Residential Load Scheduling in Smart Grid: A Cost Efficiency Perspective

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Abstract—In smart grid, residential consumers adopt different load scheduling methods to manage their power consumptions with specific objectives. The conventional load scheduling methods aim to maximize the consumption payoff or minimize the consumption cost. In this paper, we introduce a novel concept of cost efficiency-based residential load scheduling framework to improve the economical efficiency of the residential electricity consumption. The cost efficiency is defined as the ratio of consumer’s total consumption benefit to its total electricity payment during a certain period. We develop a cost-efficient load scheduling algorithm for the demand-side’s day-ahead bidding process and real-time pricing mechanism by using a fractional programing approach. Results show that the proposed scheduling algorithm can effectively reflect and affect user’s consumption behavior and achieve the optimal cost-efficient energy consumption profile. For practical consideration, we also take into account the service fee and distributed energy resources (DERs) in our framework, and analyze their impacts on the cost efficiency. Simulation results confirm that the proposed algorithm significantly improves consumer’s cost efficiency. It is shown that a higher service fee will decrease the cost efficiency, while the integration of DERs can effectively improve the cost efficiency.

Index Terms—Cost efficiency, demand-side management (DSM), fractional programing (FP), renewable energy, smart house.

 NOMENCLATURE

- $\alpha$ Leakage rate of the battery.
- $\beta_k$ Parameter to control the penalty/refund.
- $\gamma^a$ Predetermined parameter for appliance $a$ in the utility function.
- $A$ Set of household electrical appliances.
- $OC$ Intended operation cycle for the system.
- $S$ Feasible set of $s$.
- $T_a$ Set of task cycles of appliance $a$.
- $\omega^a$ Parameter that represents the value of consumption satisfaction level for appliance $a$.
- $\chi^{(+)}$, $\chi^{(-)}$ Charging and discharging efficiencies.
- $a$ An appliance in $A$.
- $B_k^a$ Upper bound for $x_k^a$.
- $b_k^a$ Lower bound for $x_k^a$.
- $c$ Capacity of the battery.
- C.E. Cost efficiency.
- $d_n^a$ Total demand for appliance $a$ to fulfill a task in cycle $n$.
- $D_{n,max}^a$ Upper bound for $d_n^a$.
- $D_{n,min}^a$ Lower bound for $d_n^a$.
- $d_l_k$ Consumer’s total demand load at the supply side in time slot $k$.
- $g_k$ Total generation capacity of the smart house in time slot $k$.
- $k$ A time slot in $OC$.
- $l_k$ Consumer’s total consumption load in time slot $k$.
- $p_k^{add}$ Additional payment for the consumption difference in hour $k$.
- $p_k^s$ Service fee for a day.
- $p_k$ Electricity retail price of the hour $k$.
- $q_k$ Stored energy in time slot $k$.
- $s^{(max)}$ Maximum energy that can be stored in one time-slot.
- $s_k$ Energy storage profile in time slot $k$.
- $s_k^{(+)} \geq 0$ Per-slot charging profile.
- $s_k^{(-)} \geq 0$ Per-slot discharging profile.
- $T_n^a$ Task cycle for appliance $a$ to fulfill its nth task.
- $T_{l_a}$ Time length as a tolerance for the latest operation on $a$.
- $U_{a,n}(d_n^a)$ Utility of appliance $a$ in its task cycle $n$.
- $x_k^a$ Load of appliance $a$ in time slot $k$.
- $s$ Storage device profile vector in an operation cycle.
- $X$ Vector of all appliances’ consumption in one day’s operation.
- $x^a$ Consumption vector of appliance $a$ in one day’s operation.
\( P^a_k \) Probability for the smart meter to turn-on or turn-off the appliance \( a \) in current time slot \( k \).

\( \text{Pay} \) Total payment of a day without service charge.

\( \text{Pay}^{\text{total}} \) Total daily payment of the smart house consumer.

I. INTRODUCTION

DEMAND-SIDE management (DSM) has been developed since early 1980s to balance the time-varying demand load of consumers and maximum power generation capacity in the power system. In DSM, the pricing mechanisms and direct control strategies are employed by the energy suppliers to affect consumers’ consumption behaviors and reshape the total load [2]–[4]. The time-of-use pricing strategy sets different prices during the day to encourage consumers to shift their demand to off-peak hours [5]–[7]. Similar to the time-of-use pricing, the critical peak pricing applies a prespecified high price during the designated critical peak periods [8], [9]. The real-time pricing adopts the time-varying price according to the wholesale price of electricity and the cost of power generation to enable consumers to adjust their demand in response to supply [10]–[15].

In addition to the pricing mechanisms imposed by the supplier, the consumer side can also exploits the residential load scheduling technique to achieve consumers’ consumption goals [13], [16]–[19], [22]. The consumption cost and consumption payoff are usually been specified as the metrics to evaluate consumer’s consumption behavior. As for consumption cost, there are scheduling algorithms designed to minimize the consumer’s expenses on electricity consumption [17]–[19], [21], [34]. These programs help the consumer to develop a conservative consumption mode. Moreover, it behooves the electricity consumer to take its consumption satisfaction level into account beyond simply minimizing the consumption expenses in managing its consumption behavior. To meet this requirement, Fahrioglu and Alvarado [23] have introduced the concept of utility function to quantify the consumer’s consumption benefit (consumption satisfaction level), and chosen the consumption payoff, defined as the difference between the consumer’s utility function and its electricity cost, as a novel metric to evaluate consumers’ consumption behaviors. Guided by this metric, a number of the existing load scheduling methods are based on this concept of payoff functions and design algorithms to maximize the payoff in the scheduling optimization [22], [23]. These programs enable the consumer to pursue the best consumption benefit within the consumption limits.

Meanwhile, living in a resource-saving society, people are likely to develop highly economical efficient consumption habits to consume the power in moderation and exploit it to the best utilization. The two existing metrics have limited capability to directly reflect the effectiveness of consumers’ consumption in terms of consumption benefits per unit of cost and thus may not be the most effective way to affect consumers’ consumption behaviors. Therefore, a specific metric that represents the consumption efficiency is required. The metric should be able to significantly indicate the effectiveness of the consumer’s electricity consumption and imply the consumption habits of different kinds of residents. Motivated by this requirement, in this paper, we propose a novel concept of consumption cost efficiency as the metric of consumption efficiency, defined as the ratio of the consumption benefit to the consumption cost. We study the effect of simple power shifting of specific appliances on the consumption cost efficiency to demonstrate the direct relation between consumption load shape and cost efficiency. Moreover, we develop a residential load scheduling algorithm to maximize the cost efficiency of residential consumption. The proposed scheduling algorithm can effectively improve the economical efficiency of the residential consumption, helping the consumer to save its expenditure while fully utilize the consumed power.

Along with the DSM techniques, the integration of distributed energy resources (DERs) into the grid can also effectively increase the grid’s capacity and reduce the emission of CO₂ [24], [25]. Equipped with the distributed energy generation, the residential customers can also participate in the electricity market as an energy supplier. This leads to a more flexible electricity trading. In [19], the distributed energy generation and storage has been optimized to reduce consumers’ expenses via noncooperative and cooperative optimization approaches. In [26], an intelligent agent-based energy management system has been proposed to facilitate power trading among microgrids and allow customers’ participation in demand response. The integration of DERs with smart house will have a significant effect on the consumption cost efficiency. We have included the DERs in the smart house consumption scenario and study the effect.

The contributions of this paper can be summarized as follows.

1) We propose a new concept named cost efficiency to metric the consumer’s consumption efficiency of its electricity expenses in practice. The consumer can adopt the cost efficiency as an indicator to adjust its consumption behavior, or choose it as the objective to optimize in the consumption scheduling.

2) We study the cost efficiencies of different household consumption patterns and analyze the effect of load shifting on the cost efficiency. These conduce to consumers’ better understanding of their consumption habits.

3) To adopt the cost efficiency as an optimization objective, we develop a novel load scheduling algorithm to optimize the cost efficiency by using the advanced fractional programing (FP) tools. The DERs are also taken into account in the algorithm design and optimization. Simulation results demonstrate that the proposed algorithm is highly effective in improving the consumption economical efficiency. It also shows that the application of DERs can significantly improve the consumption cost efficiency.

4) For practical considerations, we also consider the effects of service fee of electricity consumption in practical systems and analyze its effects on cost efficiency. It is shown that a higher service fee will considerably
decrease the consumption cost efficiency, motivating consumers to adjust their consumption behavior.

The rest of this paper is organized as follows. In Section II, we introduce the concept of cost efficiency, analyze the cost efficiencies of different consumption patterns, and study the effect of load shifting on the cost efficiency. Smart house consumption model for the proposed cost efficient scheduling algorithm is described in Section III. The proposed cost-efficient residential load scheduling algorithm is presented in Section IV. Simulation results and analysis are provided in Section V, and the conclusion is drawn in Section VI.

II. UNDERSTANDING COST EFFICIENCY

A. Concept of Cost Efficiency

The development of modern society is being transformed into a resource-saving and environment friendly mode. Following the trends of sustainable development, people gradually change their consuming habits from pursuing the best benefit into the best consumption efficiency by increasing the effectiveness of utilizing the expenditure. Similarly for electricity consumption, it is necessary to find a metric that directly measures the consumption efficiency to guide the consumers in managing their consumption behaviors.

In economics, there is a well-known metric to measure the best production efficiency by maximizing the ratio of production output and production cost [28], [29]. To achieve the best production efficiency, one has to optimally utilize the available resources to maximize the production output, so as to effectively save resources and reduce production cost. Similarly, when applying this concept to the household consumption, increasing the consumption benefit created over per unit of cost can significantly improve the utilization of the expenditure on electricity consumption and thus save unnecessary expenses and energy resources. We introduce a concept of consumption cost efficiency, defined as the ratio of consumer’s expenditure and energy resources. We introduce a concept of cost efficiency, analyze the cost efficiency of appliance in different operation modes.

B. Modeling Consumption Behavior

Before analyzing the cost efficiencies of different consumption patterns and the effect of load scheduling, we introduce the electricity consumption model. Consider a set of household electric appliances, denoted by \( A = \{1, 2, \ldots, a, \ldots, N\} \), where each single appliance is denoted by a sequential number \( a \). The intended operation cycle for the system is 24 h, denoted by \( OC \), and is divided into 24 hourly time slots. \( x^a_k \) denotes the load of appliance \( a \) in time slot \( k \).

The utility originating from the economics is adopted to represent the consumption satisfaction level and is a function of the demand load. The utility function is generally defined to be a concave quadratic function due to the decreasing effectiveness of consumption [23]. In practice, many appliances need to finish one task at the expense of several time slots (not necessarily occupy the entire time slot), i.e., an oven to bake a chicken. We define task cycle \( T^a_n \) for appliance \( a \) to fulfill its \( n \)-th task, and define \( T^a \) as the set of task cycles of appliance \( a \). We note that one appliance may have two or more task cycles in a day corresponding to consumer’s requirement. The total demand for appliance \( a \) to fulfill a task in cycle \( n \) is defined as

\[
d^a_n = \sum_{k \in T^a_n} x^a_k.
\]

We calculate the utility value based on the task cycle of every appliance. The utility of appliance \( a \) in its task cycle \( n \) is expressed as

\[
U_{a,n}(d^a_n) = U(d^a_n, \omega^a)
\]

where \( d^a_n \) is the total demand for appliance \( a \) to fulfill a work in cycle \( n \) and \( \omega^a \) is a parameter that represents the value of consumption satisfaction level for appliance \( a \). In this paper, we choose quadratic utility functions as [33]

\[
U(d^a_n, \omega^a) = \begin{cases} \omega^a d^a_n - \frac{\gamma^a}{2} (d^a_n)^2 & \text{if } 0 \leq d^a_n < \omega^a \\ \frac{\omega^a \gamma^a}{2} & \text{if } d^a_n \geq \omega^a \end{cases}
\]

where \( \gamma^a \) is a predetermined parameter.

As for the consumption payment, the electricity retail price changes from hour to hour. We define \( p_k \) as the electricity retail price of the hour \( k \). Consumer’s consumption load in time slot \( k \) is expressed as

\[
l_k = \sum_{a \in A} x^a_k.
\]

Hence, the total payment of a day without service charge is expressed as

\[
\text{Pay} = \sum_{k \in OC} p_k l_k.
\]
C. Case Study 1: Cost Efficiencies of Different Consumption Patterns

To better understand the concept of cost efficiency and how it can be used to indicate the effectiveness of consumption behaviors, in this section, we study the cost efficiencies of different consumption patterns. We choose three typical household consumption patterns according to the clustering method in [34] and present the load shapes in Fig. 1. The typical pattern of types 1 and 2 correspond to the days when most of the family members stay home and have two or three meals at home while the only key difference between these two types is whether people go out for lunch or not. Type 3 corresponds to the day when all the family members were out during the day. We assign six typical electrical appliances with different amount of tasks to fill the load shapes.

The capability of an appliance to create utility depends on the settings of $\omega$ and $\gamma$. The consumer needs to set unique $\omega$ and $\gamma$ values for each appliance and each setting is determined by the consumer’s estimated utility-creating capability of the corresponding appliance. People can decide the parameters for every appliance according to their preferences. However, once the settings are determined, all comparisons have to be done under the same criterion. In this analysis, we assume that the cooking appliances and the appliances for entertainment will generate higher utilities than the others. The energy profiles of different appliances are presented in Table I. We note that the same kind of appliances may have different rated powers due to different designs. The settings of power are based on [10] and [34] and the information provided by the online electric appliance retailers.

To understand the settings and energy profile of an appliance, we take the refrigerator for example. The refrigerator operates 24 h every day. We regard its 24-h operation as one task which will create the best utility of four for the consumer. Therefore, the best consumption case is that the refrigerator works 24 h at the cost of 3.6 kWh electricity and create a utility value of 4. Referring to (4), we have $\omega^B/\gamma^B = 3.6$ and $(\omega^B)^2/2\gamma^B = 4$. To satisfy the two equations, we have $\omega^B = 20/9$ and $\gamma^B = 50/81$.

We adopt the pricing method that reflects the fluctuation of the wholesale electricity market, referring to the New York Independent System Operator daily report [35] and the pricing mechanisms of NOCO energy [36]. The electricity retail prices are presented in Fig. 2. In this analysis, we do not include the service charge in the payment. Thus, the cost efficiency of a consumption pattern is expressed as

$$C.E. = \frac{\sum_{a \in A} \sum_{n \in T^a} U(d_n^a, \omega^a)}{\sum_{k=1}^{24} p_k h_k}.$$  

The comparison of the three consumption patterns on values of consumption indicators are presented in Table II. The cost efficiency of type 3 is much lower than that of types 1 and 2. This is because in type 1, only fridge is operating, while appliances with capabilities to create higher utilities are not in operations. The cost efficiency of type 2 is higher than that of type 1, which indicates that the case when family members have three meals in a day is more cost efficient than the case when they only have two meals. This is because we give high utility generation capabilities to the kitchen appliances.

D. Case Study 2: Effect of Load Shifting on Cost Efficiency

In this section, we choose two consumption cases to analyze the effect of load shifting on the cost efficiency. The settings of the appliances and the electricity prices are the same as those in the previous section.

In the first case, the consumer spends 4 h in watching TV with air conditioning. We have select three periods for the activity: around the noon, in the evening, and in the midnight. We consider only the loads of the A.C. and the TV, and assume that in the three cases the appliances, respectively, consumes an equal amount of electricity. The load shapes of the three
cases are presented in the top sub-figure of Fig. 3 and the corresponding indicators are presented in Table III. Enjoying TV shows in the prime time achieves the lowest cost efficiency among the three choices due to high electricity prices for the rush hours of household activities in the prime time. Having a late-night entertainment creates the highest cost efficiency as the electricity prices go down dramatically into the midnight. In the second case, we analyze the effects of delaying cooking activities on the cost efficiency, represented by the operations of microwave oven and the kitchen ventilator. The cases are set as starting on time, delaying 30 min, delaying 60 min, and delaying 90 min. We note that each former case can also be regarded as the case ahead of time to the latter one. The load shapes of the four cases are presented in the bottom sub-figure of Fig. 3 and the corresponding indicators are presented in Table IV. According to the numerical results, a relatively early cooking activity is more cost efficient due to the avoidance of consumption in on-peak hours. Taking these indicators as references, the consumer can adjust its consumption behaviors to improve the economic efficiency of household electricity consumption.

The above case studies have demonstrated that the cost efficiency as a metric can directly reflect consumer’s consumption efficiency under a specific criterion, and is a reliable indicator for the consumers to manage their consumption behaviors. Similar to the residential electricity consumption, the cost efficiency can be adopted by the commercial consumer for economical business operations and the industrial consumer to achieve highly efficient production. In the latter part of this paper, we are going to introduce a residential consumption scheduling algorithm aiming to optimize the cost efficiency and analyze the effect of service fees and DERs on the cost efficiency.

III. Smart House Consumption Model

In this section, we describe the smart house consumption and DER model, and introduce the pricing mechanisms for the residential consumption. As shown in Fig. 4, in the residential consumption scenario, the smart house applies a smart meter to monitor all household electric appliances and manage the household electricity consumption automatically [22], [27]. The smart meter facilitates the exchange of measurements and control information between the energy supplier (i.e., a distribution company, DISCO) and the residential consumer. Besides power line access, there is a two-way communication access between power suppliers and residential consumers. The existing communication networks is able to guarantee the communication conditions for applying the proposed scheduling algorithm, ensuring the data and control information exchange [27].

To manage the load shapes of all the appliance, we consider the following constraints developed in [22] for the appliance consumption model in Section II-B. Since there are appliances that require consistent power all the time, or basic power to keep the lowest working level, we set a lower bound for each $x_a^k$, denoted as $b_a^k$. $b_a^k \neq 0$ means that the appliance is supposed to be in operation in time slot $k$ (not necessarily occupy the entire time slot). For appliances such as fridge, $b_a^k$ corresponds to its must-run load, while for the A.C., $b_a^k$ represents the load to maintain the lowest cooling capacity. $b_a^k = 0$ means that the appliance is not necessarily occupied in the time slot. Similarly, the upper bound, denoted as $B_a^k$ for appliance $a$ in time slot $k$, is determined based on appliance’s maximum demand and consumer’s requirement. $B_a^k$ can be used to limit the appliance’s working time in a time slot. For example, if we decide that the microwave oven should not operate more than 30 min in time slot $k$, $B_a^k$ should be set as 0.6 kWh. $B_a^k = 0$ means that the appliance is definitely off during the entire time slot of $k$. As an appliance may be in different working levels in different task cycles due to the consumer’s requirement, the consumption amounts of different task cycles can be different. For each appliance, we define $D_{a, \text{min}}^k$ and $D_{a, \text{max}}^k$ respectively,
as the minimum consumption requirement and the maximum consumption limit for appliance \( a \) in task cycle \( n \). We have

\[
D^a_{n,min} \leq d^n_k \leq D^a_{n,max}. \tag{7}
\]

For appliances such as the washing machine, they have a specific amount of power consumption \( W \) for every task. The consumer can set \( D^a_{n,min} = D^a_{n,max} = W \). For appliances such as A.C. and TV, their consumption demands for a task can change due to different working levels or operation time. Hence, \( D^a_{n,min} \) and \( D^a_{n,max} \) can be regarded as the consumer’s tolerances of the consumption amount in one day’s operation. Let \( x^a_k = (x^a_1, x^a_2, \ldots, x^a_{24})^T \) denote the consumption vector of appliance \( a \) in one day’s operation, and \( X = (x^1, x^2, \ldots, x^n) \) denotes the vector of all appliances’ consumption in one day’s operation. The feasible consumption set is defined as: \( \mathcal{X} = \{X | x^a_k \in [b^a_k, B^a_k], \forall k \in OC, n \in T^a, a \in A \} \).

### A. Distributed Generation and Storage Model

The smart house has renewable energy resources such as photovoltaic system and wind turbine, which can be modeled as nondispatchable energy generators [19], [37]. Let \( g_k \) denote the total generation capacity of the smart house in time slot \( k \). The maximum available power by the renewable generators depends on the weather conditions, and cannot be scheduled by specific strategies.

We assume that the smart house has an energy storage device,\(^1\) and the energy storage profile in time slot \( k \) is denoted by \( s_k = s_k^{(+)} - s_k^{(-)} \), where \( s_k^{(+)} \geq 0 \) is the per-slot charging profile and \( s_k^{(-)} \geq 0 \) is the per-slot discharging profile [19], [37]. \( s_k > 0, s_k < 0, \) and \( s_k = 0 \), respectively, implies that the storage device is being charged, being discharged, and inactive. In each time slot, the storage device can only be in one of the three states. Let \( s = ((s_k^{(+)} H)_{k=1}^H, (s_k^{(-)} H)_{k=1}^H) \) denote the storage device profile vector in an operation cycle. Let \( 0 < \zeta^{(+)} \leq 1 \) and \( \zeta^{(-)} \geq 1 \), respectively, denote the charging and discharging efficiencies. For a given charging efficiency \( \zeta^{(+)} \), when an amount of \( \hat{s}_k^{(+)} \) energy has been used for charging the battery, only \( \beta^{(+)} \hat{s}_k^{(+)} \) is charged. Similarly, to provide an amount of \( \hat{s}_k^{(-)} \) energy, the storage device has to release \( \beta^{(-)} \hat{s}_k^{(-)} \) amount of energy. \( 0 < \alpha \leq 1 \) denotes the leakage rate of the battery. The stored energy \( q_k \) in time slot \( k \) decreases to \( \alpha q_k \) in time slot \( (k + 1) \). The capacity of the battery is denoted by \( c \), and the maximum energy that can be stored in one time-slot is denoted by \( s^{(max)} \).

The storage scheduling profile is bounded by the following constraints. Elements in \( s \) have to satisfy

\[
-\alpha q_{k-1} \leq \zeta^{(+)} s_k^{(+)} - \zeta^{(-)} s_k^{(-)} \leq c - \alpha q_{k-1} \tag{8}
\]

and

\[
\zeta^{(+)} s_k^{(+)} - \zeta^{(-)} s_k^{(-)} \leq s^{(max)}. \tag{9}
\]

\(^1\)We note that currently the deployment of storage device could be at great expense for common residential consumers. However, we believe that with the development of energy storage techniques and the support on DEPs by the government, storage device is likely to become a common appliance in the future residential consumers.

Moreover, it is reasonable to assume that the final storage level \( q_H \) is approximately the same as the initial level \( q_0 \) [18], that is

\[
|q_H - q_0| \leq \delta \tag{10}
\]

where the positive constant \( \delta \) is sufficiently small. We rewrite the constraints in a matrix form. Then, the feasible set \( S \) of \( s \) contains the following constraints [18]:

\[
S = \left\{ s \in \mathbb{R}^{2H}_+ : \Delta \beta s \leq s^{(max)} I_H \right\}
\]

\[
(1 - \alpha^H)q_0 - \delta \leq a^T \Delta \beta s \leq (1 - \alpha^H)q_0 + \delta \tag{11}
\]

where \( \Delta \beta = (\beta^{(+)1} I_H - \beta^{(-)1} I_H) \), \( A \) is a \( H \)-dimensional lower triangular matrix where \( [A]_{ij} = \alpha^{-j} \). \( H \)-dimensional vector \( a \) and \( b \) satisfies, respectively, \( \Delta a_k = \alpha^{H-k} \) and \( b_k = \alpha^k \). \( S \) is a compact and convex set.

Given \( k, g_k \), and \( s_k \), the consumer’s demand load at the supply side \( d_k \) in time slot \( k \) is expressed as

\[
dl_k = l_k - g_k + s_k. \tag{12}
\]

### B. Pricing Mechanism for Electricity Consumption

We adopt the electricity market model that includes day-ahead bidding and real-time charging mechanisms for the household consumption scenario. That is, we use the day-ahead bidding for major energy consumption scheduling of the next day and real-time charging for the consumption uncertainty in real consumption [26], [37].

1) Day-Ahead Bidding: In day-ahead bidding, the energy supplier informs the residential consumers the per-unit prices \( p_k, k = 1, 2, \ldots, 24 \) and service fee \( p^{sf} \) for the next day which are set based on the historical state information and the corresponding estimations. The residential consumers decide their hourly consumption demands according to the per-unit prices and service fee, and upload the information \( d_i = (d_{1i}, d_{2i}, \ldots, d_{24i})^T \) to bid for the electric energy. After receiving all the consumers’ demand information, the supplier sends confirmations to all consumers to make all contracts with consumers to take effect. We adopt the real-time pricing method with inclining block rates (IBR), where the future pricing parameters are known for the users ahead of time [13]. Let \( p_k \) denote the price per unit in time slot \( k \). \( p_k \) is defined as

\[
p_k(d_k) = \begin{cases} 
\eta_k & \text{for the part of } d_k \text{ within } c_k \\
\tau_k & \text{for the part of } d_k \text{ in excess of } c_k
\end{cases} \tag{13}
\]

where the predetermined \( \eta_k \) and \( \tau_k \) are two unit prices of different levels, and \( c_k \) is the threshold amount of the higher price. If the demand amount \( d_k > c_k \), the exceeding part \( d_k - c_k \) will be charged at the cost of \( \tau_k \) per unit. The daily service fee is denoted by a constant \( p^{sf} \).

Smart house consumer’s daily payment can be expressed as

\[
\text{Pay}(dl) = \sum_{k \in OC} \max\{\eta_k d_k, \tau_k d_k + (\eta_k - \tau_k) c_k\} + p^{sf}. \tag{14}
\]
their bid scheduling in real time. Referring to the pricing model for consumption uncertainty [17], we set an additional payment for the consumption difference as

\[ p_k^{\text{add}} = p_k(d_k)(\hat{d}_k - \hat{d}_k) \cdot \beta_k \]  

(15)
in which \( \beta_k \) is a parameter to control the penalty/refund, and is expressed as

\[ \beta_k = \begin{cases} 
1.5 & \text{if } \frac{d_k}{\hat{d}_k} \geq 1.05 \\
\frac{d_k}{\hat{d}_k} & \text{if } 0.95 < \frac{d_k}{\hat{d}_k} < 1.05 \\
0.5 & \text{if } \frac{d_k}{\hat{d}_k} \leq 0.95 
\end{cases} \]  

(16)

where \( \beta_k \) is adjustable by the supply side. If \( d_k < \hat{d}_k, p_k^{\text{add}} \) will be negative. In this case, \( \beta_k < 1 \) guarantees that the refund of the reduction in consumption is a fraction of the previous bidding expense. The total daily payment of the smart house consumer is given by

\[ \text{Pay}_{\text{total}}(\hat{d}, d) = \text{Pay}(d) + \sum_{k \in \mathcal{OC}} p_k^{\text{add}}. \]  

(17)

IV. COST-EFFICIENT RESIDENTIAL LOAD SCHEDULING

A. Scheduling in Day-Ahead Bidding

In the day-ahead bidding process, it is feasible for the smart house consumer to optimize its consumption demand allocation in bidding \( \hat{X} \) based on the given electricity prices \( p_k, k \in \mathcal{OC} \), and \( p_k^{\text{df}} \).

In smart house consumption scenario, the bidding in the proposed scheduling method is to pursue the best cost efficiency, defined as follows:

\[ \max_{\hat{X} \in \chi} \frac{\sum_{a \in \mathcal{A}} \sum_{n \in \mathcal{T}^a} U(\hat{a}_n, \omega^a)}{\sum_{k \in \mathcal{OC}} \max \{ \eta_k \hat{d}_k, \tau_k \hat{d}_k + (\eta_k - \tau_k) c_k \} + p_k^{\text{df}}} \]  

(18)

where \( \hat{X} \) is the consumption allocation of all appliances in bidding, \( U(\hat{a}_n, \omega^a) \) is defined in (4), \( p_k(\cdot) \) is defined in (13), and \( \hat{d}_k = \hat{d}_k - \hat{g}_k + \hat{s}_k \). \( \hat{g}_k \) is obtained based on the experience. We note that the optimization (18) only considers the case that the total daily demand of all appliances is greater than the experience-based total daily renewable generation. If the predetermined lowest daily consumption demand is smaller than the experience-based total daily renewable generation, the scheduling will be inactive due to the randomness of renewable generation that can cause the uncertainty of buying or selling electricity. Therefore, the optimal cost efficient consumption load allocation for the smart house consumer can be achieved by solving the following optimization problem:

\[ \hat{X} = \arg \max_{\hat{X} \in \chi} \frac{\sum_{a \in \mathcal{A}} \sum_{n \in \mathcal{T}^a} U(\hat{a}_n, \omega^a)}{\sum_{k \in \mathcal{OC}} \max \{ \eta_k \hat{d}_k, \tau_k \hat{d}_k + (\eta_k - \tau_k) c_k \} + p_k^{\text{df}}} \text{ s.t.} \]

C1 : \( b_k^a \leq \hat{d}_k^a \leq b_k^a, \forall k \in \mathcal{OC}, a \in \mathcal{A} \)

C2 : \( D_n^{a, \text{min}} \leq \hat{d}_n^a \leq D_n^{a, \text{max}}, \forall n \in \mathcal{T}^a, a \in \mathcal{A} \)

C3 : \( \hat{s} \in \mathcal{S} \)  

(19)

where C1 is the consumption boundaries for all appliances in every time slot, C2 is the consumption requirements for each appliance in fulfilling its tasks, and C3 is the feasible set of the storage scheduling profile. We note that \( \sum_{a \in \mathcal{A}} \sum_{n \in \mathcal{T}^a} D_n^{a, \text{min}} > \sum_{k \in \mathcal{OC}} \hat{s}_k \). With the scheduling result, the smart meter will provide the recommended start time for all the appliances to the consumer. In problem (19), the term \( \max(\eta_k \hat{d}_k, \tau_k \hat{d}_k + (\eta_k - \tau_k) c_k) \) is a pointwise maximum function. Since \( \eta_k \hat{d}_k \) and \( \tau_k \hat{d}_k + (\eta_k - \tau_k) c_k \) are both linear functions of \( \hat{d}_k \), this pointwise maximum function is a convex function [39]. When \( \hat{X} \) is calculated, the smart house consumer determines \( \hat{d}_l \) according to (12) and submits the demand information \( \hat{d}_l \) to the supply side.

Proposition 1: Problem (19) is a convex–concave FP problem.

Proof: See Appendix A.

Optimization problem (19) could be solved by the FP, given in Appendix A. The proposed scheduling algorithm is based on the Dinkelbach method (see Algorithm 1 in Appendix A). In the day-ahead bidding part of the algorithm, the Dinkelbach method iteratively calculates the best cost efficiency \( \lambda_n \) in step n. The sequence of \( \lambda_n \) is a strictly increasing sequence, and thus it will converges to the optimal cost efficiency \( \lambda^+ \) as shown in the following proposition.

Proposition 2: The sequence \( \{ \lambda_i \} \) calculated by Dinkelbach’s algorithm converges superlinearly to \( \lambda^+ \), \( \forall \lambda_i \leq \lambda^+ \), that is

\[ \lim_{i \to \infty} (\lambda^+ - \lambda_{i+1}) = 0, \text{if } \{ \lambda_i \} \text{ is infinite.} \]

Proof: See Appendix A.

Given \( p_k(\hat{d}_k) \) and \( \hat{d}_l \), consumer’s bidding payment is calculated as

\[ \text{Pay}(\hat{d}) = \sum_{k \in \mathcal{OC}} p_k(\hat{d}_k) \hat{d}_k + p_k^{\text{df}}. \]  

(20)

B. Management for Real-Time Consumption

In real-time consumption, there can be slight changes in the consumer’s consumption due to its temporary decisions. The renewable generation uncertainty may also cause the consumer to adjust its demand load. These can affect the electricity payment. We assume that the storage device will obey the charging and discharging arrangement by bidding strategy in the real consumption, that is \( s_k = \hat{s}_k, \forall k \in \mathcal{OC} \). As for \( \hat{g}_k \), the renewable generators will always operate in the maximum output unless the consumer has restrained the generation capacity. As for the consumer’s temporary decision to add extra consumption tasks or reduce the consumption of working appliances that effect \( l_k \), we divide it into two cases. If the consumer directly turns-on or turns-off some appliance without considering effectiveness issues, the smart meter will only do the recording. If the consumer lets the smart meter control the appliance, it has to provide an extra information for the smart meter. Here, we are interested only in determining when to turn-on or turn-off the appliance. To determine when to turn-on an appliance, the smart meter tends to choose a start time with relatively low retail price. To decide when to turn-off an appliance, the smart meter tends to choose an end time with relatively high retail price. For appliances with interruptible
load, the management problem can be decomposed into several “when to turn-on or turn-off” problems with extra settings.

The consumer has to specify an appliance \(a\) to turn-on or turn-off. Moreover, it has to set a time length \(TL_a\) as a time tolerance within which the smart meter’s operation on \(a\) must take place. The smart meter will activate the trial of controlling appliance \(a\) according to a decision-making mechanism immediately and repeat the trial periodically until the decision has been executed or is executed after a time length of \(TL_a\), or is canceled. The cycle length corresponds the cycle length of refreshing retail price, i.e., an hour. We take the washing machine for example, the consumer allows the washing machine starting to operate within \(TL_a = 1.5\) h. The smart meter will immediately run the decision-making mechanism to decide whether to start the washing machine right now. If the decision is not, it will repeat the trial an hour later. If the second trial also fails, it will start the washing machine after another 30 min as the deadline arrives.

We provide a probability-based decision-making mechanism based on the consumer’s requirement for the smart meter to control the appliances. We assume that the smart meter can record the latest 24 h retail prices \(\{p_k, k=23, \ldots, p_k-1, p_k\}\). The smart meter sorts the price sequence in ascending order and get the ascending sequence order \(a_{sk}\) of \(p_k\). The probability \(P_a^k\) for the smart meter to turn-on or turn-off the appliance \(a\) in current time slot \(k\) is given by

\[
P_a^k = \begin{cases} \frac{a_{sk}-1}{24} & \text{for turning off} \\ \frac{24-a_{sk}}{25} & \text{for turning on}. \end{cases} \tag{21}
\]

Given the real consumption information \(g_k, l_k\), the total payment of the day is calculated as

\[
\text{Pay}^\text{total}(\hat{d}, \hat{d}) = \text{Pay}(\hat{d}) + \sum_{k \in \mathcal{OC}} p_{k}^{\text{add}} \tag{22}
\]

where \(\text{Pay}(\hat{d})\) is as in (20), \(p_{k}^{\text{add}}\) is calculated as

\[
p_{k}^{\text{add}} = p_k(d_k l_k - \hat{d}_k) \beta_k = p_k(d_k) [(l_k - g_k + \hat{s}_k) - \hat{l}_k + \hat{s}_k] \beta_k. \tag{23}
\]

### C. Summary of the Proposed Scheduling Approach

For clearly understanding the proposed algorithms, we have summarized the proposed scheduling algorithm in Table V. The day-ahead scheduling algorithm adopts the Dinkelbach method to achieve the optimal solution via \(I\) iterations. In the \(n\)th iteration, obtaining the optimal consumption allocation with \(\lambda_n\) is a convex quadratic optimization problem that can be solved by iterative methods such as interior point method [39]. In each inner iteration, the algorithm has to calculate the value of a function with \(K\) variables, where \(K\) is the number of time slots and \(N\) is the number of appliances. The upper bound on the number of inner iteration steps is \(\sqrt{m}\), where \(m\) is the number of inequality constraints [39]. Hence, the algorithmic complexity of the convex quadratic optimization problem is \(O(\sqrt{mKN})\). The algorithmic complexity of the day-ahead scheduling algorithm is \(O(I\sqrt{mKN})\). The application of the proposed algorithm in a smart house with more appliances and more time slots corresponds to the increase of \(\sqrt{m}, K, \) and \(N\).

In the real consumption, let \(v\) denote the number of temporary tasks. Let \(\bar{t}i\) represent the average time tolerance of the \(v\) tasks. In each trial of starting the task, the program has to sort the given \(24\) electricity prices for calculating the probability. The complexity of calculating the probability with the worst sorting case is \(O(24^2 + 1)\). Hence, the algorithmic complexity of real consumption algorithm is \(O((24^2 + 1)\bar{v}i) \approx O(\bar{v}i)\). For a larger system with multiple consumers, each consumer will independently adopt the scheduling algorithm to achieve its best cost efficiency respectively, leading to a distributed algorithm that protects the consumer privacy.

### V. Simulation Results and Discussion

In this section, we present the simulation results of the proposed load scheduling algorithm, analyze the effects of DERs, and service fee on the cost efficiency and test the performance of the real-time consumption management.

Let us first present the system settings. The hourly electricity prices \(\{p_k, k=1, 2, \ldots, 24\}\) in the day-ahead bidding are set based on the NYISO daily report [35] and the pricing mechanisms of NOCO energy [36], which are presented in Table VI. We set \(t_k = \eta_k \cdot 1.5\). We assume that the forecast errors are within 5% [35]. Hence, the real-time prices fluctuate above and below the day-ahead prices within 5%.

All appliances are described by different utility parameters according to their power and task fulfillment. One appliance may have several tasks in a day and the total utility it creates is the sum of the utilities of every single task fulfillment. For example, if the oven operates for both lunch and supper in a day, the number of its tasks is 2. Similar to Section II-C, we adopt ten typical appliances for the consumption scheduling, of which the settings are presented in Table VII [10], [34]. For the storage device settings, we refer to [18], \(\alpha = \sqrt{0.9}, \beta(+) = 0.9, \beta(−) = 1.1, \epsilon_n = 4 \text{ kWh}, s^{(\text{max})} = 0.5 \text{ kWh/h,}\)

<table>
<thead>
<tr>
<th>TABLE V</th>
<th>COST-EFFICIENT CONSUMPTION SCHEME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Day-ahead Bidding</td>
<td></td>
</tr>
<tr>
<td>1.1 Refreshing Process</td>
<td>* At the beginning of each day, the smart meter updates the bidding information of the day by using the bidding information of the next day.</td>
</tr>
<tr>
<td>1.2 Day-ahead Bidding Process</td>
<td>* The supplier releases the hourly per-unit prices for the next day according to (13).</td>
</tr>
<tr>
<td>* The smart meter sets the experienced household generation prediction (\hat{g}) for the next day.</td>
<td></td>
</tr>
<tr>
<td>* The smart meter calculated the lowest daily demand of all the appliances. If it is smaller than (\sum_{k \in \mathcal{OC}} g_k), the scheduling for bidding is inactive. Otherwise, the smart meter calculates the bidding consumption distribution and payment by solving (19).</td>
<td></td>
</tr>
<tr>
<td>* The smart meter confirms the bidding with the supply side, and the bidding takes effect.</td>
<td></td>
</tr>
<tr>
<td>* The smart meter updates the bidding information of the next day by using the new bidding information.</td>
<td></td>
</tr>
<tr>
<td>2. Real-time Consuming</td>
<td>* Consumer and the smart meter adjust the real-time consumption according to temporary decisions.</td>
</tr>
<tr>
<td>* Total payment of the day is calculated according to (12) and (22).</td>
<td></td>
</tr>
<tr>
<td>* The smart meter updates the real consumption information and conforms the real payment with the supply side.</td>
<td></td>
</tr>
</tbody>
</table>

---

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.
TABLE VI
ELECTRICITY PRICES IN THE DAY-AHEAD BIDDING

<table>
<thead>
<tr>
<th>Hour</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($/kWh)</td>
<td>0.4239</td>
<td>0.4133</td>
<td>0.4364</td>
<td>0.4439</td>
<td>0.4832</td>
<td>0.5572</td>
</tr>
<tr>
<td>Hour</td>
<td>7th</td>
<td>8th</td>
<td>9th</td>
<td>10th</td>
<td>11th</td>
<td>12th</td>
</tr>
<tr>
<td>Price ($/kWh)</td>
<td>0.5339</td>
<td>0.5734</td>
<td>0.4933</td>
<td>0.5106</td>
<td>0.6206</td>
<td>0.6672</td>
</tr>
<tr>
<td>Hour</td>
<td>13th</td>
<td>14th</td>
<td>15th</td>
<td>16th</td>
<td>17th</td>
<td>18th</td>
</tr>
<tr>
<td>Price ($/kWh)</td>
<td>0.5739</td>
<td>0.5639</td>
<td>0.5472</td>
<td>0.5806</td>
<td>0.7372</td>
<td>0.8072</td>
</tr>
<tr>
<td>Hour</td>
<td>19th</td>
<td>20th</td>
<td>21st</td>
<td>22nd</td>
<td>23rd</td>
<td>24th</td>
</tr>
<tr>
<td>Price ($/kWh)</td>
<td>0.7808</td>
<td>0.8106</td>
<td>0.7339</td>
<td>0.7073</td>
<td>0.6061</td>
<td>0.5061</td>
</tr>
</tbody>
</table>

TABLE VII
SMART HOUSE APPLIANCE SETTINGS II

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Power (KW)</th>
<th>Utility per kWh</th>
<th>Demand per task (kWh)</th>
<th>$\omega$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>12</td>
<td>2</td>
<td>1/6</td>
</tr>
<tr>
<td>B</td>
<td>0.15</td>
<td>10/9</td>
<td>3.6</td>
<td>20/9</td>
<td>50/81</td>
</tr>
<tr>
<td>C</td>
<td>0.15</td>
<td>20/3</td>
<td>1.8</td>
<td>40/3</td>
<td>200/27</td>
</tr>
<tr>
<td>D</td>
<td>0.3</td>
<td>23/18</td>
<td>2</td>
<td>23/18</td>
<td>25/18</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>8/3</td>
<td>1.5</td>
<td>16/3</td>
<td>32/9</td>
</tr>
<tr>
<td>F</td>
<td>0.2</td>
<td>10</td>
<td>0.3</td>
<td>20</td>
<td>200/3</td>
</tr>
<tr>
<td>G</td>
<td>1.5</td>
<td>6</td>
<td>0.75</td>
<td>8</td>
<td>32/3</td>
</tr>
<tr>
<td>H</td>
<td>0.3</td>
<td>3</td>
<td>0.5</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>J</td>
<td>0.3</td>
<td>1</td>
<td>2.5</td>
<td>8</td>
<td>16/5</td>
</tr>
</tbody>
</table>

TABLE VIII
SMART HOUSE APPLIANCE CONSTRAINTS

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Origin Demand</th>
<th>$D_s^{(1)}$</th>
<th>$D_s^{(2)}$</th>
<th>$D_s^{(3)}$</th>
<th>$D_s^{(4)}$</th>
<th>Task number</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td>11.0</td>
<td>13.0</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>3.60</td>
<td>3.60</td>
<td>3.60</td>
<td>3.60</td>
<td>3.60</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>1.27</td>
<td>1.27</td>
<td>1.27</td>
<td>1.14</td>
<td>1.14</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>3.0</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>0.6</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>1.5</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>I</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>J</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.4</td>
<td>2.6</td>
<td>1</td>
</tr>
</tbody>
</table>

*Unit of measurement: kWh
$q_0 = 1 $kWh, and $\delta = 0$. The simulations are executed by MATLAB 2012a in a computer with Intel i5 CPU and 8 GB memory. In one of the valid simulations, the computation time of day-ahead scheduling for ten appliances in 24 h is 2.3613 s.

A. Cost Efficient Scheduling for Day-Ahead Bidding

We first investigate the case without considering DER and service fee. We consider a 24-h consumption requirement with ten typical appliances listed in Table VIII. To adopt the proposed scheduling algorithm, we test it with two different constraints. In case I, we let the requirements of all the appliances equal to the origin ones, while in case II, we assume that the consumer allows some of the appliances to increase or decrease a bit of their consumptions, as presented in Table VIII. The load shape of the requirement without scheduling is labeled by the black dotted line in Fig. 5. In the figure, the value of a stair corresponds to the consumption amount of the corresponding hourly time slot. The load shape of the scheduled consumption in case I is achieved by the day-ahead bidding part of the proposed algorithm in Table V, and is labeled by the red dashed line. The load shape of the scheduled consumption in case II is achieved by the same algorithm with relatively flexible demand constraints, and is labeled by the blue dash-dotted line. The indicators of the three load shapes are presented in Table IX.

With the proposed consumption scheduling and fixed requirements, the cost efficiency has increased by 7.37% compared with the origin ones. Moreover, if we run the scheduling with a flexible requirements as case II, the cost efficiency can increase by 11.60%. In case II, besides load shifting, the proposed algorithm has adjusted the consumption amounts of some appliances. The total demands of heating and of computer have been respectively reduced to 11.0 and 2.40 kWh, while the demand of TV has been increased to 1.40 kWh. The total demand and the cost have decreased while the utility and the cost efficiency have increased. We can see that the proposed algorithm manages to increase the high-efficient consumption part while reduce the relatively low-efficient part, resulting in saving and better utilizing the power.

B. Impact of the Distributed Energy Resources

In this paper, we have enabled the distributed renewable generation and the storage device by referring to [19]. The consumption constraints for the appliances are the same as in case I in Table VIII. The corresponding load shapes are presented in Fig. 6. With the renewable energy and storage device, the total demand in bidding has decreased from 28.3200 to 26.7884 kWh and the cost has decreased to 15.4264 dollars. The cost efficiency is 4.39. In real consumption, we assume that the consumer obeys the planed consumption while we allow the randomness of consumption due to fluctuation of power. A higher renewable generation in real time has pushed the cost efficiency to 4.4140. The renewable generation...
directly lead to a reduction of the expenses on the retail electricity, causing the cost efficiency to decrease significantly. Analyzing the load shape of the storage device, we can see that the discharging process occurs in the hour with the highest retail price and the major charging process occurs in the hours close to the discharging hour and with relatively low prices. This has implied two trade-offs in the deployment of storage device. One is that, we can significantly reduce the total demand in the hours with the highest retail prices by the discharging process of storage device, at the expense of extra demand in the other hours for charging the storage device. The other is that to charge the storage device in the hours with the lowest prices can reduce the expense, while the major charging hours should not be too far from the discharging hours in order to avoid the cost of power leakage.

Since that the IBR has two different rates for the consumption, we can intuitively judge that the effect of DER on the cost efficiency will be interfered by the IBR due to the transition of different rates. Therefore, we had simulations to demonstrate. As for the settings of the simulations, in most of the time slots, the total demands of the appliances are set beyond the thresholds for the highest retail price and the major charging process occurs in the hours close to the discharging hour and with relatively low prices. This has implied two trade-offs in the deployment of storage device. One is that, we can significantly reduce the total demand in the hours with the highest retail prices by the discharging process of storage device, at the expense of extra demand in the other hours for charging the storage device. The other is that to charge the storage device in the hours with the lowest prices can reduce the expense, while the major charging hours should not be too far from the discharging hours in order to avoid the cost of power leakage.

Since that the IBR has two different rates for the consumption, we can intuitively judge that the effect of DER on the cost efficiency will be interfered by the IBR due to the transition of different rates. Therefore, we had simulations to demonstrate. As for the settings of the simulations, in most of the time slots, the total demands of the appliances are set beyond the thresholds for the higher rates. As in Fig. 7, the blue dashed curve is the C.E.—renewable generation curve without IBR and the red dash-dotted line is that with IBR. Because of the penalty by IBR, the cost efficiency with IBR is smaller than the other at the start point. With the increase of renewable generation, the demands of retail electricity will gradually decrease. Because of the penalty by IBR, the cost efficiency of the increase rate of cost efficiency under IBR is lower than that of the other curve. However, as the demands of retail electricity come to the transition regions of different rates, an increasing number of hourly bills are calculated based on the lower unit prices. The two curves come closer to each other. After the hourly demands have all fallen into the region below the thresholds, pricing are equal for the two cases. The two curves are overlapping. This implies that a proper amount of renewable generation can be allocated for the consumer whose hourly consumption demands are close to the transition regions of IBR rates, to effectively avoid the penalty by the higher rate.

C. Impact of the Service Fee

In this case study, the settings of all the appliances remain the same as in Table VII and the prices remain the same as in Table VI. We have chosen four different service fees while running the simulations. In each case, the total consumption amount of a day increase from 0 to 40 kWh. Fig. 8 has demonstrated the C.E.—consumption amount curves with different service fees. In the left sub-figure, for a fixed service fee, as the consumption amount increases, the cost efficiency by the proposed algorithm first increases to a peak, and then drops. This is because, in both algorithms, at the beginning the marginal growth of utility function is much higher than that of the cost. As a result, the cost efficiency increases sharply. Since the cost efficiency is the ratio of a concave and a convex functions, it reaches the maximum value when the derivatives of the two functions are equal, and then it starts to drop from the maximal value.

The right sub-figure has presented the C.E.—service fee curves of different bidding methods with the same consumption demand of 28.4300 kWh. The proposed scheduling algorithm has a stable performance in achieving the best cost efficiency. We also found that under the same consumption constraints with different service fees, the peak point of a cost efficiency curve with higher service fee corresponds to a larger
consumption demand. This indicates that the service fee can affect the amount of most cost efficient consumption demand. In order to guarantee a relatively high cost efficiency, the consumer can determine an interval of the feasible consumption demand within which the cost efficiency value is relatively high and acceptable.

D. Real Consumption Management

We have adopted the bidding strategy for the consumption pattern B in Section V-A to predetermine the load shape in this paper. In Fig. 9, we present a real-time price curve in a time length of two days [35], [36]. The real time consumption is supposed to take place in the second day.

We test the real time management in two different cases. In the first case, we assign the temporary consumption requirements to time slots with relatively high retail price, which is considered as a “bad choice.” We assign an 1-h temporary task creating three units of utility at the cost of 1 kWh to the seventh time slot with a 6-h tolerance for the appliance’s latest start. We assign another half-hour temporary task creating two units of utility at the cost of 0.5 kWh to the 18th time slot with a 6-h tolerance for the appliance’s latest start. We have run 5000 times of the simulations with the real-consumption management for the two temporary task requirements and plotted the distribution of the cost efficiency in the top picture of Fig. 10. The average cost efficiency of the managed consumption is 4.0244, while the average cost efficiency without the real-time management is 3.9733. With a bad choice of the start time, the real-time management can effectively improve the cost efficiency by delaying the start time of the task to hours with lower price. In the second case, we assign the temporary consumption requirements to time slots with relatively low retail price, which is considered as a “good choice.” We assign an 1-h temporary task creating three units of utility at the cost of 1 kWh to the 14th time slot with a 6-h tolerance for the appliance’s latest start. The cost efficiency distribution of the 5000 simulations is presented in the bottom picture of Fig. 10. The average cost efficiency of the managed consumption is 4.0920, while the average cost efficiency without the real-time management is 4.0858. With a good choice of the start time, in most cases the real-time management have managed to maintain or even improve the cost efficiency by starting the task immediately or at the hours with low price. With the two case studies, we can say that the proposed real consumption management is effective in maintaining or improving the cost efficiency in real consumption compared to cases without the management.

VI. Conclusion

In this paper, we proposed a cost efficiency concept as a metric of the economical efficiency of the electricity consumption to optimize the consumption benefit per unit cost. The cost efficiency can vary with different consumption patterns, and is sensitive to behaviors of load shifting. The investigations of the effect of service fees and the DERs on consumption cost efficiency have shown that the higher service fees will reduce the cost efficiency and the DERs can effectively reduce the cost for buying bulk generation electricity and increase the consumption cost efficiency. Simulation results have confirmed that the proposed cost efficient algorithms significantly improve the cost efficiency in the day-ahead bidding, and the real time management is effective in maintaining or improving the cost efficiency in real consumption.

The proposed cost efficiency metric can be applied in the consumption scenarios with flexible pricing mechanism. Besides residential consumptions, it can be extended to the commercial consumption scenarios and industrial consumption scenarios to achieve a better economical efficiency for business companies and industrial producers.
APPENDIX A

PROOFS OF THE CONVERGENCE ANALYSIS

A. Proof of Proposition 1

The numerator in (19) is the sum of different concave utility functions as in (4) and is differentiable over \( X \). The denominator in (19) is a linear function and is differentiable over \( X \).

Let \( X_1, X_2 \in X \), respectively, denote two different feasible consumption vectors. We define a real number \( \tau \in [0, 1] \). Since

\[
\tau \cdot b^k \leq \tau \cdot x_{1,k}^a \leq \tau \cdot b^k, \forall k \in OC, a \in A
\]

\[
\tau \cdot D^a \leq \tau \cdot \sum_{n \in T^a} d^a_{(1)n} \leq \tau \cdot D^a_{max}, \forall a \in A
\]

\[
(1 - \tau) \cdot b^k \leq (1 - \tau) \cdot x^a_{(2)k} \leq (1 - \tau) \cdot b^k, \forall k \in OC, a \in A
\]

\[
(1 - \tau) \cdot D^a_{min} \leq (1 - \tau) \cdot \sum_{n \in T^a} d^a_{(2)n} \leq D^a_{max} \forall a \in A.
\]

Therefore

\[
b^k \leq \tau \cdot x^a_{1,k} + (1 - \tau) \cdot x^a_{(2)k} \leq b^k, \forall k \in OC, a \in A
\]

\[
\rho^a_{min} \leq \tau \cdot \sum_{n \in T^a} \rho^a_{1,n} + (1 - \tau) \cdot \sum_{n \in T^a} \rho^a_{(2,n)} \leq \rho^a_{max}, \forall a \in A
\]

which implies that the feasible domain \( X \) is a convex set. \( \tilde{g}^k \) is predetermined and the strategy set of \( \tilde{s} \) is convex [18]. Thus, problem (19) is a concave–convex FP problem.

B. Proof of Proposition 2

The rate of convergence of \( \lambda_n \) is analyzed as follows. Let \( 
\lambda', \lambda'' \in \mathbb{R} \) be any optimal solution of steps \( s' \) and \( s'' \), respectively.

**Lemma 1:** For \( \lambda', \lambda'' \in \mathbb{R} \) one has

\[
f(x'') - f(x') \geq -f(\lambda'') \left[ \frac{1}{h_2(x')} - \frac{1}{h_2(x'')} \right]
\]

(24a)

and

\[
f(x') - f(x') \leq \left[ -f(\lambda') \right]
\]

\[
+ \left( \lambda' - \lambda'' \right) h_2(x') \left[ \frac{1}{h_2(x')} - \frac{1}{h_2(x'')} \right].
\]

(24b)

**Proof:** Since \( x'' \) is optimal for step \( s'' \)

\[
h_1(x'') - \lambda'' h_2(x'') \geq h_1(x') - \lambda'' h_2(x').
\]

Therefore

\[
\frac{h_1(x'')}{h_2(x')} - \lambda'' \frac{h_2(x'')}{h_2(x')} \geq \frac{h_1(x')}{h_2(x')} - \lambda''.
\]

Hence

\[
f(x') - f(x') \geq \frac{h_1(x'')}{h_2(x'')} - \frac{h_1(x')}{h_2(x')} + \lambda'' \left[ \frac{h_2(x'')}{h_2(x')} - 1 \right]
\]

\[
= -f(\lambda') \left[ \frac{1}{h_2(x')} - \frac{1}{h_2(x'')} \right].
\]

Thus (24a) holds. The inequality (24b) can be verified analogously via the optimality of \( \lambda' \).

With (24a) and (24b), it follows Lemma 2.

**Lemma 2:** For \( \lambda' < \lambda'', h_2(x') \geq h_2(x'') \).

This is true for any optimal solution of steps \( s' \) and \( s'' \).

With Lemma 2 (24a) and \( F(\lambda'') > 0 \) for \( \lambda'' \leq \lambda^+ \), we have Lemma 3.

**Lemma 3:** If \( \lambda' < \lambda'' \leq \lambda^+ \), then \( f(x') \leq f(x'') \).

Assume that \( \lambda'' = \lambda^+ \). Considering \( F(\lambda^+) = 0 \), it follows from (24b) and Lemma 2.

**Lemma 4:** For \( \lambda' < \lambda^+ \) one has

\[
\lambda^+ - f(x') \leq (\lambda^+ - \lambda_i) \left( 1 - \frac{h_2(x^+)}{h_2(x)} \right)
\]

where \( 0 \leq 1 - h_2(x^+)/h_2(x') < 1 \). Lemma 4 implies that

\[
\lambda^+ - \lambda_{i+1} \leq (\lambda^+ - \lambda_i) \left( 1 - \frac{h_2(x^+)}{h_2(x_i)} \right)
\]

\[
\forall q_i < q^+, i = 1, 2, \ldots (25)
\]

where \( 0 \leq 1 - h_2(x^+)/h_2(x_i) < 1 \).

Considering Lemma 2, \( 1 - h_2(x^+)/h_2(x_i) \) is nonincreasing. Thus, \( \lambda_i \) converges linearly at least to \( \lambda^+ \).

Because of the continuity of \( h_1, h_2 \), and compactness of \( D \), there exist an optimal solution \( x^+ \) of (19) and a subsequence \( \{x_i\} \) of \( \{x_i\} \) such that

\[
\lim_{i \rightarrow \infty} h_2(x_i) = h_2(x^+).
\]

Since \( \{h_2(x_i)\} \) is decreasing according to Lemma 2, it implies that

\[
\lim_{i \rightarrow \infty} h_2(x_i) = h_2(x^+).
\]

Thus (25) supports Proposition 2.

APPENDIX B

FRACTIONAL PROGRAMMING

Fractional programs deal with the nonlinear optimization problem in which the objective function is a ratio of two real functions [38]. This mathematical framework is able to solve the fractional cost efficiency optimization problems in the household electricity consumption scenario. Consider the general form of the nonlinear fractional program

\[
\text{maximize } \frac{f(x)}{h_2(x)} \quad \forall x \in D
\]

where \( D \in \mathbb{R}^n \), \( h_1, h_2 : D \rightarrow \mathbb{R} \) are both differentiable, and \( h_2(x) > 0 \). When \( h_1 \) is concave, \( h_1 \geq 0 \), \( h_2 \) is convex, and \( D \) is a convex set, problem (26) turns into a concave–convex fractional program. Since \( h_1 \) and \( h_2 \) are differentiable, the objective function is pseudoconcave [32], indicating that all the stationary points are global maximums. In this case, the Karush–Kuhn–Tucker conditions are feasible if a constraint qualification is satisfied. Thus, we can directly solve problem (26) by convex programing algorithms [32]. In the following, we introduce the parametric convex program, which has been adopted in the proposed scheme.
For parametric convex programming, we use the equivalent form [39] of problem (26)

\[
\begin{align*}
\text{maximize} & \quad \lambda \\
\text{subject to} & \quad h_1(x) - \lambda \cdot h_2(x) \geq 0. \quad (27)
\end{align*}
\]

Since \( h_2 > 0 \), we rewrite the constraint, and obtain the following form:

\[
\begin{align*}
\text{maximize} & \quad \lambda \\
\text{subject to} & \quad h_1(x) - \lambda \cdot h_2(x) \geq 0. \quad (28)
\end{align*}
\]

Problem (28) is not jointly convex in \( \lambda \) and \( x \), but for a fixed \( \lambda \), there exists a feasibility problem in \( x \) which is convex, if \( h_1 \) is concave and \( h_2 \) is convex. This problem is feasible if

\[
\text{max} \quad h_1(x) - \lambda \cdot h_2(x) \geq 0. \quad (29)
\]

As described in [39], we can use a bisection method to find the optimal value of \( \lambda \) to solve the feasibility problem at each step of the algorithm.

Let \( F(\lambda) \) represent the left side of problem (29), and

\[
F(\lambda) = \max_{x \in D} h_1(x) - \lambda \cdot h_2(x). \quad (30)
\]

Reference [31] proved that \( F(\lambda) \) is convex, continuous, and strictly decreasing in \( \lambda \). \( F(\lambda) \) indicates a bi-criterion scalar optimization problem which maximizes \( h_1(x) \) and minimizes \( h_2(x) \), with \( \lambda \) being the relative weight of \( h_2 \). \( x^* \) is Pareto-optimal for the bi-criterion optimization, if it is optimal for the scalar problem [39]. We can find the optimal solution for problem (26) in the set of Pareto optimal values for the bi-criterion scalar optimization via iterative algorithms. A detailed analysis on the set of Pareto optimal values can be found in [39].

Let \( f^* \) denote the optimum value of problem (26). We have the following equivalent relations [32]:

\[
\begin{align*}
F(\lambda) > 0 & \iff \lambda < f^* \\
F(\lambda) = 0 & \iff \lambda = f^* \\
F(\lambda) < 0 & \iff \lambda > f^*. \quad (31)
\end{align*}
\]

Thus, we can solve problem (26) by finding the root of the nonlinear function \( F(\lambda) \). The condition for optimality is

\[
\begin{align*}
F(\lambda^*) = \max_{x \in D} h_1(x) - \lambda^* \cdot h_2(x) = 0. \quad (32)
\end{align*}
\]

In [40], there are several iterative algorithms to find the root of \( F(\lambda) \). We adopt the Dinkelbach method [31] which is based on the application of Newton’s method. The method is shown in Algorithm 1, and the update in Newton’s method is calculated as

\[
\begin{align*}
\lambda_{n+1} = & \lambda_n \frac{F(\lambda_n)}{F'(\lambda_n)} \\
= & \lambda_n \left( \frac{h_1(x_n^*) - \lambda_n \cdot h_2(x_n^*)}{-h_2(x_n^*)} \right) = \frac{h_1(x_n^*)}{h_2(x_n^*)}. \quad (33)
\end{align*}
\]

This iteration has a superlinear convergence rate. A detailed convergence analysis is presented in [41]. We can choose any \( \lambda_0 \) that satisfies \( F(\lambda_0) \geq 0 \) to be the initial point. Therefore, solving an optimization problem with inequality constraints on \( h_1 \) and \( h_2 \) reduces to solve the unconstrained problem and determine whether \( \lambda \) falls within the interval \([\lambda_{\min}, \lambda_{\max}]\) that corresponds to the upper and lower bounds of \( h_1 \) and \( h_2 \). If not, \( \lambda^* \) will be replaced by the endpoint.

**Algorithm 1 Dinkelbach Method**

| Data: \( \lambda_0 \) satisfying \( F(\lambda_0) \geq 0 \), tolerance \( \epsilon \); |
| step = 0; |
| while \( |F(\lambda_{\text{step}})| > \epsilon \) do |
| Use \( \lambda = \lambda_{\text{step}} \) in (27) to obtain \( x_{\text{step}}^* \); |
| \( \lambda_{\text{step}} = \frac{h_1(x_{\text{step}}^*)}{h_2(x_{\text{step}}^*)} \); |
| step = step + 1; |
| end while |

**References**


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