A simple near-optimal path selection scheme for multi-hop wireless relay networks based on Viterbi algorithm

Qimin You*, Yonghui Li, Zhuo Chen and Md Shahriar Rahman

1 School of Electrical and Information Engineering, The University of Sydney, Sydney, New South Wales, Australia
2 Wireless and Networking Technologies Laboratory, CSIRO ICT Centre, Sydney, New South Wales, Australia

ABSTRACT

In a multi-hop wireless relay network, the optimal path selection scheme with exhaustive path search entails high-implementation complexity and is impractical when the number of hops is large. In this paper, we propose a near-optimal low-complexity path selection scheme, based on amplify-and-forward protocol, whose outage performance is close to the optimal path selection scheme. The proposed scheme draws on the analogy between the node distribution of a commonly used relay network model and the trellis of a convolutional code and applies the Viterbi algorithm in selecting a path to minimise the end-to-end outage probability. To save memory and reduce the time delay in distributed implementation, the sliding window Viterbi algorithm is assumed. For reference purpose, an asymptotic lower bound of the outage probability is first derived for the optimal path selection scheme. Then an approximated outage probability expression of the proposed scheme is derived. These analytical expressions are verified by simulations to be accurate at high signal-to-noise ratios. It is revealed that when the window size is reasonably large, the proposed scheme achieves almost the same outage performance as the optimal path selection scheme at high signal-to-noise ratios. The proposed scheme has polynomial complexity and small memory storage requirement. Therefore, it is very efficient for large-scale relay networks. Copyright © 2014 John Wiley & Sons, Ltd.

*Correspondence
Q. You, School of Electrical and Information Engineering, the University of Sydney, Sydney, New South Wales, Australia.
E-mail: qiminyou@gmail.com

Received 2 December 2013; Revised 30 May 2014; Accepted 2 June 2014

1. INTRODUCTION

As an effective technique to increase signal coverage, enhance system capacity and strengthen system robustness with a low deployment cost, relay transmission [1–3] has been utilised as the underlying technology in a variety of future wireless applications, including LTE-advanced [4] and IEEE 802.16j [5]. Relay selection (RS) [2, 6, 7] is a simplified technique to achieve these advantages, and it obtains significant performance and capacity improvement compared with all participation relaying scheme [8].

In multi-hop relay networks, several papers (e.g. [9–12]) proposed a few path selection schemes based on RS. A multi-hop line network using decode-and-forward (DF) relay protocol in Rayleigh fading channel is considered in [9]. An optimal path is found based on exhaustive search to minimise the end-to-end (E2E) outage probability. However, the general relay network model is not considered in [9]. A four-hop linear network model where each cluster consists of four relays is considered in [10]. Two transmitting nodes and two receiving nodes are selected in each hop, and Alamouti space–time coding is employed for transmission. However, the extension of this scheme to general multi-hop relay networks is not considered. No performance analysis is provided by Butcharoen and Pirak [10]. A best branch with the highest E2E SNR is selected in a multi-hop communication system with $M$ multi-hop branches [13]. The network structure with arbitrary connection between a source, several relays and a destination is considered in [11] for amplify-and-forward (AF) relay protocol. An optimal selection scheme was investigated, in which the best path is selected based on exhaustive search to maximise the exact E2E signal-to-noise ratio (SNR). To simplify the selection process, a sub-optimal selection scheme was also proposed in [11], where the inverse of the E2E SNR is approximated by the summation of the inverse instantaneous SNRs along the path. It is shown analytically in [11] that at high SNRs, the outage probability of the
sub-optimal selection scheme asymptotically approaches that of the optimal selection scheme. An idealised linear network structure, which has the same number of relays per cluster, was considered in [12] for DF relay protocol. In specific, an optimal routing scheme\(^1\) was proposed to minimise the E2E outage probability. Note that the implementation of the optimal routing schemes in [11, 12] requires high computational complexity and large memory storage requirement. Therefore, the optimal routing schemes in [11, 12] are impractical for large-scale networks where the number of hops and relays per cluster is large. The sub-optimal selection scheme in [11] is implemented using the distributed shortest path algorithm. Each node maintains a routing table and periodically sends path vector tuples (i.e. destination and the corresponding distance) to all its immediate neighbour nodes. Upon receiving any update, a node updates its routing table in turn until the routing table stabilises. Therefore, the implementation of the sub-optimal path selection scheme incurs long convergence delay and large communication overhead.

To reduce implementation complexity, two schemes, namely, ad hoc and N-hop, were also proposed in [12]. For ad hoc routing scheme, RS is performed in a hop-by-hop manner. However, it does not achieve a good outage performance, and the performance loss relative to the optimal routing scheme in [12] increases with the number of hops. The N-hop routing scheme divides the network into several subnetworks, implements the optimal routing scheme in each subnetwork, and connects the selected paths in the subnetworks together. It achieves a full diversity order for idealised linear networks. However, it suffers from some power gain loss compared with the optimal routing scheme. Furthermore, the N-hop routing scheme cannot achieve the full diversity order for networks with different numbers of relays per cluster, which will be shown by simulation in Section 6.

To overcome the disadvantages of the schemes described earlier, we propose in this paper a path selection scheme based on AF relay protocol. It achieves near-optimal outage performance with low implementation complexity and small memory storage requirement. This scheme draws on the analogy between the node distribution of a commonly used relay network model and the trellis of a convolutional code. For a multi-hop relay network, it has been shown in [14] that the inverse of the equivalent SNR along a multi-hop path at high SNRs is tightly lower bounded by the summation of the inverse instantaneous SNRs along that path. The inverse of the equivalent SNR is analogous to the accumulative squared Euclidean distance in the Viterbi decoding of a convolutional code. This analogy inspires us to utilise the Viterbi algorithm (VA) in selecting the best path in multi-hop relay networks with RS\(^2\). In specific, the network topology is first mapped to the trellis diagram of a convolutional code. The branch metric in the resulted trellis is the inverse of the instantaneous SNR of the corresponding channel connecting two relays in two adjacent clusters. The path metric at hop \(i\) is defined as the accumulative branch metric from source to hop \(i\) on this path. Then, VA is applied to select a path from the source to the destination. By selecting the best path at each node of the cluster and discarding all the other paths, our proposed scheme not only guarantees the selection of the path with the best metric but also saves a lot of computation and memory resources compared with the optimal path selection scheme. A sliding window VA is proposed to reduce memory storage and shorten time delay of distributed implementation.

In this paper, an approximated expression is derived for the outage probability of the proposed path selection scheme based on sliding window VA with a window size of \(w\). Based on these theoretical analyses, we could reach several conclusions. First, by computing the limit of the outage probability of the proposed scheme based on sliding window VA, as the window size increases, we find that the proposed scheme with VA (i.e. no memory truncation) achieves almost the same outage performance as the optimal path selection scheme \(^5\) at high SNRs. Second, the SNR loss of the proposed scheme relative to the optimal path selection scheme is negligible, when the window size is no less than five times the total encoder memory. This theoretically proves that the proposed scheme based on sliding window VA achieves outage probability close to that of the optimal path selection scheme when the window size is sufficiently large. Third, the proposed scheme achieves a full diversity order. These analytical expressions are verified by simulation to be accurate in the high SNR regime in Section 6. We also discuss the implementation issues and compare the computational complexity and memory storage requirement of the proposed scheme with some existing path selection schemes [11, 12].

The rest of this paper is organised as follows. A brief introduction of convolutional codes and VA is included in Section 2. In Section 3, the system model is introduced. The proposed path selection scheme is described in Section 4. In Section 5, the outage probability expressions of the proposed and the optimal path selection schemes

\(^1\)Note that in the development of the optimal routing scheme in [12], it was assumed that the E2E SNR is equal to the minimum SNR of all the branches along the path. This was shown in the third equation in [12, (3)], which, in fact, should be a bound. As a result, the optimal routing scheme in [12] is actually a sub-optimal one.

\(^2\)Note that in the development of the optimal routing scheme in [12], the optimal routing can be realised using VA by treating the relays as states. In the optimal routing scheme in [12], the minimum SNR of each path is first found, and then the largest minimum SNR is chosen. In fact, finding the largest minimum SNR of all paths is not analogous to finding the smallest accumulative Euclidean distance in the Viterbi decoding process. Therefore, the implementation of the optimal routing scheme in [12] is irrelevant to VA, and it is different from our proposed scheme.

\(^5\)Hereafter, the optimal path selection scheme refers to the path selection scheme with exhaustive search based on the exact E2E SNR in [11].
are derived and compared. Simulation results are provided in Section 6 to substantiate the accuracy of the theoretical analysis. Section 7 discusses the implementation issues and compares the proposed scheme with several well-known path selection schemes in terms of the computational complexity and the memory storage. Section 8 summarises and concludes the paper.

2. BACKGROUND INFORMATION OF CONVOLUTIONAL CODES AND VITERBI ALGORITHM

As a powerful coding scheme to achieve reliable data transfer, convolutional codes have been extensively applied in numerous practical applications, such as cellular mobile, digital video, deep-space and satellite communications [15]. They are also widely employed as component codes in concatenated coding systems, for example, turbo coding [16, 17]. In this section, the encoding and decoding of convolutional codes are briefly reviewed, which will help understand our proposed scheme in this paper.

2.1. Encoding

A convolutional code is characterised by three parameters, which are the number of input symbols k, the number of output symbols n, and the encoder memory m. A convolutional encoder generates n encoded symbols for each k information symbols, denoted by \((n,k)\) convolutional code. A convolutional code is specified by \(k \times n\) generator polynomials, which form the generator matrix \(G(x)\)

\[
G(x) = \begin{bmatrix}
g_{1,1}(x) & g_{1,2}(x) & \cdots & g_{1,n}(x) \\
g_{2,1}(x) & g_{2,2}(x) & \cdots & g_{2,n}(x) \\
\vdots & \vdots & \ddots & \vdots \\
g_{k,1}(x) & g_{k,2}(x) & \cdots & g_{k,n}(x)
\end{bmatrix}
\]  

(1)

If the input sequence is denoted by \(b(x) = [b^1(x), b^2(x), \ldots, b^k(x)]\) and the output sequence is denoted by \(c(x) = [c^1(x), c^2(x), \ldots, c^n(x)]\), the encoding operation can be represented by

\[
c(x) = b(x)G(x)
\]  

(2)

2.2. The Viterbi algorithm

The VA performs maximum likelihood decoding of convolutional codes [18–20]. This algorithm involves a search through the trellis for the most probable sequence. The trellis diagram of an \((n,k)\) convolutional code has \(2^k\) branches diverging from and merging into each state.

If the information sequence is \(b = (b(1), b(2), \ldots, b(L))\), where \(L\) is the length. It is encoded by an \((n,k)\) convolutional code into a codeword \(c = (c(1), c(2), \ldots, c(L+m))\) of length \((L+m)\), where \(c(i) = (c^1(i), c^2(i), \ldots, c^n(i))\). The received sequence is denoted by \(y = (y(1), y(2), \ldots, y(L+m))\), based on which a maximum likelihood decoder chooses a path \(\hat{c}\). The criterion is the maximisation of the log-likelihood function of \(\log p(y | \hat{c})\) [21].

\[
\log p(y | \hat{c}) = \sum_{i=1}^{L+m} \log p(y_i | \hat{c}_i)
\]  

(3)

where \(\log p(y | \hat{c})\) is the path metric and \(\log p(y_i, | \hat{c}_i)\) is the branch metric for branch \(i\).

The VA compares the metrics of all paths entering each state and stores the path with the largest metric together with its metric. This path is called the survivor. At the last step, the survivor is decoded. The VA is summarised in Algorithm 1.

Algorithm 1: The Viterbi algorithm

1 begin
2 At time \(t = 1\), compute the branch metric for the single branch entering each state. Store the branch (the survivor) and its metric for each state.
3 for \(t = 2: L + m\) do
4 Compute the path metric for each path entering a state by adding the branch metric entering that state to the metric of the connecting survivor at the preceding trellis depth. For each state, store the path with the largest metric (the survivor), together with its metric, and eliminate all other paths.
5 end
6 Keep the final survivor and decode the code sequence.
7 end

3. SYSTEM MODEL

We consider a general \(M\)-hop relay network topology model in Figure 1, where \(M = 1\) relay clusters are equally spaced between the source (S) and the destination (D). Note that for the convenience of exposition, the relay clusters are assumed to be equally spaced. In fact, the proposed scheme works for networks with random node locations. In addition, the number of nodes in each cluster could be arbitrary. This model has been widely used in the multi-hop path selection scenario in the literature [10, 12, 22]. AF relay protocol is assumed. Note that in many practical wireless networks, for example, a wireless sensor
network, nodes are often grouped into disjoint and non-overlapping clusters [23]. A rich body of literature exists on clustering techniques [24–27], and we omit the details here. The i-th cluster consists of $N_i$ ($N_0 = N_M = 1$) relay nodes. The nodes within a cluster do not cooperate with each other, each node selects and keeps a best path from the source independently and does not know the paths at other nodes. This is different from the multi-hop multiple-input multiple-output (MIMO) relay model, in which nodes within a cluster are seen as one MIMO node [28]. It is assumed that the radio transmission of a relay within a cluster can only reach the relays in the adjacent clusters. It is further assumed that the distance between adjacent relay clusters is much larger than that between the nodes within a cluster. Therefore, the channel coefficients between two nodes in any two adjacent clusters are independent and identically distributed (i.i.d.).

The channels between two nodes in any two adjacent clusters are assumed to experience flat Rayleigh fading, where the fading coefficients are independent circular symmetric complex Gaussian random variables with zero mean and unit variance. We consider time-division multiple-access transmission during the path selection process, that is, there is only one node transmitting during each time slot.

Let $e_{ij}(m)$ denote the branch connecting relay $i$ of cluster $m - 1$ and relay $j$ of cluster $m$, $m = 1, 2, \ldots, M$. Then a path is defined as the sequence of branches $(e_{i_1,j_1}(1), e_{i_2,j_2}(2), \ldots, e_{i_{M-1},j}(M))$, where $i_n \in (1, 2, \ldots, N_0)$, $n = 1, 2, \ldots, M - 1$. Denote the number of branches joining cluster $m - 1$ and cluster $m$ as $L_m$, $L_m = N_m - 1 N_m$. In a relay selection system, the design target is to select an $M$-hop path out of $I$ paths to maximise the E2E SNR with the constraint of reasonable complexity.

It is assumed that all the nodes have the same transmit power $P_t$. Then we define average SNR of each node as $\bar{\gamma} = \frac{P_t}{N_0}$, where $N_0$ is the power spectral density of the additive white Gaussian noise. Let $h_{ij}(m)$ denote the channel coefficient of branch $e_{ij}(m)$, then the corresponding instantaneous SNR can be written as $\gamma_{ij}(m) = \bar{\gamma}|h_{ij}(m)|^2$.

Let $\gamma_i(L), L = 1, 2, \ldots, M$, denote the equivalent SNR of a path $(e_{1,j_1}(1), e_{i_2,j_2}(2), \ldots, e_{i_{M-1},j}(L))$. According to [14, (36)–(41)], we have

$$
\gamma_i^{-1}(L) \geq \sum_{m=1}^{L} \gamma_{L-m,i,m}(m) = \gamma_i^{-1}(L-1) + \gamma_{L-m,i,m}(L)
$$

(4)

The branch metric of the conventional VA in decoding at time $t$ is the Euclidean distance between the output symbol and the received symbol, that is, $v_t = ||y_t - x_t||^2$. Its path metric $u_t$ is given by $u_t = u_{t-1} + v_t$ [29, (5.23)]. The VA is an efficient way to find the minimum path metric, that is, the minimum accumulative Euclidean distance, because only the minimum path metric at time $t - 1$ needs to be saved, while all other paths resulting in larger $u_{t-1}$ can be discarded. In our proposed scheme, the branch metric is different from the conventional Viterbi decoding algorithm, and it is the inverse of the instantaneous SNR of the branch connecting relay $i$ of cluster $m - 1$ and relay $j$ of cluster $m$, $m = 1, 2, \ldots, M$, that is, $\gamma_i^{-1}(m) = \frac{1}{\bar{\gamma}|h_{ij}(m)|^2}$. Its path metric, which is the inverse of the equivalent SNR of a path $(e_{1,i_1}(1), e_{i_2,i_2}(2), \ldots, e_{i_{M-1},i}(L))$, is calculated as $\gamma_i^{-1}(L) = \gamma_i^{-1}(L-1) + \gamma_{L-m,i,m}(L)$. By selecting the optimal path at each node with the minimum path metric $\gamma_i^{-1}(L - 1)$ and discarding all the other paths, we can guarantee that the final surviving path has the maximum equivalent SNR. Therefore, the VA, widely used for convolutional code decoding, provides an effective solution to path selection problem in a multi-hop relay network.

The equivalent SNR of the selected path at hop $L$ can be written as

$$
\gamma_i(L) \leq \frac{1}{\sum_{m=1}^{L} \gamma_{L-m,i,m}(m)} = \min_{m=1,\ldots,L} \{\gamma_{L-m,i,m}(m)\}
$$

(5)
Both (4) and (5) are asymptotically tight at high SNRs [14].
Throughout this paper, outage probability will be used as the performance metric. A path is in outage if the received SNR is below a given threshold $\gamma_{th}$. The outage probability of a path $i$, denoted by $P_{out,i}$, is given by

$$P_{out,i} = \Pr \{ \gamma_i < \gamma_{th} \} \geq \Pr \left\{ \min_{m=1,2,...,M} \{ Y_{in-1,i,m}(m) \} < \gamma_{th} \right\} \quad (6)$$

It is shown by (6) that the outage probability of a path is dominated by the instantaneous SNR of the worst branch among all branches along that path.

4. DESCRIPTION OF THE PROPOSED PATH SELECTION SCHEME

Following the previous discussion, now we can describe the proposed path selection scheme as follows.

First, the network topology is mapped to the trellis of a convolutional code. The total encoder memory $K$ is selected to be the minimum integer satisfying $2^K \geq \max_{i=1,2,...,M} \{ N_i \}$. The number of inputs $k$ to the convolutional encoder is equal to $K$. Then a convolutional code is chosen based on these two parameters, and the trellis is generated by the corresponding convolutional encoder. Note that various convolutional codes with different code rates can be used as long as they satisfy the required $K$ and $k$. For simplicity, we could choose a convolutional code with code rate $R = k/n = k/(k+1)$. Also note that the distance property of the selected convolutional code is irrelevant to the system performance. Figure 2 shows the mapped trellis of the network in Figure 1.

According to (4), the inverse of the equivalent SNR of a path is tightly lower bounded by the sum of the inverse instantaneous SNRs of all the branches along the path. Therefore, the inverse of the instantaneous SNR of a branch is selected as its branch metric. Path metrics are defined as the accumulative branch metrics. The path metric at relay $R_j$ of cluster $n$ is defined as $\text{metr}(R_j,n) = \sum_{m=1}^{n} Y_{ij}^{-1}(m)$. For networks in which $N_i < 2^K$, some branches and states in the resulted trellis are redundant because there are no corresponding channels and relays in the relay network. The branch metrics that do that have corresponding channels in the relay network are set to infinity so that they will never be selected. This has also been shown in Figure 2 for illustration purpose. To reduce the complexity, the sliding window VA is utilised, and only $w$ blocks of path metrics and branch metrics are stored at each node. A relay of the first relay cluster within the window is selected when the memory is full.

Let $m-1$ denote the cluster number where the window begins, and $q$ is the number of the current hop within the window. Based on the trellis mapping, the path selection process is described as follows:

**Algorithm 2: A near-optimal path selection scheme based on Viterbi algorithm**

1. **begin**
2. **begin** at cluster 0, $m = 1, q = 1$. Compute the path metric from source to all relays in cluster 1, i.e. $\text{metr}(R_1, 1) = \gamma_{in-1}^{-1}(1)$.
3. **for** $m = 1 : M$ **do**
4. **for** $q = 1 : w$ **do**
5. $q = q + 1$, compute the accumulative path metric for all the paths entering each node in cluster $\{m + q - 1\}$ by adding the branch metric, which is the inverse of the instantaneous SNR of the branch, to the accumulative path metric of the surviving path at the previous hop, i.e.

$$\text{metr}(R_j, m + q - 1) = \text{metr}(R_i, m + q - 2) + \gamma_{ij}^{-1}(m + q - 1).$$

For each node, store a single path with the minimum metric (the largest equivalent SNR), together with its metric, and eliminate all other paths.
6. **end**
7. Trace back: when $q = w$, the surviving path with the smallest path metric is selected, and the relay within the first relay cluster on this path within the window is chosen.
8. **Let** $m = m + 1$ (window slides one hop forward), $q = 1$. Compute the path metric from the previous selected relay to all the relays at the first cluster within the window.
9. **end**
10. **end**

**Output:** The selected near-optimal path.
5. OUTAGE PERFORMANCE ANALYSIS

In this section, we derive the outage probability expressions of the optimal path selection scheme and the proposed path selection scheme based on sliding window VA. An approximation of the SNR loss of the proposed scheme based on sliding window VA relative to the optimal path selection scheme is also derived. It is shown to be negligible when the window size is five times the total encoder memory or more. Throughout this section, the generalised network topology model in Figure 1 with AF is assumed. For two-hop AF relay networks, the path selection problem has been widely studied [30, 31]. Therefore, we focus on networks with $M > 2$.

5.1. The maximum achievable diversity order for a multi-hop relay network

Definition 1. Two paths in a multi-hop relay network are independent if they share no common branch.

Lemma 1. The maximum achievable diversity order of an AF multi-hop relay network, denoted by $d$, is upper bounded by $d_{\text{max}} = \min_{m=1,2,\ldots,M} \{L_m\}$, where $L_m = N_{m-1}N_m$.

Proof. The diversity order corresponds to the number of independently faded paths that a symbol passes through [32]. This means that if a network has a diversity order $d$, there should exist $d$ independent paths between the source and the destination when spatial diversity is considered only. The bottleneck hop in a network is defined as the hop with the smallest number of independent branches, that is, there are $\min_{m=1,2,\ldots,M} \{L_m\}$ branches in the bottleneck hop.

If we assume that $d > \min_{m=1,2,\ldots,M} \{L_m\}$, at least one of the branches in the bottleneck hop is shared by more than one path. So the $d$ paths are not independent. This proves that the assumption $d > \min_{m=1,2,\ldots,M} \{L_m\}$ is invalid.

5.2. Outage probability of the optimal path selection scheme

For reference purpose, we will investigate the outage probability of the optimal path selection scheme in this subsection.

We rearrange $L_m$ in an ascending order, denoted by $f^{(i)}$, such that $d_{\text{max}} = L_m^{(1)} \leq L_m^{(2)} \leq \cdots \leq L_m^{(M)}$. It is assumed that there are $T$ bottleneck hops that have $d_{\text{max}}$ distinct branches, $d_{\text{max}} = f^{(1)} = f^{(2)} = f^{(3)} = \cdots = f^{(T)}$, $T \in \{1,2,\ldots,M\}$.

Theorem 1. The E2E outage probability of the optimal path selection scheme, denoted by $P_{\text{out}}^{\text{opt}}$, can be expressed as

$$P_{\text{out}}^{\text{opt}} \geq P_0 = T \left[ \begin{array}{c} 1 - \exp \left( -\frac{\gamma_{th}}{\gamma} \right) \end{array} \right]^{d_{\text{max}}} + \left( \frac{\gamma_{th}}{\gamma} \right)^{d_{\text{max}}}$$

where we write $f(x) = o(g(x)), x \to x_0$, if $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$.

By using Taylor series expansion, (7) can be further written as

$$P_{\text{out}}^{\text{opt}} \geq P_0 = T \left( \frac{\gamma_{th}}{\gamma} \right)^{d_{\text{max}}} + o \left( \frac{\gamma_{th}}{\gamma} \right)^{d_{\text{max}}}$$

See Appendix 9.1.

The lower bound (8) is asymptotically tight at high SNRs, as will be verified by simulations in Section 6. According to (8), at high SNRs, the optimal path selection scheme can always achieve a full diversity order, which equals the minimum number of independent channels per hop, $d_{\text{max}} = \min_{m=1,2,\ldots,M} \{L_m\}$. This means that the number of channels in the bottleneck hop determines the diversity order. It is also seen that the number of bottleneck hops determines the power gain. Therefore, the outage probability is independent of the number of hops $M$ at high SNRs.
A similar conclusion was made in [12] for DF protocol and an idealized linear network model, where each cluster has the same number of relays. Our analysis earlier is for AF protocol, and the number of relays in each cluster is arbitrary.

5.3. Outage probability of the proposed scheme

In this subsection, we derive the outage probability of the proposed path selection scheme with sliding window VA (w ≥ 2).

**Lemma 2.** The probability of not selecting the optimal branch at hop j, denoted by $p_{tr,j}$, due to a truncation window size w, can be upper bounded by

$$p_{tr,j} \leq p_0 = \exp \left( -\frac{w}{K} \right)$$

See Appendix 9.3.

Let $d_h$ be the minimum number of hops of all finite-length paths in the trellis that diverge from and remerge with the optimal path. Note that this $d_h$ is different from the minimum free Hamming distance of a convolutional code. $d_h$ only depends on the trellis structure and is irrelevant to the distance property of the selected convolutional code.

**Theorem 2.** At high SNRs, the outage probability of the proposed path selection scheme based on sliding window VA, denoted by $P_{SWVA}$, can be approximated by

$$P_{SWVA} \approx T \left\{ 1 - \exp \left( -\frac{w}{K} \right) \right\} \left( \frac{d_{max}}{d_h} \right)^{d_{max}}$$

$$+ T \exp \left( -\frac{w}{K} \right) \left( \frac{d_{max}}{d_h} \right)^{d_{max}}$$

See Appendix 9.3.

Note that (10) is an approximation of the outage probability of the proposed path selection scheme based on sliding window VA. It will be verified by simulation to be accurate at high SNRs in Section 6. It is shown in (10) that the proposed path selection scheme is able to achieve a full diversity order. When there is no memory truncation ($w \to \infty$), the sliding window VA reduces to the VA. Let $w \to \infty$ in (10), we get the outage probability of the proposed scheme with VA, denoted by $P_{VA}$. And we have

$$P_{VA} \approx T \left\{ \frac{d_{max}}{d_h} \right\}$$

Comparing (8) and (11), we observe that the outage performance of the proposed scheme based on VA without memory truncation approaches the optimal path selection scheme as the window size increases in the high SNR regime. The comparison between (10) and (8) indicates that the proposed path selection scheme based on sliding window VA, when compared with the optimal path selection scheme, suffers an approximate SNR loss $\Delta \gamma_0$ because of memory truncation. And we have

$$\Delta \gamma \approx \Delta \gamma_0 = \frac{10}{d_{max}} \log_{10} \left\{ 1 - \exp \left( -\frac{w}{K} \right) + \exp \left( -\frac{w}{K} \right) \left( \frac{d_h}{d_{max}} \right)^{d_{max}} \right\} \text{dB}$$

Figure 3 presents the approximate SNR loss $\Delta \gamma_0$ given in (12). It is shown that when the window size is five times the total encoder memory ($w/K = 5$), the SNR loss is about 0.043 dB. According to [33], the SNR loss of the sliding window VA relative to the maximum likelihood decoding could be regarded as acceptable when it is less than 0.05 dB. Therefore, the proposed path selection scheme achieves an outage probability close to that of the optimal path selection scheme when the window size is five times the total encoder memory or more.

6. SIMULATION RESULTS

In this section, we present simulation results to validate our proposed scheme. We use binary phase-shift keying modulation. To simulate our proposed scheme for different network topologies, we first map the network topologies to trellises of convolutional codes, whose generator matrices are denoted by G and are listed in Table I. Note that the

![Figure 3: SNR loss approximation $\Delta \gamma_0$ in (12).](image)

**Table I.** Simulation parameters.

<table>
<thead>
<tr>
<th>Figure</th>
<th>R (code rate)</th>
<th>K</th>
<th>G (octal)</th>
<th>No. of hops</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1/2</td>
<td>1</td>
<td>(1 2)</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>2/3</td>
<td>2</td>
<td>(2 3 3)</td>
<td>(3 1 2) 100</td>
</tr>
<tr>
<td>6, 7</td>
<td>2/5</td>
<td>2</td>
<td>(3 3 1 0 2)</td>
<td>(3 1 2 3 0) 100</td>
</tr>
<tr>
<td>8</td>
<td>1/2</td>
<td>1</td>
<td>(1 2)</td>
<td>6</td>
</tr>
</tbody>
</table>
performance of our proposed scheme is independent of the number of hops in the network. For exposition purpose, we run simulations for networks with 6 and 100 hops to show the performance of our proposed scheme in networks with a small number and a large number of hops, respectively. The convolutional code parameters in Table I are from [34]. The SNR threshold is assumed to be $\gamma_{th} = 1$ dB throughout this section.

Figure 4 shows the outage probability of a 100-hop relay network with two relays at each of the intermediate clusters based on the proposed path selection scheme. We have $K = 1$ and $k = 1$. The rate 1/2 convolutional code in Table I is selected to generate the trellis of Figure 4. In specific, the sliding window VA is employed for different window sizes ($1 \leq w \leq 5$). For comparison purpose, the outage probability of the optimal path selection scheme is also included, in terms of both simulation and analytical lower bound in (8). The solid lines show the simulation results, and the dashed lines are from analyses (8) and (10). It is shown in Figure 4 that the lower bound of the outage probability of the optimal path selection scheme (8) is asymptotically tight at high SNRs. For all window sizes ($1 \leq w \leq 5$), the approximation (10) asymptotically approaches the simulation results at high SNRs. It is also shown that the outage probability of the proposed path selection scheme based on sliding window VA improves as the window size increases. The SNR loss of the proposed path selection scheme relative to the optimal path selection scheme are 6.7 and 1.16 dB when the truncation window sizes are 1 and 2, at the outage probability of $10^{-4}$. The SNR loss decreases slightly when the window size increases to 3, 4 and 5. When the window size increases to 5, this SNR loss is negligible. This figure also shows that the proposed path selection scheme achieves a full diversity order.

Figures 5 and 6 are based on a network with 100 hops whose number of relays in each cluster is uniformly distributed between 2 and 4 with $N_1 = 2, N_99 = 3$. $K = 2$ and $k = 2$ are selected so that $2^K$ equals to the largest number of relays per cluster. The rate 2/3 convolutional code in Table I is selected to generate the trellis of Figure 5. The rate 2/5 convolutional code in Table I is selected to generate the trellis of Figure 6. The comparison between Figures 5 and 6 reveals that the outage probability of these two networks are almost the same using the proposed path selection scheme although the rate convolutional codes are utilised to generate the trellises. This is because these two codes have the same underlying trellis structure. The SNR loss of the proposed path selection scheme based on sliding window VA relative to the optimal path selection scheme are 1 and 0.32 dB when the window sizes are 2 and 4, at the outage probability of $10^{-4}$. The SNR loss decreases slightly when the window size increases to 6, 8 and 10. When the window size increases to 10, which is five times the total encoder memory ($K = 2$), the SNR loss is negligible. It is clear from Figures 5 and
6 that (8) and (10) asymptotically approach the simulation results in the high SNR regime and that the proposed scheme achieves a full diversity order.

Figure 7 compares the outage probabilities of the ad hoc routing scheme [12, 22], the N-hop routing scheme [12], the optimal path selection scheme [11] and the proposed path selection scheme. The simulation is based on a 100-hop relay network whose number of relays in each cluster is uniformly distributed between 2 and 4 with $N_1 = 3$ and $N_{99} = 3$. $K = 2$ and $k = 2$ are selected. The rate 2/5 convolutional code in Table I is selected to generate the trellis of Figure 7. It is observed that the ad hoc routing scheme achieves the worst outage performance. Both ad hoc routing and N-hop routing achieve a diversity order of 2. When $N$ increases from 2 to 10, the N-hop routing scheme achieves some power gains with no improvement on the diversity order. Note that the simulation and analysis in [12] are based on idealised linear networks, which have fixed number of relays per cluster, the ad hoc and N-hop routing scheme can achieve a full diversity order in this scenario. However, for networks with variable number of relays per cluster, the ad hoc and N-hop routing scheme may not achieve the full diversity order. For the proposed path selection scheme, it achieves a full diversity order of 3 when the window size is 2. For $w = 10$, which is five times the total encoder memory ($K = 2$), the proposed scheme achieves almost the same outage probability as the optimal path selection scheme. The proposed scheme achieves a significant outage performance improvement compared with the ad hoc and N-hop routing schemes. At the outage probability of $10^{-3}$, the proposed path selection scheme with $w = 10$ achieves 8, 6 and 4 dB SNR gains over ad hoc scheme, N-hop scheme with $N = 2$ and N-hop scheme with $N = 10$, respectively.

The simulation results earlier show the outage performance of the optimal path selection scheme and the proposed path selection scheme in the high SNR regime. To demonstrate the outage performance of these two schemes in the low-to-medium SNR regime, Figure 8 is included.

It shows the outage probability of a 6-hop relay network with two relays at each of the intermediate clusters. $K = 1$ and $k = 1$ are selected. The rate 1/2 convolutional code in Table I is selected to generate the trellis. It is shown that the outage probability of the proposed scheme improves as the window size increases. The proposed scheme achieves almost the same outage probability as the optimal path selection scheme when $w = 5$, which is five times the total encoder memory.

Figure 9 shows the outage probability of a 50-hop network with four relays per cluster ($K = 2$) using the proposed path selection scheme with different window sizes ($1K \leq w \leq 8K$) at the SNR of 19 dB. It is clear from Figure 9 that the outage probability decreases dramatically as $w/K$ increases from 1 to 3. The outage probability decreases gradually when $w/K$ increases from 4 to $\infty$. Note that in Figure 9, a window size of $w = 4 = 2K$ degrades the performance by about 3 dB, and $w = 8 = 4K$ is almost as good as $w = 16 = 8K$. 

Figure 7. Outage performance comparison between different multi-hop path selection schemes.

Figure 8. Outage performance of a network with two relays per cluster over 6 hops.

Figure 9. Outage performance of the network with four relays per hop over 50 hops using different truncation window sizes at the SNR of 19 dB.
7. IMPLEMENTATION ISSUES, COMPUTATIONAL COMPLEXITY AND MEMORY STORAGE COMPARISONS

The proposed path selection scheme can be implemented either in a centralised or distributed way. If a central controller is available, it can collect the accumulative path metrics of the surviving paths when the memory is full and select a best relay on the first cluster within the window. If a central controller is not available, the proposed path selection scheme can be performed in a distributed way. In [35], a best relay is selected distributedly from a set of available relays to help the communication from the source to the destination. In specific, a timer is started based on the instantaneous channel information. The best relay has its timer reduced to zero first and starts transmitting. In our path selection scheme based on sliding window VA, the best relay at the first relay cluster within the window can be selected in a similar way. When the memory is full, each relay in the last relay cluster within the window computes the accumulative path metrics and sets a timer proportional to the metrics. The smaller the path metric is, the shorter the time should be. In this way, the timer of the relay with the best path within the window expires first. Then the relay sends a flag signal, and the index of its selected relay of its previous relay cluster. All the other relays in the last relay cluster within the window, while awaiting the timer to reduce to zero, are in listening mode. As soon as they hear the flag, they keep silent. When the relays at the previous relay cluster hear the signal, they check the received index and keep silent if it does not match its own index. The selected relay in the penultimate relay cluster then sends the index of its selected relay at its previous relay cluster. This process continues until the relays at the first relay cluster within the window receive the selection signal, and the best relay is selected. The best relay at the first relay cluster within the window sends a finish signal and the path selection process is repeated. This mode requires that there is a low-rate feedback channel from the destination to the source and that all the relays in each relay cluster could hear each other.

The computational complexity (in terms of the number of comparisons) and memory storage requirements (in terms of the sum of the memory for branch metrics and path metrics) of the ad hoc routing [12, 22], N-hop routing [12], optimal path selection [11] and the proposed scheme based on sliding window VA are listed in Table II. In this table, $N_{\text{max}} = \max_{i=1,2,\ldots,M-1} \{N_i\}$. The ad hoc routing scheme achieves the lowest computational complexity and the smallest memory storage requirement. The proposed path selection scheme has the second lowest computational complexity and memory storage requirement. The number of comparisons of the proposed scheme increases polynomially as the number of nodes in the network. The N-hop routing scheme has a relatively high complexity and large memory storage compared with the ad hoc and the proposed scheme. The computational complexity of the optimal path selection scheme increases exponentially with the number of nodes in the network. All the path metrics and branch metrics have to be stored in order to choose the optimal path. The high computational complexity and large storage requirement make the implementation of the optimal path selection scheme impractical.

Based on the discussions in Sections 6 and 7, it is clearly shown that the proposed path selection scheme can achieve outage performance close to the optimal path selection scheme but has significantly reduced complexity and a much lower requirement for memory storage.

8. CONCLUSION

In this paper, we proposed a near-optimal path selection scheme for multi-hop wireless relay networks. It draws on the analogy between the distribution of relay nodes of a commonly used relay network model and that of a convolutional code trellis and applies the VA in selecting a path to maximise the E2E SNR. The network structure is first mapped into a convolutional code trellis. The VA is then applied to select a path from the source to the destination. To reduce implementation delay and memory storage, the sliding window VA is employed. An asymptotic lower bound of the outage probability of the optimal path selection scheme is derived. An approximation of the outage probability of the proposed path selection scheme based on sliding window VA is derived. These analyses are verified by simulation to be accurate at high SNRs. Both simulation and analytical results show that the SNR loss of the proposed scheme based on sliding window VA relative to the optimal path selection scheme is negligible when the window size is five times the total encoder memory or

<table>
<thead>
<tr>
<th>Schemes</th>
<th>Ad hoc</th>
<th>N-hop</th>
<th>Optimal</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational complexity</td>
<td>$MN_{\text{max}}$</td>
<td>$(\frac{M}{2} + 1)N_{\text{max}}^{\text{w}}$</td>
<td>$N_{\text{max}}^{\text{w}}$</td>
<td>$(M - w)wN_{\text{max}}^{\text{w}}$</td>
</tr>
<tr>
<td>Memory for branch metrics</td>
<td>$N_{\text{max}}^{\text{w}}$</td>
<td>$(N_{\text{max}}^{\text{w}})^{\text{w}}$</td>
<td>$N_{\text{max}}^{\text{w}}$</td>
<td>$wN_{\text{max}}^{\text{w}}$</td>
</tr>
<tr>
<td>Memory for path metrics</td>
<td>$N_{\text{max}}^{\text{w}}$</td>
<td>$(N_{\text{max}}^{\text{w}})^{\text{w}}$</td>
<td>$N_{\text{max}}^{\text{w}}$</td>
<td>$wN_{\text{max}}^{\text{w}}$</td>
</tr>
</tbody>
</table>
more. The proposed scheme can be implemented with low-complexity and small-storage requirement and is efficient for large-scale networks with a large number of hops and relays per cluster.

APPENDIX

Proof of Theorem 1

Let $\gamma_{jl}$ be the instantaneous SNR of the bottleneck hop $j$, $l = 1, 2, \cdots, T, l = 1, 2, \cdots, d_m$. According to [36, (2.1.1)], the cumulative distribution function of $\gamma_{jl}$ is given by

$$F_X(x) = 1 - \exp \left(-\frac{1}{\gamma}x\right) \quad \text{(A1)}$$

The probability density function of $\gamma_{jl}$ can be expressed as

$$f_X(x) = \frac{1}{\gamma} \exp \left(-\frac{1}{\gamma}x\right) \quad \text{(A2)}$$

The probability that $\gamma_{jl}$ is below the threshold $\gamma_{th}$, denoted by $P_{jl}$ and is given by

$$P_{jl} = \Pr \{ \gamma_{jl} < \gamma_{th} \} = \int_{0}^{\gamma_{th}} f_X(x) dx = \left\{ 1 - \exp \left(-\frac{\gamma_{th}}{\gamma} \right) \right\} \quad \text{(A3)}$$

The bottleneck hop is in outage when the largest instantaneous SNR of the channels within the hop is smaller than $\gamma_{th}$. The probability can be expressed as

$$P_j = \Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \gamma_{jl} < \gamma_{th} \right\} = \left\{ 1 - \exp \left(-\frac{\gamma_{th}}{\gamma} \right) \right\}^{d_{\text{max}}} \quad \text{(A4)}$$

Because the equivalent SNR of path $i$ can be approximated by the minimum instantaneous SNR of all branches along that path. The outage performance of a path is dominated by the worst branch on it. The E2E communication between the source and destination fails when at least one of the hops in the network fails. The outage probability can be lower bounded by the summation of the probabilities that one of the bottlenecks is in outage, but other hops are not in outage; and the probability of none of the bottleneck hops is in outage, but one of the other hops is in outage.

$$P_{\text{out}}^{\text{opt}} \geq P_0$$

$$= \Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \min_{m=1, \cdots, M} \{ \gamma_{im-1,im}(m) \} < \gamma_{th} \right\}$$

$$+ \Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \{ \gamma_{1, l} > \gamma_{th} \}, \max_{l=1, \cdots, d_{\text{max}}} \{ \gamma_{2, l} < \gamma_{th} \} \right\}$$

$$\vdots$$

$$+ \Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \min_{m=1, \cdots, M} \{ \gamma_{im-1,im}(m) \} < \gamma_{th} \right\}$$

$$\Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \{ \gamma_{1, l} > \gamma_{th} \}, \cdots, \max_{l=1, \cdots, d_{\text{max}}} \{ \gamma_{T, l} < \gamma_{th} \} \right\}$$

where $\gamma_{im-1,im}(m)$ is the instantaneous SNR of the branch connecting relay $i_{m-1}$ of cluster $m-1$ and relay $i_m$ of cluster $m$. Note that the probability of two or more hops in outage can be neglected when compared with the probability of one of the bottleneck hops is in outage in the high SNR regime. Therefore, (A5) is tight in the high SNR regime. The $d_m$ branches in the first bottleneck hop are shared by all $I$ paths, the first term in (A5) is equal to

$$P_1 = \Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \{ \gamma_{1, l} > \gamma_{th} \} \right\} \quad \text{(A6)}$$

The $d_{\text{max}}$ branches at the second bottleneck hop are also shared by all the $I$ paths. Considering that the $d_m$ branches at the second bottleneck hop are independent of the $d_{\text{max}}$ branches in the first bottleneck, the second term in (A5) can be written as

$$P_2 = (1 - P_1) \Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \{ \gamma_{2, l} < \gamma_{th} \} \right\}$$

$$= \Pr \left\{ \max_{l=1, \cdots, d_{\text{max}}} \{ \gamma_{2, l} < \gamma_{th} \} \right\} + o \left( \frac{\gamma_{th}}{\gamma} \right) \quad \text{(A7)}$$

Similarly, we can obtain the values of the first \(T\) terms. It is difficult to derive the exact expression of the last term in (A5). However, in this case, none of the bottleneck branches is in outage. If \(T = M\), this means that all the branches are bottleneck branches, then last term equals to 0. If \(T < M\), there are at least \(L_{max}^{(M)} > d_{max}\) distinct bottleneck branches. At high SNR regions, the last term is given by

\[
P_{T+1} = \begin{cases} 
0, & T < M \\
\text{A line here}^n_{\text{A line here}} & T = M 
\end{cases} 
\] (A8)

Combining all the terms together, we obtain

\[
P_{\text{out}}^{\text{opt}} \geq P_0 = \sum_{j=1}^{T} \Pr \left( \max_{l=1,2,\cdots,d_{\text{max}}} \gamma_{j,l} < \gamma_h \right) + o \left( \left( \frac{\gamma_h}{\gamma} \right)^{d_{\text{max}}} \right)
\] (A9)

\[
P_{\text{out}}^{\text{opt}} \geq P_0 = T \left( \frac{\gamma_h}{\gamma} \right)^{d_{\text{max}}} + o \left( \left( \frac{\gamma_h}{\gamma} \right)^{d_{\text{max}}} \right) 
\] (A10)

At high SNRs, the asymptotic average outage probability can be lower bounded by

\[
P_{\text{out}}^{\text{opt}} \geq P_0 = T \left( \frac{\gamma_h}{\gamma} \right)^{d_{\text{max}}} + o \left( \left( \frac{\gamma_h}{\gamma} \right)^{d_{\text{max}}} \right) 
\] (A11)

The tightness of this bound will be verified by simulation in Section 6.

This proves Theorem 1.

**Proof of Theorem 2**

In the proposed path selection scheme based on sliding window VA, the network is in outage when at least one of the hops is in outage. Similar to the derivation of (A5), the outage probability of the proposed path selection scheme can be lower bounded by the summation of the probabilities that one of the bottleneck hops is in outage, but other hops are not in outage; and the probability of none of the bottleneck hops is in outage, but one of other hops is in outage. Let \(\gamma_{j,l}^{'}\) denote the instantaneous SNR of bottleneck hop \(j, l = 1,2,\cdots, T\), \(m = 1,2,\cdots, d_{max}\) of the proposed path selection scheme based on sliding window VA. The outage probability of the proposed path selection scheme based on sliding window VA, denoted by \(P_{\text{out}}^{\text{SWVA}}\), is lower bounded by

\[
P_{\text{out}}^{\text{SWVA}} \geq \Pr \left\{ \max_{l=1,\cdots, T} \left\{ \min_{m=1,\cdots, M} \gamma_{l,m}^{'} \right\} < \gamma_h \right\}
\]

\[
+ \Pr \left\{ \max_{l=1,\cdots, T} \left\{ \min_{m=1,\cdots, M} \gamma_{l,m}^{'} \right\} > \gamma_h \right\} \right\} \right) 
\]

\[
+ \Pr \left\{ \max_{l=1,\cdots, T} \left\{ \min_{m=1,\cdots, M} \gamma_{l,m}^{'} \right\} > \gamma_h \right\} \right\} \right) 
\]

\[
+ \Pr \left\{ \max_{l=1,\cdots, T} \left\{ \min_{m=1,\cdots, M} \gamma_{l,m}^{'} \right\} > \gamma_h \right\} \right\} \right) 
\]

\[
\max_{l=1,\cdots, T} \left\{ \gamma_{T,l}^{'} \right\} < \gamma_h \right\} \right) 
\]

\[
+ \Pr \left\{ \max_{l=1,\cdots, T} \left\{ \min_{m=1,\cdots, M} \gamma_{l,m}^{'} \right\} > \gamma_h \right\} \right) \right) 
\]

\[
\max_{l=1,\cdots, T} \left\{ \gamma_{l,T}^{'} \right\} < \gamma_h \right\} \right) 
\]

\[
+ \Pr \left\{ \max_{l=1,\cdots, T} \left\{ \min_{m=1,\cdots, M} \gamma_{l,m}^{'} \right\} > \gamma_h \right\} \right) 
\]

\[
\max_{l=1,\cdots, T} \left\{ \gamma_{T,l}^{'} \right\} < \gamma_h \right\} \right) 
\]

\[
\max_{l=1,\cdots, T} \left\{ \gamma_{l,T}^{'} \right\} < \gamma_h \right\} \right) 
\]

where \(E(r)\) is the error exponent at convolutional code rate \(r\). According to [38], the error exponent can be expressed as

\[
E(r) = \frac{1}{nK} \log_2 P_d 
\] (A13)

where \(P_d\) denotes the probability of the first event error. For fully connected networks, the free distance equals to 2. If \(p\) denotes the binary symmetric channel transition probability, \(p \leq \frac{1}{2}\). According to [21, (11.10)], the first event error is made with probability

\[
P_d = \frac{1}{2} \left( \frac{1}{2} \right) + \left( \frac{1}{2} \right) = p \leq \frac{1}{2} 
\] (A14)

Substituting (A13) and (A14) into (A12), the probability of not selecting the optimal branch at an arbitrary hop \(j\), due to a truncation window size \(w\), is upper bounded by

\[
p_{\text{tr},j} \leq \exp \left\{ \frac{w}{K} \log_2 P_d \right\} \leq p_0 = \exp \left\{ \frac{-w}{K} \right\} 
\] (A15)

This proves Lemma 2.
The $d_{\text{max}}$ branches in the $j$th bottleneck hop are shared by all $I$ paths. Similar to the derivation of (A7), the $j$th term, $j = 1, 2, \ldots, T$, in (A16) can be written as

$$p_{\text{SWVA}} = \Pr \left\{ \max_{l=1,2,\ldots,d_{\text{max}}} \left\{ \frac{Y'_{j,l}}{\gamma_{th}} \right\} < \gamma_{th} \right\} + \alpha \left( \frac{\gamma_{th}}{\beta} \right)^{d_{\text{max}}}$$  \hspace{1cm} (A17)

**When no truncation error happens**

If the optimal branch is selected by the proposed path selection scheme, $Y'_{j,l}$ is the same as the $Y_j$ in (A5). Therefore, the outage probability of the $j$th bottleneck hop when no truncation error happens, denoted by $p_{\text{SWVA}}^{\text{trunc error}}$, can be written as

$$p_{\text{SWVA}}^{\text{trunc error}} = \left\{ 1 - \exp \left( \frac{\gamma_{th} d_{\text{max}}}{\beta} \right) \right\}^{d_{\text{max}}}$$  \hspace{1cm} (A18)

**When a truncation error happens**

When there is a truncation error at hop $j$, $d_{\text{bottleneck}}$ consecutive branches selected diverge from those selected by the optimal path selection scheme. These $d_{\text{bottleneck}}$ branches are optimal in the $d_{\text{bottleneck}}$ hop subnetwork. In all these diverged paths, the ones with the first event error dominates. Therefore, we approximate $p_{\text{SWVA}}^{\text{error}}$ by

$$p_{\text{SWVA}}^{\text{error}} = \Pr \left\{ \max_{l=1,2,\ldots,d_{\text{bottleneck}}} \left\{ \min \left\{ Y_{j,l}, \ldots, Y_{j+d_{\text{bottleneck}}-1} \right\} \right\} < \gamma_{th} \right\}$$

$$= \left\{ 1 - \exp \left( - \frac{\gamma_{th} d_{\text{bottleneck}}}{\beta} \right) \right\}^{d_{\text{max}}}$$  \hspace{1cm} (A19)

Combining the previous two cases together, we obtain

$$p_{j}^{\text{SWVA}} \approx \left\{ 1 - p_{\text{trunc}} \right\} p_{\text{SWVA}}^{\text{trunc error}} + p_{\text{trunc}} p_{\text{SWVA}}^{\text{error}}$$  \hspace{1cm} (A20)

By using Taylor series expansion, at high SNRs, (A22) can be written as

$$p_{\text{SWVA}}^{\text{out}} \approx T \left\{ 1 - \exp \left( - \frac{\gamma_{th}}{\beta} \right) \right\}^{d_{\text{max}}} + T \exp \left( - \frac{\gamma_{th} d_{\text{max}}}{\beta} \right)$$  \hspace{1cm} (A23)

This proves Theorem 2.

Note that (A15) is an upper bound and (A19) is a lower bound. By substituting (A15), (A18) and (A19) into (A20), we obtain (A21). By substituting (A21) into (A16), an approximation of the outage probability of the proposed path selection scheme is derived. It will be verified by simulation to be accurate at high SNR regions in Section 6.

**REFERENCES**


