Design of Space-Time Turbo Trellis Coded Modulation For Fading Channels

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***Abstract*-This paper[1] presents the design of space-time turbo trellis coded modulation (ST turbo TCM). We introduce new recursive space-time trellis coded modulation (STTC) which outperform feedforward STTC proposed in [1],[3]. A substantial improvement in performance can be obtained by constructing parallel concatenation of recursive STTC and making use of iterative decoding. The new recursive STTCs can be used directly in this scheme. ST turbo TCM outperforms the best known STTC by about 2 dB on slow fading channels and by up to 8 dB on fast fading channels.**

I. Introduction

Space-time coding techniques have received a lot of attention lately in efforts to develop spectrum efficient transmission and access techniques. Code design criteria based on the rank and the determinant of the codeword difference matrix for trellis based ST codes were derived in [1], [2]. In this approach, multiple transmit antennas and error correction coding are combined with higher level modulation schemes. An ST encoder takes as input a block of b binary data, and maps them into n_T modulation symbols from a signal set of 2^b points. Each output modulation symbol feeds a separate transmit antenna. The symbols from n_T antennas are transmitted simultaneously in one symbol interval. The scheme gives a maximum spectral efficiency of b bits/s/Hz which is equal to the spectral efficiency of the reference uncoded systems. In [1], feedforward STTCs with two transmit antennas were designed. In [3], a set of improved feedforward 4-PSK STTCs relative to those in [1] were proposed.

Space-time trellis coded modulation (STTC) can benefit from both diversity and coding gains. However, when the number of transmit antennas gets larger, the complexity of the receiver structure and of the code construction becomes prohibitive. Further improvements in bit error rate performance can be achieved while maintaining the same bandwidth efficiency by constructing a parallel concatenation of STTCs. This, however, will require recursive STTCs. Yuan *et al.* proposed a new set of design criteria for STTCs in [4]. Using these criteria, we design and construct a set of robust recursive 4-PSK and 8-PSK STTCs which perform well on slow and fast Rayleigh fading channels and are superior to those in [1] and [3]. Furthermore, we use the new recursive STTCs directly in a parallel concatenation scheme to take advantage of interleaver gain and iterative decoding. The ST turbo TCM scheme outperforms the best known STTC with two transmit and two receive antennas by about 2 dB on slow fading channels and by up to 8 dB on fast fading channels. We also evaluate the decoder convergence and iterative decoding threshold of the new ST turbo TCM scheme. We show that parallel concatenation of the new recursive STTC is more efficient than a similar scheme proposed in [8] as it converges faster at a lower value of E_b/N_0.

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II. STTC System Model

The system under consideration employs STTC with n_T transmit and n_R receive antennas. While the transmitter has no knowledge about the channel, it is assumed that the receiver can recover the channel state information perfectly. Information bits are encoded into n_T streams of M-PSK symbols by the ST encoder. A space-time symbol $\mathbf{x}_t$ consists of n_T M-PSK symbols, and can be written as $\mathbf{x}_t = (x_t^1, x_t^2, \dots, x_t^{n_T})$. At any given time t, an M-PSK symbol x_t^i is transmitted through the i'th antenna, $i = 1, 2, \dots, n_T$.

At the receiver, each antenna receives a noisy superposition of n_T transmitted symbols which have been subjected to independent fading. After matched filtering and assuming ideal timing information, the received signal r_t^j at the j'th receive antenna at time t can be expressed as

$$r_t^j = \sqrt{E_s} \sum_{i=1}^{n_T} h_{i,j}(t) x_t^i + n_t^j \qquad (1)$$

where $h_{i,j}(t)$ models the complex fading gain from transmit antenna i to receive antenna j at time t, $i = 1, 2, \dots, n_T$, $j = 1, 2, \dots, n_R$, and E_s is the energy per symbol. On a fast fading channel, we assume the fading coefficients change independently from symbol to symbol. On a slow fading channel, we assume that the fading coefficients remain the same. The fading gains are modeled as independent samples of a complex Gaussian random variable with a zero mean and a variance of 0.5 per dimension. The noise n_t^j at the j'th receive antenna at time t is modeled as an independent sample of a zero mean complex Gaussian random variable with a noise spectral density of N_0.

III. Performance Analysis and Design Criteria

Following the derivation in [4], let us consider an $n_T \times n_T$ codeword distance matrix $\mathbf{A}(\mathbf{x}, \hat{\mathbf{x}}) = \mathbf{B}(\mathbf{x}, \hat{\mathbf{x}}) \cdot \mathbf{B}^{\mathrm{H}}(\mathbf{x}, \hat{\mathbf{x}})$ between two codewords of length l, $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_t, \dots, \mathbf{x}_l)$ and $\hat{\mathbf{x}} = (\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, ..., \hat{\mathbf{x}}_t, \dots, \hat{\mathbf{x}}_l)$. The matrix $\mathbf{B}^{\mathrm{H}}$ denotes the Hermitian of a matrix $\mathbf{B}$, and $\mathbf{B}(\mathbf{x}, \hat{\mathbf{x}})$ is a codeword difference

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matrix, defined as

$$\mathbf{B}(\mathbf{x},\hat{\mathbf{x}}) = \begin{bmatrix} x_1^1-\hat{x}_1^1 & x_2^1-\hat{x}_2^1 & \cdots & x_l^1-\hat{x}_l^1 \\ x_1^2-\hat{x}_1^2 & x_2^2-\hat{x}_2^2 & \cdots & x_l^2-\hat{x}_l^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{n_T}-\hat{x}_1^{n_T} & x_2^{n_T}-\hat{x}_2^{n_T} & \cdots & x_l^{n_T}-\hat{x}_l^{n_T} \end{bmatrix}. \quad (2)$$

For the purpose of our discussion, let us define r the minimum rank of the matrix $\mathbf{A}(\mathbf{x},\hat{\mathbf{x}})$ over all possible codeword pairs, and δ_H the minimum symbol Hamming distance, defined as

$$\delta_{H_{\min}} = \begin{matrix}\min \\ \mathbf{x},\hat{\mathbf{x}}\end{matrix} |\upsilon(\mathbf{x},\hat{\mathbf{x}})|, \quad (3)$$

taken over all codeword pairs, where $\upsilon(\mathbf{x},\hat{\mathbf{x}})$ denotes the set of time instances $t \in \{1,2,\ldots,l\}$, such that $\|\mathbf{x}_t - \hat{\mathbf{x}}_t\| \neq 0$.

The analysis in [4] shows that when $r \cdot n_R \geq 4$ and $\delta_H \cdot n_R \geq 4$, the design criteria for STTC on slow and fast fading channels are identical. Under these conditions, the pairwise error probability is dominated by the squared Euclidean distance d_E^2, defined as

$$d_E^2 = \sum_{t \in \upsilon(\mathbf{x},\hat{\mathbf{x}})} \|\mathbf{x}_t - \hat{\mathbf{x}}_t\|^2, \quad (4)$$

Therefore, provided that $r \cdot n_R \geq 4$ and $\delta_H \cdot n_R \geq 4$, we can construct recursive STTCs which best satisfy the design criterion and perform well on both types of fading channels, and can be directly used as component codes in a parallel concatenation structure.

IV. Recursive Space-Time Trellis Codes

For a recursive 4-PSK STTC, the binary input bits are c_t^0 and c_t^1. The new variables $\hat{c}_t^k, k \in \{0,1\}$ are modulo-2 sum of the current input bit c_t^k and the delayed modified input bits $\hat{c}_{t-j_k}^k$ which can be written as

$$\hat{c}_t^k = c_t^k + \sum_{j_k=1}^{\nu_k} \hat{c}_{t-j_k}^k \mod 2. \quad (5)$$

The output symbol to be transmitted from transmit antenna i, $i \in \{1,\ldots,n_T\}$, at time t can be expressed algebraically as

$$x_t^i = \sum_{j_0=0}^{\nu_0} \hat{c}_{t-j_0}^0 a_{j_0}^i + \sum_{j_1=0}^{\nu_1} \hat{c}_{t-j_1}^1 b_{j_1}^i \mod 4, \quad (6)$$

where the code's memory order $\nu = \nu_0 + \nu_1$, $a_{j_k}^i, b_{j_k}^i \in \{0,1,2,3\}$, and $j_k \in \{0,1,\ldots,\nu_k\}$. Fig. 1 depicts the encoder structure for a recursive 16-state 4-PSK STTC with two transmit antennas.

For a recursive 8-PSK STTC, the input bits at time t are c_t^0, c_t^1 and c_t^2. The encoder is also implemented as a feedback shift register with a memory order of ν. With the new variable

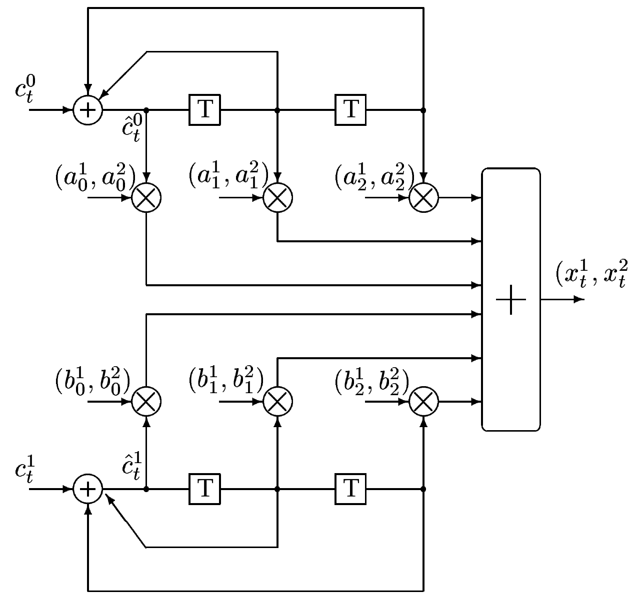

Fig. 1. Recursive 16-state 4-PSK encoder structure

$\hat{c}_t^k$ defined as in (5), the output symbol for transmit antenna i can be written as

$$x_t^i = \sum_{j_0=0}^{\nu_0} \hat{c}_{t-j_0}^0 a_{j_0}^i + \sum_{j_1=0}^{\nu_1} \hat{c}_{t-j_1}^1 b_{j_1}^i + \sum_{j_2=0}^{\nu_2} \hat{c}_{t-j_2}^2 d_{j_2}^i \mod 8, \quad (7)$$

where $\nu = \nu_0 + \nu_1 + \nu_2$, $a_{j_k}^i, b_{j_k}^i, d_{j_k}^i \in \{0,1,\ldots,7\}$, $k \in \{0,1,2\}$, and $j_k \in \{0,1,\ldots,\nu_k\}$.

With this code structure, we would like to find a set of coefficients $a_{j_k}^i, b_{j_k}^i$ for recursive 4-PSK STTCs, and $a_{j_k}^i, b_{j_k}^i, d_{j_k}^i$ for recursive 8-PSK STTCs with two transmit antennas for a given memory order which best satisfy the design criterion, given in Section III. When $r \cdot n_R \geq 4$ and $\delta_H \cdot n_R \geq 4$, the design criteria for recursive STTC on slow and fast fading channels coincide. Therefore, assuming that at least two receive antennas ($n_R \geq 2$) is available to the system, we would like to find a set of robust recursive STTCs, performing well on slow and fast fading channels. Tables I and II list recursive 4-PSK and 8-PSK STTCs, respectively, which best satisfy the design criterion on slow and fast fading channels, provided that $n_R \geq 2$. Each entry in both tables are of rank $r = 2$ and minimum symbol Hamming distance $\delta_H = 2$ or larger, satisfying the condition on the design criterion.

V. Performance of Recursive STTC

Fig. 2 shows the performance of the new 8-state and 32-state 4-PSK recursive STTC in comparison with feedforward STTC of the same memory order proposed in [1] and [3] with four receive antennas on slow fading channels. The 8-state STTCs in [1] and [3] have virtually the same performance, while the new 8-state recursive STTC offer a 0.5 dB gain over the other

TABLE I
RECURSIVE 4-PSK STTC FOR SLOW AND FAST FADING CHANNELS, BANDWIDTH EFFICIENCY 2 BITS/SEC/HZ

ν	2	3	4	5	6
d_E^2	10.0	12.0	16.0	16.0	18.0
(a_0^1, a_0^2)	(0,2)	(2,2)	(1,2)	(0,2)	(0,2)
(a_1^1, a_1^2)	(1,2)	(2,1)	(1,3)	(2,3)	(3,1)
(a_2^1, a_2^2)	-	-	(3,2)	(1,2)	(3,3)
(a_3^1, a_3^2)	-	-	-	-	(3,2)
(b_0^1, b_0^2)	(2,3)	(2,0)	(2,0)	(2,2)	(2,2)
(b_1^1, b_1^2)	(2,0)	(1,2)	(2,2)	(1,2)	(2,2)
(b_2^1, b_2^2)	-	(0,2)	(2,0)	(2,3)	(0,0)
(b_3^1, b_3^2)	-	-	-	(2,0)	(2,0)

TABLE II
RECURSIVE 8-PSK STTC FOR SLOW AND FAST FADING CHANNELS, BANDWIDTH EFFICIENCY 3 BITS/SEC/HZ

ν	3	4	5
d_E^2	7.172	8.0	8.586
(a_0^1, a_0^2)	(2,1)	(2,4)	(0,4)
(a_1^1, a_1^2)	(3,4)	(3,7)	(4,4)
(b_0^1, b_0^2)	(4,6)	(4,0)	(0,2)
(b_1^1, b_1^2)	(2,0)	(6,6)	(2,3)
(b_2^1, b_2^2)	-	-	(2,2)
(d_0^1, d_0^2)	(0,4)	(7,2)	(3,0)
(d_1^1, d_1^2)	(4,0)	(0,7)	(2,2)
(d_2^1, d_2^2)	-	(4,4)	(3,7)

two STTCs at a frame error rate (FER) of 10^{-3}. The new recursive 32-state STTC offers a 0.5 dB gain over the 32-state STTC in [1] at the same frame error rate. On fast fading channels the new recursive STTCs offer up to 2 dB gain compared to previously known feedforward STTCs [5].

VI. SPACE-TIME TURBO TCM

Fig. 3 shows the encoder structure of a space-time turbo TCM with two transmit antennas, consisting of two recursive STTC encoders in the upper and lower branches, and linked by a pairwise interleaver and a symbol deinterleaver [6]. Each encoder operates on a message block of L groups of b information bits, where L is the interleaver size. The message sequence $\mathbf{c}$ is given by $\mathbf{c} = (\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_t, \dots, \mathbf{c}_L)$ where $\mathbf{c}_t$ is a group of b information at time t, given by $\mathbf{c}_t = (c_{t,0}, c_{t,1}, \dots, c_{t,b-1})$.

The upper recursive STTC encoder in Fig. 3 maps the input sequence into two streams of L M-PSK symbols, $\mathbf{x}_1^1$, $\mathbf{x}_1^2$, where $\mathbf{x}_1^i = (x_{1,1}^i, x_{1,2}^i, \dots, x_{1,L}^i)$, $i \in \{1, 2\}$ and $M = 2^b$. Prior to encoding by the lower encoder, the information bits are interleaved by a pairwise symbol interleaver. The pairwise symbol interleaver operates on groups of b bits instead of on single bits. The interleaver maps even positions to even positions, and odd ones to odd ones. The lower encoder also produces two streams of L M-PSK symbols. Each stream is deinterleaved, resulting in $\mathbf{x}_2^1$ and $\mathbf{x}_2^2$, where $\mathbf{x}_2^i = (x_{2,1}^i, x_{2,2}^i, \dots, x_{2,L}^i)$. Assuming L is even, the first streams of symbols generated by the upper and lower encoders, $\mathbf{x}_1^1$ and $\mathbf{x}_2^1$, are alternately punctured into $\mathbf{x}^1 = (x_{1,1}^1, x_{2,2}^1, x_{1,3}^1, x_{2,4}^1, \dots, x_{1,L-1}^1, x_{2,L}^1)$ and transmitted through the first transmit antenna. The second streams of symbols generated by the upper and lower encoders, $\mathbf{x}_1^2$ and $\mathbf{x}_2^2$, are alternately punctured into $\mathbf{x}^2 = (x_{1,1}^2, x_{2,2}^2, x_{1,3}^2, x_{2,4}^2, \dots, x_{1,L-1}^2, x_{2,L}^2)$ and transmitted through the second transmit antenna.

At the receiver, we employ an iterative symbol-by-symbol log-MAP decoder whose operation is similar to that for binary turbo codes except the symbol probability is used as the extrinsic information instead of the bit probability [7]. It is worth noting that for symbol-by-symbol log-MAP decoding, each component decoder should avoid to use the same systematic information twice in each iteration. In turbo TCM, each decoder alternately receives the noisy output of its own encoder

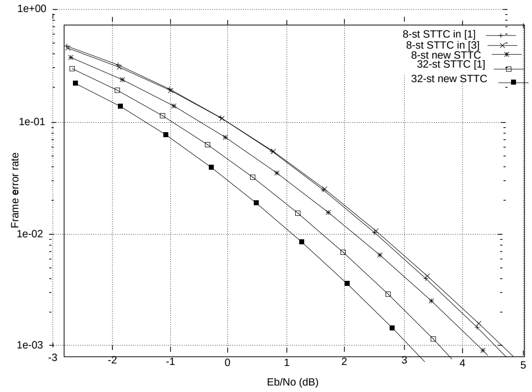

Fig. 2. Performance comparison of the 8-state and 32-state recursive 4-PSK STTC on slow fading channels, bandwidth efficiency 2 bits/sec/Hz.

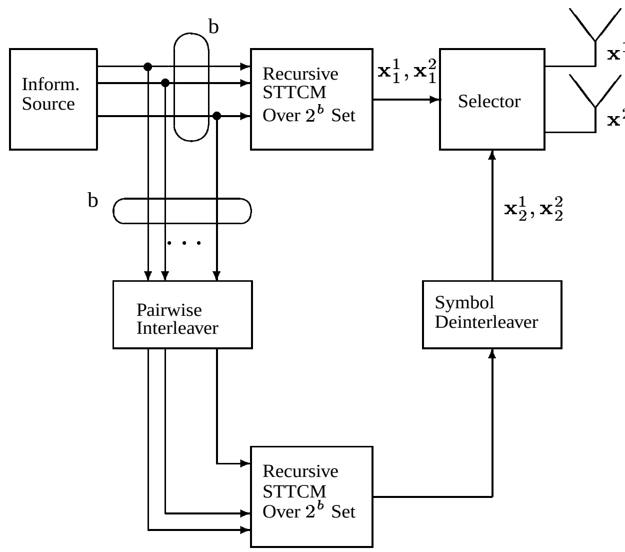

Fig. 3. Space time turbo TCM encoder

and that of the other encoder.

For example, let us consider the first decoder. For every odd received signal, the decoding operation proceeds as for the binary turbo codes when the decoder receives the symbol generated by its own encoder. The only difference is that the extrinsic information is replaced by the joint extrinsic and systematic information. However, for every even received signal, the decoder receives the punctured symbol in which the parity digit is generated by the other encoder. The decoder in this case ignores this symbol by setting the branch transition metric to zero. The only input at this step in the trellis is the a priori component obtained from the other decoder.

VII. Code Performance

In all simulations, two receive antennas is assumed at the receiver. Fig. 4 shows the FER performance of the new 32-state 8-PSK ST turbo TCM in comparison with that of the 32-state 8-PSK STTC on a fast fading channel. With ten iterations the ST turbo TCM scheme offers more than 7 dB gain at FER=10^{-3}, compared to the 32-state recursive 8-PSK STTC in Table II. When the number of iteration is reduced to six, the performance degrades by about 0.3 dB.

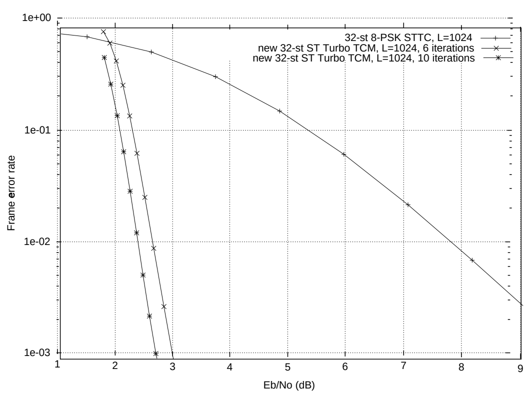

Fig. 4. BER performance comparison between a 32-state 8-PSK STTC and a 32-state 4-PSK ST Turbo TCM with bandwidth efficiency 3 bits/sec/Hz on fast fading channels

Fig. 5 shows the FER performance comparison between the new 16-state 4-PSK ST turbo TCM and a 16-state turbo TCM scheme from [6] on a fast fading channel. With the same interleaver size of 1024 and ten iterations, the ST turbo TCM offers a 2.5 dB gain at a frame error rate of 10^{-2}. The bandwidth efficiency in all cases is 2 bits/sec/Hz. Note that to achieve the same bandwidth efficiency, the scheme by Robertson *et al.* has to use 8-PSK signal set.

Fig. 6 shows the performance of the 16-state 4-PSK ST turbo TCM on a slow fading channel. The performance curves show that with ten iterations and an interleaver size of 1024, at FER=10^{-2} it offers more than 2 dB gain over the 16-state recursive 4-PSK STTC. Since the fading coefficients remain constant over a frame on a slow fading channel, the interleaver gain can not be realized and, thus, the performance of ST turbo

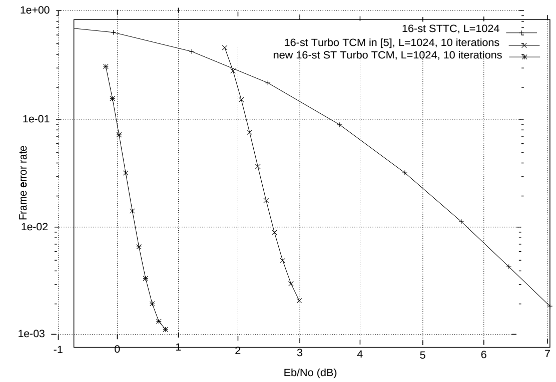

Fig. 5. FER performance comparison between the 16-state 4-PSK ST turbo TCM and a 16-state 8-PSK turbo TCM [5], bandwidth efficiency 2 bits/sec/Hz on fast fading channels

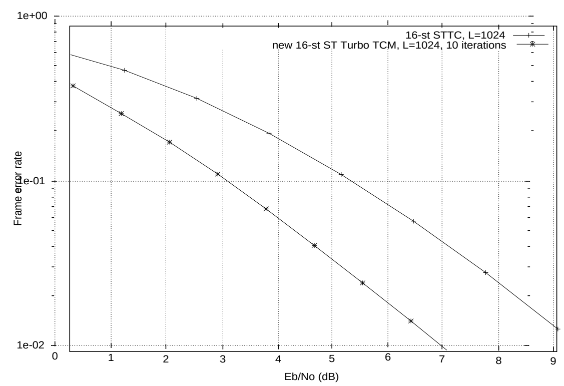

Fig. 6. FER performance of a 16-state 4-PSK STTC and a 16-state ST Turbo TCM, bandwidth efficiency 2 bits/sec/Hz on slow fading channels

TCM is worse than on fast fading channels.

Fig. 7 compares the sensitivity of the new ST turbo TCM and that of the ST turbo TCM with Tarokh's STTC as its constituent code [8] to the fading rate. The performance curves show that the new ST turbo TCM is less sensitive to the change in the fading rate. When the fading rate is such that 100 consecutive symbols are affected by the same fading coefficients, at FER=10^{-2}, the new ST turbo TCM loses 1.7 dB, while the ST turbo TCM in [8] suffers a 2.1 dB loss. Under both fading conditions the new ST turbo TCM outperforms ST turbo TCM in [8].

Divsalar, Dolinar and Pollara proposed a method of analyzing iterative decoding by modeling the density of extrinsic information in iterative turbo decoders by a Gaussian density function and computing the corresponding mean μ_i and variance σ_i^2 in the i'th iteration in the Gaussian density evolution [9]. Assuming perfect interleaving, each extrinsic information message is independent and identically Gaussian distributed with mean μ_i and variance σ_i^2 at end of the i'th iteration. The SNR$_i$ of the extrinsic information at the i'th iteration is defined as $SNR_i = \mu_i^2/\sigma_i^2$. The convergence of an iterative decoder can be evaluated by plotting the SNR$_i = \mu_i^2/\sigma_i^2$ at the output

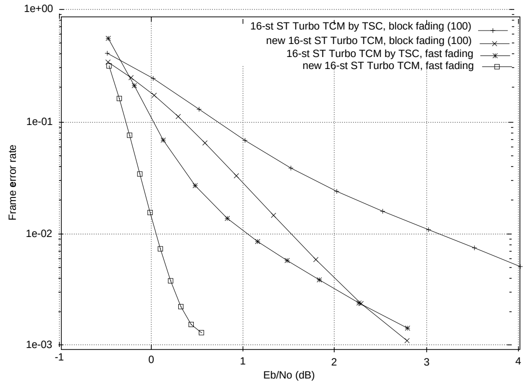

Fig. 7. FER performance comparison between the new 16-state ST Turbo TCM and Tarokh's, bandwidth efficiency 2 bits/sec/Hz on block and fast fading channels

of decoder 1 as a function of the SNR_i at the input of decoder 1, and the SNR_i at the input of decoder 2 as a function of the SNR_i at the output of SNR_i at decoder 2 as the iteration progresses. In the first iteration, the SNR_i at the input of the first iteration is 0 dB. The SNR_i at the input of decoder 2 always equals to the SNR_i at the output of decoder 1, and the SNR_i at the input of decoder 1 when $i > 1$ always equal to the SNR_{i-1} at the output of decoder 2. This technique can also be used to determine the E_b/N_o threshold, beyond which the decoder converges and the bit error rate goes to zero as the number of iteration increases. The threshold is the E_b/N_o value at which the two curves touch.

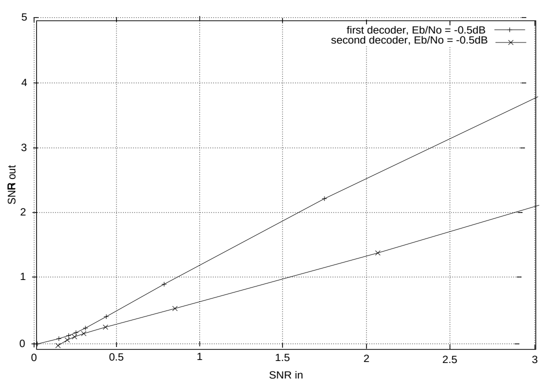

Fig. 8. Convergence and iterative decoding threshold of ST turbo TCM decoder when recursive 16-state STTC in [] is used as the component code

Fig. 8 shows the input/output SNR curves of ST turbo TCM scheme with a 16-state QPSK STTC by Tarokh as the component code [8] when $E_b/N_0 = -0.5$ dB. The two SNR curves just touch at this E_b/N_0. Fig. 9 shows the input/output SNR curves of ST turbo TCM scheme with the new recursive 16-state QPSK STTC as the component code when $E_b/N_o = -0.5$ dB. The figure shows a tunnel between the two curves through which the iterative decoding progresses. Further investigations show that the threshold for this code is -0.8 dB

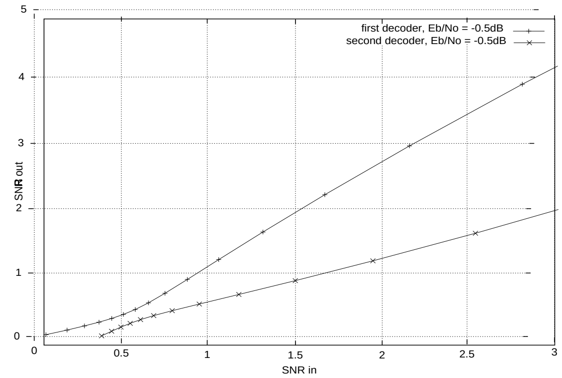

Fig. 9. Convergence of ST turbo TCM decoder when the new recursive 16-state QPSK STTC is used as the component code

while the threshold for ST turbo TCM with Tarokh's code as the component code is -0.5 dB [5]. This suggests ST turbo TCM with the new recursive 16-state QPSK STTC as the component code is more optimized than that with the 16-state QPSK STTC by TSC as the component code because it converges more quickly at a lower operating E_b/N_o.

VIII. Conclusions

New recursive STTCs which best satisfy the design criterion on slow and fast fading channels are proposed. These recursive STTCs outperform previously known feedforward STTC. Moreover, they can be used directly as component codes in a parallel concatenation structure, benefiting from interleaver gain and iterative decoding. This structure offers up to 8 dB gain compared to the traditional STTC scheme on fast fading channels and about 2 dB gain on slow fading channels.

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