Cooperative Spectrum Sharing in Cognitive Radio Networks with Multiple Antennas

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Abstract—In this paper, we consider a cognitive radio network consisting of a primary transmitter-primary receiver pair, and a secondary base station-secondary receiver pair. To improve the performance of both the primary and secondary pair, we propose an overlay spectrum sharing scheme where the primary user (PU) leases half of its time slots to the secondary user (SU) in exchange for the SU cooperatively relaying the PU’s data using the amplify and forward scheme. The proposed scheme will involve the design of antenna weights and power allocation to meet a certain error or rate design criteria for both the PU and SU. To analyze the performance of the proposed scheme, we derive new closed form expressions for the rate and bit error rate for arbitrary signal-to-noise ratio (SNR). In addition, we carry out an asymptotic analysis in the high SNR regime to obtain the diversity order. These expressions, along with numerical analysis, reveal that the proposed cooperative overlay scheme can achieve significant performance gains, for both the PU and the SU, compared to a conventional non-cooperative underlay scheme, which gives both users the incentive to cooperate.

Index Terms—Cognitive radios, cooperative relaying, overlay spectrum sharing, zero-forcing.

I. INTRODUCTION

Cognitive radio (CR) [1] has been recently proposed as a promising technology to improve the utilization efficiency of radio spectrum [2]. It allows secondary user (SU) networks to coexist with primary user (PU) networks through spectrum sharing, provided that the secondary spectrum access will not adversely affect the PU’s performance. To allow for this spectrum sharing, three models have been considered in the literature: the interweave, underlay and overlay model. The interweave model operates by the SUs first sensing the availability of spectrum holes, i.e., spectrum bands not occupied by the PUs. The SUs are then restricted to transmit over these bands [2]. However, this model is highly sensitive to sensing errors and PU traffic patterns [2].

In the underlay model, the SUs simultaneously transmit with the PUs over the same spectrum, provided that the received SU’s signal power levels at all PU receivers are kept below a predefined threshold [2], [3]. In contrast to the interweave model, the underlay model has the advantage that the SUs can directly occupy the licensed spectrum without considering the behavior of the PUs’ traffic patterns. However, a key problem with this model is that the SUs may suffer from bad performance due to (i) the power constraints imposed at the SUs, especially if the SU’s transmitters are close to the PU’s receivers and (ii) the interference from the PUs.

In the overlay model, the SUs simultaneously transmit with the PUs over the same spectrum, provided that the SUs aid the PU’s transmission by cooperative communication techniques, such as advanced coding or cooperative relaying techniques [3]. Thus in contrast to the underlay model, the overlay model does not require strict transmit power constraints at the SUs due to interference caused to the PUs. Cooperative relaying has emerged as a powerful technique for the overlay model, due to its ability to exploit user diversity and provide high reliability and capacity in wireless networks. This is achieved by the use of intermediate relay nodes, which are used to aid transmission between the source and destination nodes. The two most common cooperation protocols are Decode and Forward (DF) and Amplify and Forward (AF). DF is a simple scheme, which decodes the signal at the relay and re-encodes it before forwarding it to the destination [4]. AF is a simpler scheme, in which the signal is amplified at the relay and forwarded to the destination [4]. In this work, we consider the use of the AF protocol, due to its simple operation.

The advantages of cooperative communications has thus prompted its application to the overlay model. In this paper, we consider a CR network comprising of a primary transmitter (PT), primary receiver (PR) and a secondary receiver (SR), each equipped with a single antenna, and a secondary base station (ST) equipped with multiple antennas. We consider an overlay spectrum sharing scheme where the PU leases half of its time slots to the SU in exchange for the SU cooperatively relaying the PU’s data using the AF. In addition, we consider a conventional non-cooperative underlay scheme as a benchmark scheme, where both the PU and SU access the channel simultaneously based on the underlay model.

1The considered model may correspond to a practical scenario in which a secondary base station is serving one SU which has also been considered in [5]. In addition, although we only consider single antennas at the PT, PR and SR, the main motivation of this paper is to demonstrate the significant performance gains for both the PU and SU, compared to the underlay model with no cooperation. As such, we focus on the single-antenna scenario, but note that performance gains can also be shown for the multiple-antennas scenario.

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Related works on the use of cooperative communications in the overlay model can be found in [3], [5]–[10]. This was investigated from an information theory perspective in [3], [6], [7]. In [5], [8]–[10], various practical schemes for the overlay model were proposed where the SUs cooperatively relay the PU’s data in exchange for spectrum access. However, these schemes have predominately focused on the scenario where the SUs’ performance was not a main consideration and the PUs’ performance was either improved or guaranteed, compared to the non-cooperative scenario in which the PUs continuously transmit. In particular, [8], [9] considered the case where the PU controls the spectrum leasing process to guarantee or improve its own performance, and as such, the SU may not be allowed access to the spectrum in certain scenarios. Moreover, the scheme in [9] considered the scenario where spectrum access to the SU was only granted provided that the PU’s outage performance was not affected, leading to potential marginal performance gains for the PU compared to the underlay model. In [10], a game-theoretic approach was used to allocate spectrum access to the PUs and SUs, but resulting in worse performance for the SUs compared to the underlay model. In [5], the proposed scheme is restricted to the specific case where the number of antennas at the ST is equal to two, and assumes that the ST can support full duplex relaying.

In this paper, we consider a different approach where both the PU’s and SU’s performance is taken into account. Our main contributions can be summarized as two-fold. Firstly, we propose a novel practical overlay scheme which results in a significant improvement in the rate and error performance for both the PU and the SU, over a wide range of practical signal-to-noise ratios (SNRs), compared to a conventional underlay system with no cooperation. Our proposed scheme is in contrast to previous works in [8]–[10], which have predominately focused on the scenario where the PUs’ performance was either improved or guaranteed, while the SUs were not a main consideration and thus may have poor performance\(^3\).

The advantages of our proposed scheme are a result of the following:

- Under the proposed overlay scheme, (i) the PU performance is significantly enhanced by exploiting the benefit of cooperative diversity with the aid of the SU, and (ii) the SU performance is enhanced by the PU scheduling its transmission such that interference free transmission can be achieved for the SU. This will result in a win-win scenario for the PU and SU, and gives both the PU and SU the incentive to cooperate.
- The proposed scheme is not interference-limited, i.e., there is no interference at the primary and secondary systems from the ST and PT respectively.
- The performance of both the PU and SU can be improved by either (i) increasing the transmission power at the ST and (ii) the number of antennas at the ST.

In order to achieve high performance for both the PU and SU, we propose the design of antenna weights at the ST such that (i) the signal powers intended for the PR and SR are maximized, and (ii) the interfering signals from the ST and PT at the PR and SR are respectively cancelled. Furthermore, we will allocate the power at the ST between the PT’s signal and the ST’s signal in order to meet both the PU’s and SU’s error or rate criteria.

Secondly, we derive new closed-form bit-error-rate (BER) and rate expressions for arbitrary SNR to facilitate performance evaluation for the proposed scheme. Moreover, an asymptotic BER expression in the high SNR regime is also derived which explicitly reveals that the diversity order at both the PU and SU for the proposed cooperative scheme is significantly higher than the diversity order obtained using the non-cooperative underlay scheme. We also investigated the insights concerning the effect of different system parameters on the PU’s rate performance for the proposed scheme. In particular, we have analytically shown that the rate performance at the PU for the proposed scheme can be improved by either i) increasing the number of antennas \(N\) at the ST, ii) increasing the transmitted power at the ST \(P_{\text{ST}}\), and iii) increasing the power allocation number \(\beta\) at the ST. This will provide the designer several options to further improve the PU’s performance for the proposed scheme. Monte Carlo simulations are also provided to verify our analysis and to numerically demonstrate the significant improvements of the proposed scheme for both the PU and SU, compared to the non-cooperative underlay scheme.

The rest of this paper is organized as follows. The system model and the transmission scheme are introduced in Section II. The proposed design of the antenna weights at the ST is described in Section III. We then analyze the PU’s and SU’s received SNRs and distributions in Section IV. The error and rate performance of the proposed scheme are then investigated respectively in Sections V and VI. Finally, conclusions are drawn in Section VII.

II. SYSTEM MODEL

We consider a CR network consisting of a primary transmitter-primary receiver and a secondary transmitter base station-secondary receiver pair, where the secondary transmitter base station is equipped with \(N\) antennas and the other transmitters and receivers are equipped with single antenna. To allow for efficient spectrum sharing between the primary and secondary users, we consider a two time slots transmission scheme, as shown in Fig. 1. In the first time slot, the PT transmits its signal to the PR, which is also received by the ST and the SR. In the second time slot, the ST amplifies and forwards the PT’s signal, and at the same time, transmits its own signal while the PT remains silent.

In this transmission scheme, the PR receives two independent copies of the signal transmitted by the PT, obtained from direct transmission by the PT in the first time slot and forwarded from the ST in the second time slot. The PT’s signal is then retrieved by applying maximum ratio combining (MRC) to these two signals. The ST’s signal is retrieved at the SR from the signal obtained from the ST in the second time

\(^3\)Compared to [5], [8], [9], where the SU may not have the opportunity to transmit in some scenarios, in our work the SU transmission is always allowed.
slot, and discarding the received signal from the PT in the first time slot.

\[ P_{PU,1} = \sqrt{P_{PU} h_D x_{PU}} + n_{PU}, \tag{1} \]

where \( P_{PU} \) is the transmitted power from the PT, \( h_D \sim \mathcal{CN}(0, d_{PR,PU}^\alpha) \) is the Rayleigh channel from the PT to the PR, \( \alpha \) is the path loss exponent, \( d_{PR,PU} \) is the PT-PR distance, \( x_{PU} \) is the transmitted scalar symbol from the PT with \( E[|x_{PU}|^2] = 1 \) and \( n_{PU} \sim \mathcal{CN}(0, \sigma^2) \) is additive white Gaussian noise (AWGN) at the ST. At the ST, after applying a 1 × N weight vector \( w_1 \) to the N × 1 received signal from the PT, the resultant scalar signal at the ST in the first time slot can be written as

\[ y = \sqrt{P_{PU}} w_1^\dagger h_{PU} x_{PU} + w_1^\dagger n_1, \tag{2} \]

where \( h_{PU} \sim \mathcal{CN}_{N,1}(0, d_{PT,PU}^\alpha I_N) \) is the Rayleigh channel vector from the PT to the ST, where \( d_{PT,PU} \) is the PT-ST distance, \( n_1 \sim \mathcal{CN}_{N,1}(0, I_N) \) is the AWGN vector and \( (\cdot)^\dagger \) denotes conjugate transpose.

In the second time slot, the ST first normalizes the received PT’s signal by multiplying \( y \) in (2) by the normalization constant

\[ Q_y = 1/\sqrt{\sum w_k^\dagger (P_{PU} h_{PU} h_k^\dagger) w_k + \sigma^2 I_N} w_y. \tag{3} \]

The ST then amplifies and forwards this normalized signal, in addition to transmitting its own signal. To enable the concurrent transmission of the normalized PT’s signal and ST’s signal such that they are received without interference at the PR and SR respectively, the ST applies \( N \times 1 \) transmit weight vectors to both the normalized PT’s signal and the ST’s signal, denoted by \( w_{t_{PU}} \) and \( w_{t_{SU}} \) respectively. The received scalar signal at the PR from the ST can thus be written as

\[ r_{PU,2} = Q_r \beta g_{PU}^\dagger \left( \sqrt{\beta w_{t_{PU}} Q_y y + 1 - \beta w_{t_{SU}} x_{SU}} \right) + n_2 \]

\[ = Q_r Q_y \sqrt{P_{PU}} \sqrt{\beta g_{PU}^\dagger w_{t_{PU}} w_1^\dagger h_{PU} x_{PU}} + Q_r \sqrt{1 - \beta} g_{SU}^\dagger w_{t_{SU}} x_{SU} + Q_r Q_y \sqrt{\beta g_{PU}^\dagger w_{t_{PU}} w_{t_{SU}}^\dagger n_1} + n_2, \tag{4} \]

where \( g_{PU} \sim \mathcal{CN}_{N,1}(0, d_{PT,PU}^\alpha I_N) \) is the Rayleigh channel vector from the ST to the PR, \( g_{SU} \sim \mathcal{CN}_{ST,PU}(0, \sigma^2) \) is the ST-PR distance, \( r_{SU} \) is the transmitted scalar signal from the ST with \( E[|r_{SU}|^2] = 1 \), \( \beta \) is the power allocation number used to allocate the available power at the ST between the PT’s signal and the ST’s signal with \( 0 < \beta < 1 \). In addition, \( n_2 \sim \mathcal{CN}(0, \sigma^2) \) is the AWGN at the PR and \( Q_r \) is the normalization constant, designed to ensure that the total transmit power at the ST is constrained, and is given by

\[ Q_r = \frac{P_{SU}}{\text{Trace} \left( \beta w_{t_{PU}} w_{t_{PU}}^\dagger + (1 - \beta) w_{t_{SU}} w_{t_{SU}}^\dagger \right)}. \tag{5} \]

The received scalar signal at the SR from the ST in the second time slot can be written as

\[ r_{SU} = Q_r g_{SU}^\dagger \left( \sqrt{1 - \beta} w_{t_{SU}} x_{SU} + \sqrt{\beta} w_{t_{PU}} Q_y y \right) + n_3 \]

\[ = Q_r \sqrt{1 - \beta} g_{SU}^\dagger w_{t_{SU}} x_{SU} + Q_r Q_y \sqrt{P_{PU}} \sqrt{\beta} g_{PU}^\dagger w_{t_{PU}} w_{t_{SU}}^\dagger h_{PU} x_{PU} + Q_r Q_y \sqrt{\beta} g_{SU}^\dagger w_{t_{SU}} w_{t_{SU}}^\dagger n_3, \tag{6} \]

where \( g_{SU} \sim \mathcal{CN}_{SU}(0, d_{ST,SR}^\alpha I_N) \) is the Rayleigh channel vector from the ST to the SR, \( d_{ST,SR} \) is the ST-SR distance and \( n_3 \sim \mathcal{CN}(0, \sigma^2) \) is the AWGN at the SR. The PR then applies MRC to the two received signals, given in (1) and (4) in the first and second time slot respectively, resulting in a received signal to interference noise ratio (SINR) at the PR given by

\[ \gamma_{PU} = \frac{P_{PU}[h_D]^2}{\sigma^2} + \frac{Q_r^2 Q_y^2 P_{PU} \beta |g_{PU}^\dagger w_{t_{SU}}||h_{PU}|^2}{A + Q_r^2 Q_y^2 \beta \sigma^2 |g_{PU}^\dagger w_{t_{SU}}||w_{t_{SU}}|^2 + \sigma^2}, \tag{7} \]

where \( A = Q_r^2(1 - \beta) |g_{PU}^\dagger w_{t_{SU}}|^2 \), while the received SINR at the SR is given by

\[ \gamma_{SU} = \frac{Q_r^2(1 - \beta) |g_{SU}^\dagger w_{t_{SU}}|^2}{B + Q_r^2 Q_y^2 \beta \sigma^2 |g_{SU}^\dagger w_{t_{SU}}||w_{t_{SU}}|^2 + \sigma^2}, \tag{8} \]

where \( B = Q_r^2 Q_y^2 P_{PU} \beta |g_{SU}^\dagger w_{t_{SU}}||h_{PU}|^2 \). Note that to perform MRC at the PR, the PR requires \( \sqrt{P_{PU}} h_D \), which can be obtained via pilot training symbols [12], and the complex scalar \( Q_r Q_y \sqrt{P_{PU}} \sqrt{\beta} g_{PU}^\dagger w_{t_{PU}} w_{t_{SU}}^\dagger h_{PU} \), which is initially transmitted from the ST before the transmission procedure. The design of the weights \( w_{t_{PU}} \) and \( w_{t_{SU}} \) will be discussed in the next section. Throughout this paper, we assume that the channels \( h_{PU} \), \( g_{PU} \) and \( g_{SU} \) are known at the ST. In practice, channel state information (CSI) between the ST and SR can be obtained by the classic channel training, estimation, and feedback mechanisms as in [12], while the CSI between the PT and ST and the PT and PR can be obtained as in [6], as we assume that the PU and SU systems cooperate with each other. Finally, in a fading environment, there might

\[ 3 \]
be cases where it is difficult for the ST to perfectly estimate instantaneous channels. In such cases, the results obtained in this paper provide upper-bounds for the performance of the proposed scheme in a CR network.

![Diagram](image)

**Fig. 2.** System model for the conventional non-cooperative underlay scheme. Note there is no line drawn from the ST towards the PR as interference from the ST to the PR is canceled at the ST in this scheme.

To demonstrate the advantages of our proposed overlay scheme, we consider a conventional underlay scheme as a benchmark, illustrated in Fig. 2. Under this conventional underlay scheme, the PT continuously transmits to the PR directly. At the same time, the ST continuously transmits to the SR directly, while limiting or cancelling its interference to the PR. As a result, the SR will also receive interference from the PT transmission.

### III. PROPOSED WEIGHTS DESIGN

In this section, we will design the antenna weights at the ST to allow for efficient spectral sharing between the PU and the SU. In the first time slot of the proposed scheme, the ST receives only the PT’s signal, and thus the optimal linear weight design is MRC. The received weight \( w_r \) at the ST is thus chosen as

\[
w_r = \frac{h_{PU}}{||h_{PU}||}
\]

where \( ||\cdot|| \) is the Frobenius norm. In the second time slot, the ST amplifies and forwards the PT’s signal in addition to transmitting its own signal. The transmit weight vectors, \( w_{tPU} \) and \( w_{tSU} \), will be designed such that the interference caused by the PT’s signal and the ST’s signal to the SR and PR are respectively mitigated. There are several techniques that can be employed in the design of \( w_{tPU} \) and \( w_{tSU} \) to achieve this. Examples of such techniques for interference mitigation are dirty paper coding (DPC) [13] or zero forcing (ZF) [14]. In this paper, we consider the suboptimal ZF approach because it is a simple and practical scheme with lower complexity than other interference cancellation schemes. Thus, we trade optimality for simplicity in choosing the design of the transmit weights. As we will show later, it can also offer significant performance improvements over the non-cooperative underlay scheme. In order to apply ZF principles at the ST, we make the common assumption that \( N \geq 2 \).

According to ZF principles, the transmit weight vector \( w_{tPU} \) is chosen to lie in the orthogonal space of \( g_{SU} \) such that

\[
|g_{SU}^† w_{tPU}| = 0 \text{ and } |g_{SU}^† w_{tSU}| \text{ is maximized. Similarly, } w_{tPU} \text{ is chosen in the orthogonal space of } g_{SU} \text{ such that } |g_{PU}^† w_{tSU}| = 0 \text{ and } |g_{SU}^† w_{tSU}| \text{ is maximized.}
\]

The problem of designing the transmit weights, \( w_{tPU} \) and \( w_{tSU} \) at the ST can thus be formulated as

\[
\begin{align*}
\tilde{w}_{tPU} & = \arg \max_{w_{tPU}} |g_{PU}^† w_{tPU}| \quad \text{subject to } |g_{SU}^† w_{tSU}| = 0 \text{ and } \|w_{tPU}\| = 1. \\
\tilde{w}_{tSU} & = \arg \max_{w_{tSU}} |g_{SU}^† w_{tSU}| \quad \text{subject to } |g_{PU}^† w_{tSU}| = 0 \text{ and } \|w_{tSU}\| = 1.
\end{align*}
\]

Using projection matrix theory [15], the weights which satisfy the conditions in (10), are given by

\[
\tilde{w}_{tPU} = \frac{A_{PU}^† g_{PU}}{||A_{PU}^† g_{PU}||} \quad \text{and} \quad \tilde{w}_{tSU} = \frac{A_{SU}^† g_{SU}}{||A_{SU}^† g_{SU}||},
\]

where \( A_{PU} \) and \( A_{SU} \) are the projection matrices for the PU and SU, given respectively by

\[
A_{PU}^† = (I - g_{SU}g_{SU}^† g_{PU})^{-1}g_{PU}^† \quad \text{and} \quad A_{SU}^† = (I - g_{PU}g_{PU}^† g_{SU})^{-1}g_{SU}^†.
\]

Substituting (9) and (11) into (5) and (3), and the resultant expressions into (7) followed by some algebraic manipulation, the SNR at the PR can be written as

\[
\gamma_{PU} = \gamma_D + \gamma_R
\]

where

\[
\gamma_D = \frac{P_{PU}}{\sigma^2} |h_D|^2 \quad \text{and} \quad \gamma_R = \frac{g_1^2}{\gamma_1 + g_2 + 1}.
\]

We further have

\[
\gamma_1 = \frac{P_{PU}}{\sigma^2} \|h_{PU}\|^2 \quad \text{and} \quad \gamma_2 = \frac{P_{SU} \beta}{\sigma^2} \|A_{PU} g_{PU}\|^2.
\]

Similarly, substituting (9) and (11) into (5) and (3), and the resultant expressions into (7) followed by some algebraic manipulation, the SNR at the SR can be written as

\[
\gamma_{SU} = \frac{P_{SU} (1 - \beta)}{\sigma^2} \|A_{SU} g_{SU}\|^2.
\]

### IV. SNR ANALYSIS AND DISTRIBUTION

In this section, we compare the PU’s and SU’s received SNR for the proposed overlay scheme and the non-cooperative underlay scheme. For the proposed scheme, we also derive new expressions for the outage probability and SNR probability density function (p.d.f.) at the PU. These expressions will be useful in deriving BER and rate expressions in later sections.

To facilitate these analysis and derivations, we first present the following lemmas:

**Lemma 1:** Let \( \mathbf{h} \sim \mathcal{CN}_{N,1}(0, \mathbf{I}_N) \) and \( \mathbf{g} \sim \mathcal{CN}_{N,1}(0, \mathbf{I}_N) \). Then \( |\mathbf{A}^† \mathbf{h}|^2 \) is a chi squared random variable (RV) with \( 2(N - 1) \) degrees of freedom (d.o.f.), where \( \mathbf{A}^† = (I - \mathbf{g} \mathbf{g}^†)^{-1} \mathbf{g}^† \).

**Proof:** The proof follows by first noting that \( \mathbf{A}^† = (I - \mathbf{g} \mathbf{g}^†)^{-1} \mathbf{g}^† \) has rank \( (N - 1) \) [15, Theorem 4.21, Theorem 4.22]. As \( \mathbf{A}^† \) is a Hermitian idempotent matrix, the result is obtained by applying [16, Theorem 2 Ch. 1].
A. Received SNR

1) PU Performance: Under the conventional underlay scheme, the PT continuously transmits to the PR directly without any interference from the ST. Thus the received SNR at the PR is given by

\[ \gamma_{D}^\text{conv} = \gamma_{D}. \]  

By comparing (14) and (17), we see that the PU's SNR under the proposed overlay scheme is greater then the PU's SNR under the non-cooperative underlay scheme by an additional \( \gamma_{R} \) term. This term reflects an additional SNR gain achieved by the use of the ST as a relay. Note that this gain achieved in the overlay scheme is obtained at the expense of using one additional time slot for transmission, compared to the non-cooperative underlay scheme. We will examine this tradeoff in Section V-D and Section VI-C.

2) SU Performance: Under the conventional non-cooperative underlay scheme, the ST continuously transmits to the SR directly, while canceling the interference to the PR. In this scheme, the key detriment is the interference received at the SR by the PT. To illustrate how the proposed overlay scheme can perform better than this non-cooperative underlay scheme, we consider a simple example of non-cooperative underlay schemes, depicted in Fig. 2. The received SINR at the SR in this case is given by

\[ \gamma_{SU}^\text{conv} = \frac{\gamma_{SU}^\dagger \mathbf{w}_{SU} \bar{h}_{SU}}{\gamma_{SU}^\dagger \mathbf{h}_{PT,SR} \mathbf{w}_{SU} + 1}, \]  

where \( \gamma_{SU}^\dagger_1 = \frac{\mathbf{P}_{SU}}{\sigma^2}, \gamma_{SU}^\dagger_2 = \frac{\mathbf{P}_{SU}}{\sigma^2}, \mathbf{h}_{PT,SR} \sim \mathcal{CN}(0, \mathbf{d}_{PT,SR}) \) is the Rayleigh channel from the PT to the SR and \( \mathbf{d}_{PT,SR} \) is the PT-SR distance. Note that if there is no cooperation between the PU and the SU, canceling the interference received at the SR by the PT is not possible, regardless of how the weights are designed at the ST. Note that the design of \( \mathbf{w}_{SU} \) can be chosen based on ZF principles, such that the interference to the PR is canceled, while maximizing the SU's signal power at the SR. This can be achieved by choosing \( \mathbf{w}_{SU} \) in (11).

We see from (18) that the received SINR at the SR for the non-cooperative underlay scheme has an interference term \( \gamma_{SU}^\dagger \mathbf{h}_{PT,SR} \mathbf{w}_{SU} \mathbf{w}_{SU} \mathbf{h}_{PT,SR} \mathbf{h}_{PT,SR} \mathbf{w}_{SU} \), due to the PT transmission. In contrast, we see from (16) that the received SNR at the SR for the proposed overlay scheme does not have any interference, and thus will potentially lead to a better performance. Note, however, that these gains come at the expense of using more time slots for transmission. We will examine this tradeoff in Section V-D and Section VI-C.

B. Outage Probability and SNR p.d.f.

1) PU Distribution: The outage probability is an important quality of service measure, defined as the probability that the received SNR \( \gamma_{PU} \) drops below an acceptable threshold \( \gamma_{th} \), and is denoted for the PU by \( F_{\gamma_{PU}}(\gamma_{th}) \). To derive the distribution of \( \gamma_{PU} \) in (14), we first need to obtain the distribution of \( \gamma_{D} \) and \( \gamma_{R} \). We observe that \( \gamma_{D} \) is an exponential RV with cumulative distribution function (c.d.f.) given by

\[ F_{\gamma_{D}}(x) = 1 - e^{-\frac{x}{\gamma_{D}}}, \quad x \geq 0 \]  

and \( \gamma_{1} \) is a chi-squared RV with \( 2N \) d.o.f. with c.d.f. given by

\[ F_{\gamma_{1}}(x) = 1 - e^{-\frac{x}{\gamma_{1}}}, \quad x \geq 0, \]  

where \( \gamma_{1} = \frac{\mathbf{P}_{PU}}{\sigma_{PU}^2} \) and \( \gamma_{1} = \frac{\mathbf{P}_{SU}}{\sigma_{SU}^2} \). By applying Lemma 1 to \( \gamma_{2} \), we see that \( \gamma_{2} \) is a chi-squared RV with \( 2(N-1) \) d.o.f. with c.d.f. given by

\[ F_{\gamma_{2}}(x) = 1 - e^{-\frac{x}{\gamma_{2}}}, \quad x \geq 0, \]  

where \( \gamma_{2} = \frac{\mathbf{P}_{SU}}{\sigma_{SU}^2} \). We thus observe that \( \gamma_{R} \) is the harmonic mean of two chi-squared random variables - the distribution of which has already been derived in [17]. Although the distribution of \( \gamma_{D} \) and \( \gamma_{R} \) are known, deriving exact outage probability and SNR p.d.f. expressions for the PU is difficult, as the distribution of \( \gamma_{D} + \gamma_{R} \) is hard to obtain. To this end, we note that the SNR at the PR in (14) can be accurately upper bounded by [18], [19]

\[ \gamma_{PU} \leq \gamma_{PU}^{ub} = \gamma_{D} + \gamma_{RA} \]  

where \( \gamma_{RA} = \min(\gamma_{1}, \gamma_{2}) \). We now present the c.d.f. and p.d.f. of \( \gamma_{PU}^{ub} \) in (22), given in the following theorem:

\[ \text{Theorem 1: The c.d.f. and p.d.f. of } \gamma_{PU}^{ub} \text{ are given, respectively, by} \]

\[ F_{\gamma_{PU}^{ub}}(x) = \frac{1}{\gamma_{2} \Gamma(N)} \sum_{k=0}^{N-2} \psi_{k}(\theta_{k}, \gamma_{1}) + \frac{1}{\gamma_{1} \Gamma(N-1)} \sum_{k=0}^{N-1} \psi_{k}(\theta_{k}, \gamma_{2}) \]  

\[ f_{\gamma_{PU}^{ub}}(x) = e^{-\frac{x}{\gamma_{D}}} \left( \frac{1}{\gamma_{1} \Gamma(N)} \sum_{k=0}^{N-2} \frac{\mu_{2} - \theta_{k}}{k!} \gamma(\theta_{k}, \mu_{2} x) + \frac{1}{\gamma_{1} \Gamma(N-1)} \sum_{k=0}^{N-1} \frac{\mu_{2} - \theta_{k}}{k!} \gamma(\theta_{k}, \mu_{2} x) \right) \]  

where

\[ \psi_{k}(\lambda, \gamma) = \frac{\mu_{1} - \lambda}{k!} \gamma(\lambda, \mu_{1} x) - e^{-\frac{x}{\gamma_{D}}} \frac{\mu_{2} - \lambda}{k!} \gamma(\lambda, \mu_{2} x), \]

\[ \mu_{1} = \frac{1}{\gamma_{D}} + \frac{1}{\gamma_{2}}, \mu_{2} = \frac{1}{\gamma_{1}} + \frac{1}{\gamma_{2}}, \psi_{k} = k + N, \theta_{k} = k + N - 1 \]

and \( \gamma(\cdot, \cdot) \) is the lower incomplete gamma function [20].

\[ \text{Proof: See Appendix A.} \]

We will show in Section V and Section VI that the expressions in (23) and (24) can be used to derive accurate expressions for the BER and rate respectively at the PU.

2) SU Distribution: By applying Lemma 1 to (16), we see that \( \gamma_{SU} \) is a chi-squared RV with \( 2(N - 1) \) d.o.f. with c.d.f. and p.d.f. given respectively by

\[ F_{\gamma_{SU}}(x) = 1 - e^{-\frac{x}{\gamma_{SU}}}, \quad x \geq 0 .\]  

\[ f_{\gamma_{SU}}(x) = \frac{1}{\gamma_{SU} \Gamma(N)} \sum_{k=0}^{N-2} \frac{(x/\gamma_{SU})^{k}}{k!}, \quad x \geq 0 .\]
and
\[ f_{\gamma_{SU}}(x) = \frac{x^{N-2}e^{-x/\gamma_{SU}}}{\Gamma(N-1)\gamma_{SU}^{N-1}}, \quad x \geq 0, \]  
(26)
where \( \gamma_{SU} = \frac{P_{20}(1-\phi)}{d_{SU} \sigma^2} \). As \( \gamma_{SU} \) is a chi-squared RV with \( 2(N-1) \) d.o.f. we see that \( \gamma_{SU} \) is stochastically strictly increasing\(^4\) with \( N \).

V. ERROR PERFORMANCE

In this section, we analyze the BER performance at the PU and SU for the proposed scheme. To facilitate the analysis for the PU, we derive new BER expressions for arbitrary and high SNR. For completeness, we also present a BER expression at the SU for the proposed scheme. In order to derive expressions for the BER, let us first consider the symbol error rate (SER), defined as

\[ P_s = E \left[ aQ \left( \sqrt{2b\gamma} \right) \right] \]  
(27)
where \( Q(\cdot) \) is the Gaussian-Q function defined as \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} dy \) and \( a \) and \( b \) are parameters related to specific modulation formats. To derive expressions for the BER as a function of the SNR per bit, when using \( M \)-ary signalling, we use the common approximation [21], [22]

\[ P_b = \frac{P_s}{\log_2(M)}, \quad r_{D,b} = \frac{\gamma_p}{\log_2(M)}, \quad \gamma_{1,b} = \frac{\gamma_1}{\log_2(M)}, \]
\[ \gamma_{2,b} = \frac{\gamma_2}{\log_2(M)} \quad \text{and} \quad \gamma_{SU,b} = \frac{\gamma_{SU}}{\log_2(M)} \]  
(28)
where \( \gamma_{D,b}, \gamma_{1,b}, \gamma_{2,b} \) and \( \gamma_{SU,b} \) denotes the average received SNR per bit.

A. BER at Arbitrary SNR

1) PU's BER: We now present a new expression for the BER at the PR, given by the following corollary.

**Corollary 1:** The BER at the PR for the proposed overlay scheme can be approximated by

\[ P_b = \frac{a\sqrt{b}}{2\pi \log_2(N) M} \left( \frac{1}{\log_2(M)\Gamma(N)x_{\gamma_{1,b}}^{N-1}} \sum_{k=0}^{N-2} \varphi_k(\eta_k, \gamma_{2,b}) \right) + \frac{1}{\Gamma(N-1)\gamma_{2,b}^{N-1}} \sum_{k=0}^{N-1} \varphi_k(\theta_k, \gamma_{1,b}), \]  
(29)
where

\[ \varphi_k(\zeta, \eta) = \frac{1}{k!\log_2(M)\Gamma(N)} \frac{1}{\Gamma(\zeta+1)\Gamma(\eta+1)} \times 2F_1 \left( 1, \zeta + 1; \eta + 1; \frac{\mu_1}{\mu_1 + b} \right) \]
\[ \times 2F_1 \left( 1, \zeta + 1; \eta + 1; \frac{\mu_2}{\mu_2 + b} \right), \]
\[ \mu_1 = \frac{1}{\log_2(M)\gamma_{1,b}}, \quad \mu_2 = \frac{1}{\log_2(M)\gamma_{1,b}} + \frac{1}{\log_2(M)\gamma_{SU,b}} - 2F_1(\cdot, \cdot; \cdot; \cdot), \]  
\[ \text{and} \quad 2F_1(\cdot, \cdot; \cdot; \cdot) \]  
is the Gauss’ hypergeometric function defined in [20].

\[^4\] Let \( X \) and \( Y \) be random variables, \( X \) is said to be stochastically strictly larger than \( Y \) if \( P(X > t) < P(Y > t) \) for all \( t \in \mathbb{R} \).

**Proof:** We proceed by rewriting the SER expression, given in (27), directly in terms of the c.d.f. of the output SNR as follows

\[ P_s = \frac{a\sqrt{b}}{2\pi \sqrt{a}} \int_0^{\infty} e^{-bu} F_1(u) du. \]  
(30)
The result follows by substituting (23) into (30), solving the resultant integral using [20, Eq. 6.455.2] and applying the approximations in (28) into the resultant expression.

2) SU's BER: The BER at the SR for the proposed overlay scheme can be approximated by

\[ P_b^{SU} = \frac{a}{2\log_2(M)} \times \left( 1 - \sqrt{\pi} \sum_{k=0}^{N-2} \frac{(\log_2(M)\gamma_{SU,b})^{-k}}{k! \left( (\log_2(M)\gamma_{SU,b} + b)^{k+\frac{1}{2}} \right)} \right). \]  
(31)

**Proof:** We proceed by substituting (25) into (30), solving the resultant integral using [20] and applying the approximations in (28) into the resultant expression.

B. BER at High SNR

We now consider the BER at high SNR for the PU and SU. In the high SNR regime, the key factors governing the system performance are the diversity order \( G_d \) and array gain \( G_a \).

1) PU's BER: We first present a new expression for the BER at high SNR in terms of these parameters for the proposed overlay scheme, given by the following corollary.

**Corollary 2:** At high SNR, the BER at the PR for the proposed overlay scheme can be approximated by

\[ P_b^{\infty} = (G_a)^{-G_d} (\gamma_{2,b})^{-G_d} \]  
(32)
where the array gain is given by

\[ G_a = \left( \frac{\alpha \Gamma(N+\frac{1}{2})}{2\pi b \Gamma(N) \Gamma(\log_2(M))^{-1}} \right)^{-\frac{1}{N}} \]  
and diversity order by

\[ G_d = N \]
with \( \kappa = \frac{\gamma_{2,b}}{\gamma_{SU,b}} \).

**Proof:** See Appendix B.

By noting that the diversity order of the conventional underlay scheme is one, Corollary 2 reveals that our proposed overlay scheme, compared to the non-cooperative underlay scheme, achieves an \( N \) times increase in the diversity order.

2) SU's BER: As the received SNR at the SR is a chi-squared RV with \( 2(N-1) \) d.o.f. the diversity order achieved by our proposed overlay scheme is thus given by

\[ G_{d,a} = N - 1. \]  
(33)

By observing the received SINR in (18), we see that the diversity order of the conventional underlay scheme for the SU is zero. Therefore, (33) reveals that the diversity order of the SU in the proposed overlay scheme is greater than...
the diversity order of the SU using the conventional non-cooperative underlay scheme by an additional $N - 1$ term. This indicates that the proposed scheme can achieve significant gains in the error performance at high SNR, compared to the conventional underlay scheme.

C. Optimum Power Allocation at the ST

In this subsection, we investigate the design of the power allocation number $\beta$ at the ST to satisfy BER requirements for both the PU and the SU. In particular, $\beta$ is chosen according to (34) at top of the next page where $a_1, b_1, a_2, b_2, a_3, b_3,$ and $b_4$ are modulation parameters. Note that we have made explicit the dependence of the SNR at the PU and at the SU on $\beta$. We now present numerical results for the BER at the PU and SU for the proposed overlay scheme with $\beta$ chosen according to (34) in Section IV-A.

D. Simulation Results

We now present numerical results for the BER at the PU and SU for the proposed overlay scheme and the non-cooperative underlay scheme for various numbers of antennas at the ST. Throughout this paper, we consider a system topology where the PT, PR, ST, and SR are collinear. In particular, the ST is located between the PT and the PR and the PR is located between the ST and the SR. Therefore, we have $d_{PT,PR} = d_{ST,PR} + d_{PR,ST}$ and $d_{ST,SR} = d_{ST,PR} + d_{PR,SR}$. Furthermore, for convenience we denote $\gamma_A = \frac{P_{PT}}{\sigma^2}$ and $\gamma_B = \frac{P_{PT}}{\sigma^2}$. For a fair comparison between the two schemes, we assume that for the proposed scheme the PT and ST transmit with power $P_{PT}$ and $P_{SU}$ respectively, while for the underlay scheme the PT and ST transmit with power $P_{PT}/2$ and $P_{SU}/2$ respectively. In addition, in order to maintain the same spectral efficiency between the two schemes, we use the 16-QAM modulation scheme ($a = 3$ and $b = 0.1$) for the proposed overlay scheme, and the QPSK modulation scheme ($a = 2$ and $b = 0.5$) for the non-cooperative underlay scheme.

1) Using Fixed $\beta$: Figs. 3 and 4 show the BER vs. $\gamma_A$ at the PU and SU respectively for the proposed overlay scheme with $\beta = 0.5$ and the non-cooperative underlay scheme, for $N = 2$ and $N = 4$. In Fig. 3, the ‘PU proposed overlay (Analytical)’ curves are from (29), and closely match the exact BER curves obtained using Monte Carlo simulations. Moreover, the ‘PU overlay high SNR (Analytical)’ curves are from (32), and it is clear that these curves converge to the exact BER in the high SNR regime. In Fig. 4, the ‘SU proposed overlay (Analytical)’ curves are from (31) and closely match the exact BER curves obtained using Monte Carlo simulations. We see from Figs. 3 and 4 that the PU’s and SU’s BER performance for the proposed overlay scheme outperforms the PU’s and SU’s BER performance for the non-cooperative underlay scheme respectively, for all SNR values and antenna configurations.

This indicates that the diversity order at both the PR and SR for the proposed overlay scheme increases with the number of antennas, as shown in (32) and (33) respectively. This is in contrast to the non-cooperative underlay scheme where the diversity order at both the PR and SR is a constant.
non-cooperative underlay scheme. For all curves, \( N = 3 \) and are generated by Monte Carlo simulations. We see that the proposed overlay scheme achieves a better average BER performance than the conventional scheme, for both the PU and the SU over all values of \( d_{PT,ST} \). For the proposed overlay scheme, we observe that the PU’s BER performance improves for increasing \( d_{PT,ST} \) when \( 0.1 \leq d_{PT,ST} \leq 1.6 \), but gets worse when \( 1.7 \leq d_{PT,ST} \leq 2.9 \). This can be explained by first noting that the PU’s performance for the proposed scheme is dominated by the worst link, and that the received SNR performance for each link for the same transmitted power is affected by two factors, the diversity gain and the distance. For \( 0.1 \leq d_{PT,ST} \leq 1.6 \), the ST-PR link is the worst link because, compared to the PT-ST link, it has (i) a lower diversity gain and (ii) a longer distance i.e., \( d_{PT,PR} < d_{ST,PR} \). This implies that the ST-PR link is the bottleneck, and increasing \( d_{PT,ST} \), and thus decreasing \( d_{PT,PR} \), improves the BER at the PU. In contrast, for \( 1.7 \leq d_{PT,ST} \leq 2.9 \), although the PT-ST link has a larger diversity gain than the ST-PR link, it also has a longer distance, i.e., \( d_{PT,ST} > d_{PT,PR} \). Fig. 5 reveals that for these distance configurations, the detrimental effect of a longer distance dominates the beneficial effect of a higher diversity gain. The PT-ST link is thus the bottleneck, and increasing \( d_{PT,ST} \) degrades the BER at the PU. We also note that the optimal ST placement which achieves the best BER performance at the PU for the proposed scheme, denoted by \( d_{PT,ST}^{opt,BER} \), is at \( d_{PT,ST}^{opt,BER} = 1.6 \). Note, however, that this optimal distance may not result in the optimal BER performance at the SU.

2) Using \( \beta_{opt,BER} \): Fig. 6 shows the BER vs. \( \gamma_A \) at the PU and the SU for the proposed overlay scheme using \( \beta_{opt,BER} \) and the non-cooperative underlay scheme. All curves are for \( N = 2 \), and are generated using Monte Carlo simulations. We see from Fig. 6 that \( \beta_{opt,BER} \) can be found for all SNR values, implying that (i) the proposed overlay scheme will always achieve a better average BER performance than the conventional underlay scheme, for both the PU and SU, and (ii) the sum-BER using \( \beta_{opt,BER} \) is minimized, and hence is always less than the sum-BER obtained using a fixed \( \beta \). However, we note that that performance gains of the sum-BER for the proposed scheme with \( \beta_{opt,BER} \) is not that significant compared to the performance gains with fixed \( \beta = 0.5 \), as have been verified through simulations, but omitted due to space limitations. As \( N = 2 \) represents the antenna configuration which achieves the worst performance for both the PU and the SU, we note that \( \beta_{opt,BER} \) can be found when \( N > 2 \) for a wider range of SNR values.
VI. RATE PERFORMANCE

In this section, we analyze the rate at the PU and the SU for the proposed overlay scheme. We first derive a new rate expression at the PR. For completeness, we also present a rate expression at the SR for the proposed scheme. For a fair comparison, the rate achieved at the PU and SU, for the proposed overlay scheme, is given respectively by

\[ R_{PU} = \frac{1}{2} E \left[ \log_2 \left( 1 + \gamma_{PU} \right) \right] \]  

(35)  

and

\[ R_{SU} = \frac{1}{2} E \left[ \log_2 \left( 1 + \gamma_{SU} \right) \right], \]  

(36)

while the rate achieved at the PU and the SU, for the non-cooperative underlay scheme, is given respectively by

\[ R_{PU, D} = E \left[ \log_2 \left( 1 + \gamma_{PU}^{\text{conv}} \right) \right] \]  

(37)  

and

\[ R_{SU, D} = E \left[ \log_2 \left( 1 + \gamma_{SU}^{\text{conv}} \right) \right], \]  

(38)

Note that \( R_{SU} \) and \( R_{PU} \) are multiplied by factor of \( \frac{1}{2} \) as both these schemes use two time slots for transmission. To analyze the effect of different system parameters on the rate performance at the PU for the proposed scheme, we introduce the following proposition:

**Proposition 1:** The instantaneous rate achieved at the PR for the proposed scheme will outperform the instantaneous rate achieved at the PR for the underlay scheme when

\[ \gamma_R = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_{SD}} > \gamma_{D}^2 + \gamma_{D}. \]  

(39)

**Proof:** We note that the rate at the PU for the proposed scheme will outperform the instantaneous rate achieved at the PR for the underlay scheme only when

\[ \frac{1}{2} \log_2 \left( 1 + \gamma_{PU} \right) > \log_2 \left( 1 + \gamma_{PU}^{\text{conv}} \right). \]  

(40)

The desired result is obtained after some mathematical manipulation.

**Proposition 1** provides useful insights about the effect of different system parameters on the rate performance at the PR for our proposed scheme. In particular, **Proposition 1** shows that the PU’s rate performance for the proposed scheme can be improved by i) increasing the number of antennas \( N \) at the ST, ii) increasing the transmitted power at the ST \( P_{SU} \), and iii) increasing the power allocation number \( \beta \) at the ST. This can be justified by noting that \( \gamma_2 \) and \( \gamma_1 \) are stochastically strictly increasing with \( N \) and that \( \gamma_2 \) is stochastically strictly increasing with \( P_{SU} \) and \( \beta \) respectively. This would give the designer several options to further improve the rate performance at the PU for the proposed scheme. Note that even though these insights are obtained using the instantaneous rate expressions, they also hold for the average rate and BER performance as shown in Sections VI-C and V-D respectively.

A. Rate Evaluation

1) PU’s Rate: We now present a new closed form expression for the rate achieved at the PR for the proposed overlay scheme, given by the following corollary.

**Corollary 3:** The rate at the PR for the proposed overlay scheme can be upper bounded by

\[ R_{PU} = \frac{1}{2 \ln(2) \gamma_D} \left( \frac{1}{\Gamma(N) \gamma_1} \sum_{k=0}^{N-2} I_k(\gamma_{DK}, \gamma_2) \right. \]  

\[ + \left. \frac{1}{\Gamma(N-1) \gamma_2} \sum_{k=0}^{N-1} I_k(\gamma_{DK}, \gamma_1) \right) \]  

(41)

where

\[ I_k(\omega, \gamma) = \frac{\mu^2 \mu^{\gamma} \Gamma(\omega)}{k! \gamma^k} \left( \gamma^{-\gamma} e^{-\frac{\gamma}{\mu}} \right) \left( \frac{1}{\Gamma(\omega)} \right) \]  

\[ = e^{\mu \cdot \omega} \sum_{m=0}^{\omega-1} \sum_{p=1}^{m+1} \frac{\mu^2 \gamma^m (p - (m + 1), \gamma) \mu^p}{\mu^p \mu^p}, \]  

\( \Gamma(\cdot, \cdot) \) is the upper incomplete gamma function [20] and \( E_1(\cdot) \) is the exponential integral [23, Eq. (5.1.1)].

**Proof:** See Appendix C.

2) SU’s Rate: The rate at the SR for the proposed overlay scheme is given by

\[ R_{SU} = \frac{e^{\frac{1}{2} \gamma_{SU} \cdot k! \gamma^k}}{2 \ln(2) \gamma_D} \sum_{k=0}^{N-1} \frac{\Gamma(k - N + 1, \frac{1}{\gamma_{SU} \cdot k! \gamma^k})}{\gamma_{SU} \cdot k! \gamma^k}. \]  

(42)

**Proof:** The proof follows by substituting (26) into (36) and solving the resultant integral using [24, Eq. 78].

B. Optimum Power Allocation at the ST

In this subsection, we investigate the design of the power allocation number \( \beta \) at the ST to meet a certain rate design requirement. In particular, \( \beta \) is chosen according to (43) at the top of the next page. Note that the use of \( \beta_{opt, rate} \) based on (43), will simultaneously (i) achieve a high rate at both the PU and the SU, and (ii) guarantee that the PU’s and the SU’s rate for the proposed scheme will equal to or outperform the PU’s and the SU’s rate for the conventional underlay scheme, as shown in the next subsection. Since it is difficult to obtain a closed form expression for \( \beta_{opt, rate} \), we numerically evaluate \( \beta_{opt, rate} \) by Monte Carlo simulations in the next subsection. We will also show through simulations that \( \beta_{opt, rate} \) can always be found for all practical SNR values and antenna configurations. Note that in the case where \( \beta_{opt, rate} \) based on (43) can not be found, then both the PU and SU can operate under the underlay model, as described in Section IV-A.

C. Simulation Results

We now present the numerical results for the rate achieved at the PU and SU for the proposed overlay scheme and the non-cooperative underlay scheme.
\[ \beta_{\text{opt.rate}} = \arg \max_{\beta} \frac{1}{2} \mathbb{E} \left[ \log_2 (1 + \gamma_{\text{PU}}(\beta)) \right] + \frac{1}{2} \mathbb{E} \left[ \log_2 (1 + \gamma_{\text{SU}}(\beta)) \right] \]
subject to
\[ \frac{1}{2} \mathbb{E} \left[ \log_2 (1 + \gamma_{\text{PU}}(\beta)) \right] \geq \mathbb{E} \left[ \log_2 (1 + \gamma_{\text{conv}}) \right] \]
\[ \frac{1}{2} \mathbb{E} \left[ \log_2 (1 + \gamma_{\text{SU}}(\beta)) \right] \geq \mathbb{E} \left[ \log_2 (1 + \gamma_{\text{conv}}) \right] . \]
(43)

1) Using Fixed \( \beta \): Figs. 7 and 8 show the rate vs. \( \gamma_{\text{A}} \) at the PU and SU respectively for the proposed overlay scheme with \( \beta = 0.5 \) and the non-cooperative underlay scheme, for \( N = 2 \) and \( N = 4 \). In Fig 7, the ‘PU proposed overlay (Analytical)’ curves are from (41), and closely match the exact rate curves obtained using Monte Carlo simulations. In Fig 8, the ‘SU proposed overlay (Analytical)’ curves are from (42), and clearly match the exact rate curves obtained using Monte Carlo simulations. We observe from Figs. 7 and 8 that both the PU’s and SU’s rate for the proposed overlay scheme outperforms the PU’s and SU’s rate achieved by the non-cooperative underlay scheme, for a wide range of SNR values and antenna configurations. We also note that the gain of the proposed scheme relative to the non-cooperative underlay scheme increases with the number of antennas. This can be explained by the fact that the rate at the PR for the proposed overlay scheme increases with the number of antennas as shown in Proposition 1, and that the received SNR at the SR increases with the number of antennas as shown in Section IV-A2. Fig. 9 shows the rate vs. \( d_{\text{PR.ST}} \) at the PU and the SU for the proposed overlay scheme with \( \beta = 0.5 \) and the non-cooperative underlay scheme. All curves are generated by Monte Carlo simulations with \( N = 3 \). The conclusions and explanations for Fig. 9 are similar to the conclusions and explanations for Fig. 5.

Finally, we can see from Figs. 3, 4, 5, 7, 8 and 9 that the simple choice of fixed \( \beta = 0.5 \) for the proposed overlay scheme can achieve a significantly better rate and BER performance than the non-cooperative underlay scheme, at both the PU and SU, for all SNR values and antenna configurations. This is another advantage of our scheme as this can further reduce the complexity of our scheme, compared to the use of optimum \( \beta \) in our scheme where the ST has to perform complex processing to determine \( \beta_{\text{opt.rate}} \) and \( \beta_{\text{opt.BER}} \) to optimize the rate and BER performance respectively. Thus, the designer has the option to trade performance for simplicity.

2) Using \( \beta_{\text{opt.rate}} \): Fig. 10 shows the rate vs. \( \gamma_{\text{A}} \) at the PU and the SU for the proposed overlay scheme using \( \beta_{\text{opt.rate}} \) based on (43), and the non-cooperative underlay scheme. All curves are for \( N = 2 \), and are generated using Monte Carlo simulations. We see that the use of \( \beta_{\text{opt.rate}} \) can simultaneously (i) achieve a high rate at both the PU and the SU, and (ii) guarantee that the PU’s and SU’s rate for the proposed scheme will equal to or outperform the PU’s and SU’s rate for the conventional underlay scheme for all SNR values.

For the system configurations in Figs. 6 and 10, we note that in the low SNR regime, the values of \( \beta_{\text{opt.rate}} \) are small, as more power is allocated to the SU to improve its performance. This can be seen in Fig. 8 where we observe that in the low SNR regime, the SU performance for the underlay scheme can outperform the SU performance for the overlay scheme.
outperform the PU’s and SU’s rate and BER performance for
respectively, for the proposed scheme and the scheme in [5].

The gains of our proposed scheme come from the design of antennas weights and the power allocation at the ST. It has also been verified through simulations that the SU’s BER performance for the scheme in [5] can outperform SU’s BER performance for the proposed scheme for some SNR regimes in certain scenarios. This corresponds to the scenarios where (i) the ST is close to the PR and thus the ST can always successfully decode the PT message for the scheme in [5] and, (ii) the SR is far from the PT and thus the SR does not suffer from a significant interference from the PT for the scheme in [5]. The simulation results for these specific scenarios have been omitted due to the space limit.

with fixed $\beta = 0.5$. In addition, we note that for the SNR range $8 - 24$ dB, the values of $\beta_{\text{opt,rate}}$ and $\beta_{\text{opt,BER}}$ based on (43) and (34) respectively are similar, and lie within the range of $0.3 < \beta < 0.5$. We also note that for different system configurations, the values of $\beta_{\text{opt,rate}}$ and $\beta_{\text{opt,BER}}$ can be different at each SNR, as have been verified through simulations but omitted due to the space limit. As $N = 2$ represents the antenna configuration which achieves the worst performance for both the PU and the SU, we note that $\beta_{\text{opt,rate}}$ can be found when $N > 2$ for a wider range of SNR values.

Figs. 11 and 12 compare the rate and BER performance, respectively, for the proposed scheme and the scheme in [5] for $N = 2$. We observe from these figures that the PU’s and SU’s rate and BER performance for the proposed scheme outperform the PU’s and SU’s rate and BER performance for the scheme in [5] for all SNR values. We also note that the same conclusion can be observed for a wide range of SNR values and many different system configurations, as verified through simulations. The gains of our proposed scheme come from the design of antennas weights and the power allocation at the ST.
VII. CONCLUSIONS

In this paper, we have proposed and investigated a solution for spectrum sharing based on the idea that SU can earn spectrum access in exchange for cooperation with the PU. We have proposed the design of a spectrum sharing scheme which results in a higher rate and error performance for both the PU and SU networks, compared to a conventional underlay system where no cooperation occurs. We have provided analytical and numerical results which have confirmed the considered model as a promising paradigm for CR networks.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions which improved the quality of the paper.

APPENDIX

A. Proof of Theorem 1

Using order statistics [25, Theorem 4.1], the c.d.f of $\gamma_{RA}$ in (22) can be shown to be given by

$$F_{\gamma_{RA}}(x) = F_{\gamma_1}(x) + F_{\gamma_2}(x) - F_{\gamma_1}(x)F_{\gamma_2}(x), \quad (44)$$

where $F_{\gamma_1}(x)$ and $F_{\gamma_2}(x)$ are given in (20) and (21), respectively. By substituting (20) and (21) in (44), and after some mathematical manipulation, we obtain

$$F_{\gamma_{RA}}(x) = \frac{1}{\Gamma(N)} \sum_{k=0}^{N-2} \frac{\mu_1}{k! \gamma_2^{\nu_k}} (\eta_k, \mu_1x)$$

$$+ \frac{1}{\Gamma(N-1)} \sum_{k=0}^{N-1} \frac{\mu_1}{k! \gamma_1^{\nu_k}} (\theta_k, \mu_1x), \quad (45)$$

where $\mu_1 = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$, $\eta_k = k + N$ and $\theta_k = k + N - 1$. By taking the derivative of $F_{\gamma_{RA}}(x)$ in (45) w.r.t to $x$, the p.d.f of $\gamma_{RA}$ is given by

$$f_{\gamma_{RA}}(x) = \frac{1}{\gamma_1} x^{N-1} e^{-\frac{x}{\gamma_1}} \Gamma(N-1, \frac{x}{\gamma_2})$$

$$+ \frac{1}{\gamma_2} x^{N-2} e^{-\frac{x}{\gamma_2}} \Gamma(N, \frac{x}{\gamma_1}). \quad (46)$$

To proceed we note that the Laplace transform of $\gamma_{PU}^{ub}$ in (22) can be shown to be given by [26]

$$\mathcal{L}\{f_{\gamma_{PU}^{ub}}(x)\} = \frac{\mathcal{L}\{f_{\gamma_{RA}}(x)\}}{(s + \frac{1}{\gamma_D})^{\gamma_D}} \quad (47)$$

By utilizing [26, Eq. 1.1.13] to calculate the inverse Laplace transform of (47), and solving the resultant integral by using [20], we obtain the result for the SNR p.d.f. $f_{\gamma_{PU}^{ub}}(x)$, in (24).

The c.d.f of $\gamma_{PU}^{ub}$ in (23), $F_{\gamma_{PU}^{ub}}(x)$, can be obtained by using the differential property of the Laplace transform, utilizing [26, Eq. 1.1.14] to calculate the inverse Laplace transform and solving the resultant integral by using [20].

B. Proof of Corollary 2

In order to derive expressions for the BER at high SNR, let us first consider the SER at high SNR, defined as

$$P_s^\infty \approx (G_a\gamma_2)^{-G_a}. \quad (48)$$

Note the received SNR at the PU has the form of $\gamma_{PU} = \gamma_D + \gamma_R$. Thus, by using [27, Proposition 4], the aggregate diversity gain $G_d$ and array gain $G_a$ at the PU can be shown to be respectively given by

$$G_d = G_{dD} + G_{dR} \quad (49)$$

and

$$G_a = 2b \left( \frac{a2\sqrt{\pi\Gamma(G + \frac{1}{2})G_{dD}\Gamma(G_{dD})G_{dR}\Gamma(G_{dR})}}{\Gamma(G + 1)G_{dD}^{G_{dD}}\Gamma(G_{dD} + \frac{1}{2})G_{dR}^{G_{dR}}\Gamma(G_{dR} + \frac{1}{2})} \right)^{-\frac{1}{2}} \quad (50)$$

where $G = G_{dD} + G_{dR}, G_{dD}$, and $G_{dR}$ are the diversity order of the $\gamma_D$ and $\gamma_R$, respectively, and $G_{dD}$ and $G_{dR}$ are the array gains of $\gamma_D$ and $\gamma_R$ respectively. Since $\gamma_D$ in (14) is distributed as an exponential RV, the diversity order and the array gain of $\gamma_D$ are respectively given by [27]

$$G_{dD} = 1 \quad \text{and} \quad G_{aD} = \frac{1}{\kappa} \left( \frac{\Gamma(\frac{1}{2})}{\sqrt{\pi\Gamma(N)}} \right)^{-1}. \quad (51)$$

Since $\gamma_R$ is the harmonic mean of two chi-squared random variables, by using the results in [17], it can be shown that the diversity gain and the array gain of $\gamma_R$ are respectively given by

$$G_{dR} = N - 1 \quad \text{and} \quad G_{aR} = \left( \frac{2^{N-2}\Gamma(N-\frac{1}{2})}{\sqrt{\pi\Gamma(N)}} \right)^{-\frac{1}{N-1}}. \quad (52)$$

The final result is obtained by substituting (51) and (52) in (50), substituting (51) and (52) in (49), and applying the approximations in (28), followed by some mathematical manipulation.

C. Proof of Corollary 3

The rate at the PR for the proposed overlay scheme can be upper bounded by

$$R_{pr} = \frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \gamma_{PU}^{ub} \right) \right]$$

$$= \frac{1}{2\ln(2)} \int_0^\infty \ln(1 + x) f_{\gamma_{PU}^{ub}}(x) \, dx \quad (53)$$

where $f_{\gamma_{PU}^{ub}}(x)$ is given in (24). By substituting (24) in (53), we obtain

$$R_{pr} = I_1 + I_2 \quad (54)$$

where

$$I_1 = \frac{1}{2\ln(2)} \gamma_D^{\gamma_D} \Gamma(N)$$

$$\times \int_0^\infty \ln(1 + x) e^{-\frac{x}{\gamma_D}} \sum_{k=0}^{N-2} \frac{\mu_2^{\eta_k}}{k! \gamma_D^{\nu_k}} \gamma(\eta_k, \mu_2x) \, dx \quad (55)$$
and

$$I_2 = \frac{1}{2\ln(2)} 2^{-N-1} \Gamma(N-1) \left( \sum_{k=0}^{N-1} \frac{\mu^2 \gamma_{\mu \theta_k}}{k!^2} \right) \gamma(\theta_k, \mu x) dx$$

(56)

By utilizing the identity [23, Eq. 8.352.6] to write $\gamma(\eta_k, \mu x)$ in the series form, the integral $I_1$ can be written as

$$I_1 = \frac{1}{2\ln(2)} 2^{-N-1} \Gamma(N) \ln(2) \left( \sum_{k=0}^{N-2} \frac{\mu^2 \eta_k}{k!^2} \Gamma(\eta_k) \right) \left( \int_0^\infty \ln(1+x) e^{-\frac{\gamma_D}{2} x} dx \right)$$

(57)

By using the identity in [23] and the integral identity in [24, Eq. 78] to calculate the first and the second integral of (57), respectively, we obtain

$$I_1 = \frac{1}{2\ln(2)} 2^{-N} \Gamma(N) \ln(2) \left( \sum_{k=0}^{N-2} \frac{\mu^2 \eta_k}{k!^2} \Gamma(\eta_k) \right) \left( \int_0^\infty \ln(1+x) e^{-\frac{\gamma_D}{2} x} dx \right)$$

(58)

By using a similar procedure, $I_2$ can be shown to be given by

$$I_2 = \frac{1}{2\ln(2)} 2^{-N-1} \Gamma(N-1) \ln(2) \left( \sum_{k=0}^{N-1} \frac{\mu^2 \theta_k}{k!^2} \Gamma(\theta_k) \right) \left( \int_0^\infty \ln(1+x) e^{-\frac{\gamma_D}{2} x} dx \right)$$

(59)

The final result is obtained by substituting (58) and (59) in (54), followed by some mathematical manipulation.

REFERENCES


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