes by at least 3 dB and is only 0.25 dB away from an interference free performance when \(\text{SER}=10^{-4}\). For a \((1, 2, 4)\) system, the proposed method outperforms the existing schemes by at least 4 dB and is 4 dB away from the interference free performance for \(\text{SER}=10^{-4}\). In addition, the proposed method eliminates the dependency between the numbers of transmit and receive antennas. The complexities of the proposed method for \((1, 2, 4), (2, 2, 3), (2, 1, 3)\) are shown on average 50\%, 75\% and 70\% less than the complexities of schemes in [9]–[12] with the same configurations, respectively. The proposed method can improve the performance and capacity of co-working WLANs and cellular mobile networks. The capacity of these systems can be increased up to \(K\) times.

APPENDIX A

PROOF OF SINR EQUIVALENCE

We first calculate \(SINR_j\) using the term on the left hand side of (20). We need to prove that \(\mathbf{R}^{-1}_j \mathbf{H}_j \mathbf{H}^H_j\) only has one eigenvalue. Let us assume that \(\mathbf{H}^H_j \neq 0\). We denote \(\mathbf{R}^{-1}_j \mathbf{H}_j \mathbf{H}^H_j\) by \(\mathbf{a}\) and \(\mathbf{H}^H_j\) by \(\mathbf{b}\), where \(\mathbf{a} = [a_1...a_{N_{MS}}]^T \in C^{N_{MS} \times 1}\) and \(\mathbf{b} = [b_1...b_{N_{MS}}] \in C^{1 \times N_{MS}}\), respectively. We then express \(\mathbf{R}^{-1}_j \mathbf{H}_j \mathbf{H}^H_j\) as \(\mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j = \mathbf{a} \mathbf{b} = [b_1 a ... b_{N_{MS}} a]\). Here, \(\mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\) is a matrix that has \(N_{MS}\) columns and rows. We can see that the vectors represented by each column of matrix \(\mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\) can be rewritten using vector \(\mathbf{a}\) as a basis. This indicates that the rank of this matrix is 1 and as a consequence, there is only one eigenvalue. The receive weights vector computation in (19) can be written as \(\mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j \mathbf{r}_j = \lambda_j \mathbf{r}_j\), where \(\lambda_j\) is the eigenvalue for link \(j\). By multiplying both sides of this equation by \(\mathbf{H}^H_j\), we have \((\mathbf{H}^H_j \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j - \lambda_j \mathbf{H}^H_j) \mathbf{r}_j = 0\).

The eigenvalue of \(\mathbf{H}^H_j \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\) is the same as the eigenvalue of \(\mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\). As a consequence, \(\mathbf{H}^H_j \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\) only has 1 eigenvalue. This eigenvalue is the solution for the term on the left hand side of (20). Thus, the \(SINR_j\) for it is given as \(SINR_j = \lambda_j = \mathbf{H}^H_j \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\). We now find the \(SINR_j\) by solving the term on the right hand side of (20). The optimum receive weights vector is given by [18] as \(\mathbf{r}_j = \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j c\) where \(c = \mathbf{H}^H_j \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \). By substituting this receive weights vector into (11) and replacing its denominator with \(\mathbf{H}^H_j \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\), we obtain \(SINR_j = \mathbf{H}^H_j \mathbf{R}^{-1}_j \hat{\mathbf{H}}_j \hat{\mathbf{H}}^H_j\).

APPENDIX B

PROOF OF CONVERGENCE LEMMA 1

First, we note that in order to calculate \(SINR_i\) in (11), we need to know the receive weights vector for link \(j\), \(\mathbf{r}_j\) and all transmit weights vectors \(t_1,...,t_K\), obtained by using (16) and (15), respectively. Thus, we can write \(SINR_j\) as \(SINR_j(\mathbf{r}_j, \mathbf{T})\) since it is a function of \(\mathbf{r}_j\) and \(\mathbf{T}\). Since in the proposed iterative method, we optimize one variable at a time, while fixing the other one, we can write

\[
SINR_j(g_j(\mathbf{T}), \mathbf{T}) = \max_{a \in A_1} SINR_j(a, \mathbf{T}, g_j(\mathbf{T})) \in A_1
\]

where \(\mathbf{T}\) is fixed while the best \(\mathbf{r}_j = g_j(\mathbf{T})\) in the solution set \(A_1\) is searched and

\[
SINR_j(\mathbf{r}_j, f_1(\mathbf{R})) = \max_{a \in A_2} SINR_j(\mathbf{r}_j, a, f_1(\mathbf{R})) \in A_2
\]

where \(\mathbf{r}_j\) is fixed while the transmit weights vectors for \(K\) links, \(\mathbf{T} = f_1(\mathbf{R})\) in the solution set \(A_2\) are searched, respectively. To describe the proposed alternating optimization process, we denote the number of iterations by \(i\), the receive weights vector by \(\mathbf{r}_j^{(i)}\) and transmit weights vectors by \(\mathbf{T}^{(i)}\). First, \(\mathbf{r}_j^{(0)}, j = 1,...,K\), are arbitrarily chosen as initial vectors. \(\mathbf{T}^{(i)}\) is then calculated by using the function in (15), \(f_1(\mathbf{R})\). For \(i \geq 1\), we then have, \(\mathbf{r}_j^{(i)} = g_j(\mathbf{T}^{(i)})\), \(j = 1,...,K\) where \(\mathbf{T}^{(i)} = [t^{(i)}_1,...,t^{(i)}_K]\) and \(\mathbf{r}_j^{(i)} = f_1(\mathbf{R}^{(i-1)})\) where \(\mathbf{R}^{(i)} = \text{Diag}(\mathbf{r}_j^{(i)})\). Here, \(\mathbf{r}_j^{(i)}\) and \(\mathbf{T}^{(i)}\) are generated in the order \(r_j^{(0)} = r_j^{(1)},...r_j^{(K)}\) and so on. From (31) and (32), and by using the fact that \(SINR_j(\mathbf{r}_j, \mathbf{T})\) is non-decreasing and bounded from above by constraints in (13), we can write

\[
SINR_j(\mathbf{r}_j^{(i)}, \mathbf{T}^{(i)}) \geq SINR_j(\mathbf{r}_j^{(i)}, \mathbf{T}^{(i-1)}) \geq SINR_j(\mathbf{r}_j^{(i-1)}, \mathbf{T}^{(i-1)}).
\]

The terms on the right hand side of (33) come from the fact that since we are performing an alternate optimization of the transmit-receive weights by using (31) and (32), the

Fig. 9. Capacity Comparison for a (1,2,4) System using Various Non-Linear Precoding Algorithms
Appendix C

Proof of Convergence Lemma 2

We know from Convergence Lemma 1 that we can write the optimal solution as \( \text{det}(R^{H}HT^{*}) = \text{det}(Z) = \prod_{i} |z_{i}| \) where Z and \( z_{i} \) are a lower triangular matrix and the entry of the diagonal of Z, respectively. R* and T* indicate the optimal transmit-receive weights vectors for \( K \) links. We also need (15) to be satisfied for the optimal solution for each link \( j \), \( (H_{j}^{H}r_{j}^{(1)})^{H}t_{j}^{(1)} = 0 \), \( l = j + 1, ..., K \). The vector created by multiplying the channel matrix by the receive antenna weights vector is perpendicular to the transmit weights vector for links \( j + 1, ..., K \). As a result, there is no interference at all at link \( j \). This is so since the transmission spaces of link \( j + 1, ..., K \) do not overlap with the transmission space of link \( j \) and the interference from link \( 1, ..., j - 1 \) to link \( j \) is cancelled by THP. However, prior to finding the optimal solution, the receiver design from (16) destroys the orthogonality created by QR decomposition in (15). As a result at \( i^{th} \) iteration, for link \( j \), we have \( (H_{j}^{H}r_{j}^{(1)})^{H}t_{j}^{(i)} \neq 0, l = j + 1, ..., K \). This means the transmission space for link \( j \) intersects with the transmission spaces of link \( j + 1, ..., K \) prior to convergence. In other words, the vector generated by \( H_{j}^{H}r_{j}^{(1)} \) also has non-zero components along \( t_{j+1}, ..., t_{K} \), thus reducing the optimal signal gain for link \( j, z_{j} \). We can then conclude that \( \prod_{j} |\beta_{j}^{(i)}| \leq \prod_{j} |z_{j}| \).

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References


Wibowo Hardjawana received B. Eng degree in Electronics Engineering from the University of Melbourne and PhD degree in Electrical Engineering from the University of Sydney in 1998 and 2009, respectively. From 1999 to 2004, he was working as a Telecom Engineer at Singapore Telecome Ltd responsible for cellular and fibre networks implementation and planning. He is currently a Research Associate with Telecommunication Lab, University of Sydney, Australia. His current research interests are in the area of wireless communications, with a particular focus on MIMO, cooperative communications and coding techniques.

Branka Vucetic currently holds the Peter Nicol Russell Chair of Telecommunications Engineering at the University of Sydney. During her career she has held various research and academic positions in Yugoslavia, Australia and UK. Her research interests include wireless communications, coding, digital communication theory and MIMO systems. Prof Vucetic co-authored four books and more than two hundred papers in telecommunications journals and conference proceedings. She has been elected to the grade of IEEE Fellow for contributions to the theory and applications of channel coding.
Yonghui Li (M’04-SM’09) received his PhD degree in November 2002 from Beijing University of Aeronautics and Astronautics. From 1999 - 2003, he was affiliated with Linkair Communication Inc, where he held a position of project manager with responsibility for the design of physical layer solutions for LAS-CDMA system. Since 2003, he has been with Telecommunication Lab, University of Sydney, Australia. He is now a Senior Lecturer in School of Electrical and Information Engineering, University of Sydney. He is also currently the Australian Queen Elizabeth II fellow. His current research interests are in the area of wireless communications, with a particular focus on MIMO, cooperative communications, coding techniques and wireless sensor networks. He holds a number of patents granted and pending in these fields. He was an Associate Editor for EURASIP JOURNAL ON WIRELESS COMMUNICATIONS AND NETWORKING from 2006-2008, and Editor for the JOURNAL OF NETWORKS. He also served as the Editor for special issue on “advances in error control coding techniques” in EURASIP JOURNAL ON WIRELESS COMMUNICATIONS AND NETWORKING. He has also been involved in the technical committee of several international conferences, such as ICC, Globecom, etc.

Zhendong Zhou (S’03-M’07) received the B.S.E.E. and M.S.E.E. degrees from Zhejiang University, China, in 2000 and 2003, respectively, and the Ph.D. degree from the University of Sydney in 2007. Since 2006, he has been working as a research fellow in the Telecommunications Laboratory, University of Sydney. His research interests include adaptive modulation and coding, MIMO space-time processing, cooperative communications and network coding.